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Economics Letters 81 (2003) 285–290

**economics
letters**

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Free entry does not imply zero profits

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Received 16 September 2002; accepted 19 May 2003

Abstract

Traditional economic wisdom says that free entry in a market will drive profits down to zero. This paper shows that profits may remain bounded away from zero when firms have to make a negligible small investment to learn the demand.

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Keywords: Entry deterrence; Information acquisition

JEL classification: D43; D82

1. Introduction

Potential competition or the threat of entry is often considered to be as powerful as actual competition. Can firms make positive profits in the presence of potential entrants? When incumbent firms have no commitment power to deter entry, one would think that profits will indeed be driven down to zero. Let us illustrate this in a very simple entry game in a homogeneous good market where firms incur a fixed set-up cost K . Firms first decide whether or not to enter and incur this fixed cost. After observing the number of firms that entered, all firms play a Cournot game. Assuming symmetry amongst firms, each firm's Cournot payoff (apart from the set-up cost) will be a decreasing function of the number of firms that enters, $\pi(N)$. Ignoring the integer problem, firms will enter up to the point where profits equal set-up cost, i.e. where $\pi(N^*) = K$ and end up having zero profit. If we take the integer problem into account, the number of entrants could be fractionally less, and profits will be positive, but small (“almost zero”).

The present paper shows, however, that pure profits in a free entry equilibrium are possible, even when the market is for a homogeneous good and no firm has any advantage over any other firm. We assume that

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all firms initially face uncertainty about the demand curve, although they all have the opportunity (before entry decisions are taken) to resolve their uncertainty at a very small cost. The reader might have expected that this perturbation of the model would not overturn the zero profit result described in the previous paragraph as, intuitively, it seems probable that firms will resolve their uncertainty before entry decisions are taken. However, as will be shown below, the fact that information acquisitions are not observed implies that an entrant may optimally and rationally decide to stay out, even knowing that the market is large enough for two competing firms and that only one firm enters. More generally, the number of firms that enter may be strictly less than the number of firms needed to drive profits down to (almost) zero.

2. Entry with sunk costs and stochastic demand

There are $n \geq 2$ firms. They have to decide whether to enter in a new market. Inverse demand in this market is given by $p = x + y - q$, where q denotes the total production and where both x and y are stochastic. We assume that the two random variables are independent, and that each can take a high or low value: x is equal to x^H or x^L , with probability α and $1 - \alpha$, respectively. Similarly, y is equal to y^H or y^L , with probability β and $1 - \beta$, respectively. $x^H > x^L$, $y^H > y^L$. Firms can become informed about the realization of (some of) these variables, at a small cost of $\varepsilon > 0$ per variable. After observing the privately gathered information, each firm decides whether or not to enter this market, which implies that a cost K has to be sunk. After observing who entered, the firms that entered compete in the usual Cournot fashion. We assume that production is at zero cost.

We assume that the variables satisfy the following four assumptions:

- (A1) $(x^H + y^H)^2/16 < K$;
- (A2) $(x^H + y^L)^2/9 > K$;
- (A3) $(x^L + y^H)^2/4 > K > (x^L + y^L)^2/9$;
- (A4) $(x^L + y^L)^2/4 < K$.

These assumptions imply that the demand function is such that when x is high, two competing firms will be able to recover their entry cost. The market is never large enough to support three competing firms so we may assume without loss of generality that there are only two potential entrants, A and B¹. When x is low and y is high, two competing firms will not be able to recover the entry cost, but a single firm exercising monopoly power will recover the entry cost. Finally, in the state where both x and y are low, not even a monopolist would recover its cost.

3. The examples

Example 1. Let $\alpha = \beta = 1/2$ and $x^H = 17$, $x^L = 7$, $y^H = 11$, $y^L = 5$, $K = 50$ (note that (A1)–(A4) hold).

¹With $n \geq 3$ our results are easily extended by having firms 1 and 2 behave as A and B in our examples, and each firm $j > 2$ never acquire information and always stay out.

Proposition 1. *The information acquisition and entry game given by the parameters in Example 1 exhibits a sequential equilibrium in which only one firm enters in the high–low state. In particular, the following strategies and beliefs form such a sequential equilibrium².*

Firm A learns x , and enters only when it is high. If he does not meet a competitor, A believes that the high–low state occurred and produces the monopoly quantity $(x^H + y^L)/2 = 11$. If he does meet a competitor, A believes that the high–high state prevailed and produces the Cournot quantity $(x^H + y^H)/3 = 28/3$.

Firm B learns y and enters only when it is high. If he does not meet a competitor, B believes that the low–high state occurred and produces $(x^L + y^H)/2 = 9$. If he does meet a competitor, B believes that the high–high state prevailed and produces $(x^H + y^H)/3 = 28/3$.

Proof. Note that all information sets are reached with positive probability. Firms update their beliefs after observing whether the other firm has entered. If the other firm enters then it must mean that he has ‘good’ information; if he does not enter, it means that he has ‘bad’ information. Beliefs are, therefore, consistent. It suffices now to check that the strategies are sequentially rational, given the beliefs.

It is clear that firms make positive profits whenever they have decided to enter, and that their production levels maximize their profits given the decision of the other firm. The only deviations that might be profitable are, (1) for firm A to get complete information and enter additionally in the case of low x and high y ; (2) for firm B to get completely informed and enter additionally in the case of high x and low y . It is obvious that deviation 1 is not profitable. After observing entry in this state, firm B will believe that both variables are high and will, therefore, produce $(x^H + y^H)/3 = 28/3$. Firm A’s optimal quantity is, therefore, $(x^L + y^H - 28/3)/2 = 13/3$ for a profit of approximately³ 18.78, which is certainly not enough to cover the entry cost of 50.

Deviation 2 is not profitable either. By the same reasoning as before, entry by firm B will imply that firm A believes demand is very high and produces $(x^H + y^H)/3 = 28/3$. The optimal production level for firm B is, therefore, $(x^H + y^L - 28/3)/2 = 19/3$, yielding a profit of 40.11, which is again not enough to cover the entry cost. $\square$

This example thus shows that there exists a sequential equilibrium of the information acquisition and entry game in which in the high–low state only one firm will enter, when in fact two firms competing à la Cournot would make profits. Even zero information cost does not destroy this equilibrium. The result is driven by the fact that when firm B deviates and enters in the high–low state, firm A is deceived and overestimates demand, which in turn leads to overproduction (relative to the Cournot equilibrium) which makes entry not profitable for the deviating firm, at least for some range of the parameters.

Notice that in the equilibrium described in Proposition 1, all information sets are reached with positive probability. Beliefs are determined by Bayes’ rule and are not exogenously and carefully designed to support the equilibrium strategies. Hence, this equilibrium will satisfy any refinements of sequential equilibrium based on restrictions on out-of-equilibrium beliefs (such as the Intuitive Criterion of [Cho and Kreps, 1987](#)).

² See [Kreps and Wilson \(1982\)](#) for a formal definition.

³ We will round off all payoffs to two decimals.

Consider a small variation of our model of information acquisition. Suppose that firms can also gather information after entry has taken place but before production is determined. This variation does not destroy the validity of Proposition 1. Namely, firms will not find it optimal to spend money to do late research, since Bayesian updating along the equilibrium path already yields full information.

One might believe that the fact that firms become (at first) only partially informed is crucial. The next example shows that this is not true.

Example 2. Let $\alpha = 1/10$, $\beta = 1/2$, and $x^H = 19$, $x^L = 7$, $y^H = 13$, $y^L = 7$, $K = 70$ (note that assumptions (A1)–(A4) are satisfied).

Proposition 2. *In the information acquisition and entry game given by the parameters in Example 2, the following strategies and beliefs form a sequential equilibrium in which only one firm enters in the high–low state:*

s^A : Firm A learns x and y and enters, except in the low–low state of demand. If he does not meet a competitor, he produces the corresponding monopoly quantity (16, 13 or 10). If he does meet a competitor in the high–high state of demand, he produces $(x^H + y^H)/3 = 32/3$. If he meets a competitor in the high–low state of demand, A believes that the competitor entered without having gathered any information, and he produces $q_A^{hl} = 953/102$. If he meets a competitor in the low–high state of demand, A believes that the competitor entered without having gathered any information, and he produces $q_A^{lh} = 647/102$.

s^B : Firm B learns x and y , and enters only when both are high. If he does not meet a competitor, he produces the monopoly quantity $(x^H + y^H)/2 = 16$. If he does meet a competitor he produces the Cournot output $(x^H + y^H)/3 = 32/3$.

Proof. Let us check that the beliefs are consistent. Consider the possibility that firm B trembles with small probability δ , in which case he does not gather information and decides to enter. When he does not meet a competitor he produces $(x^L + y^L)/2 = 7$, when he does meet a competitor he produces $373/51$. Let all other pure information acquisition and entry decisions occur with probability of order δ^2 . When firm A meets a competitor when demand is high–high, he will assign probability of almost 1 to the event that firm B did not tremble. When firm A meets a competitor when demand is high–low or low–high, belief revision will imply that firm A believes with probability of almost 1 that firm B trembled and entered without having gathered information. As $\delta \rightarrow 0$ the fully mixed strategy pair converges to (s^A, s^B) , while the beliefs generated by the fully mixed strategy pair converge to the beliefs described above.

Now we have to check that the strategies s^A, s^B are sequentially rational, given the beliefs. It is clear that firm A's strategy is optimal along the equilibrium path. How much should firm A produce in case he meets a competitor when demand is high–low or low–high? When firm B has trembled and finds himself in the situation where he entered without having gathered information and meeting a competitor, he will discard the possibility of a low–low demand state. He will update his beliefs using Bayes' rule:

$$\mu^{hh} = \frac{\alpha\beta}{1 - (1 - \alpha)(1 - \beta)} = \frac{1}{11}$$

$$\mu^{hl} = \frac{\alpha(1 - \beta)}{1 - (1 - \alpha)(1 - \beta)} = \frac{1}{11}$$

$$\mu^{lh} = \frac{(1 - \alpha)\beta}{1 - (1 - \alpha)(1 - \beta)} = \frac{9}{11}$$

Now it can be verified that $q_B = 373/51$ maximizes $q_B(\mu^{hh}(x^H + y^H - (x^H + y^H)/3 - q_B) + \mu^{hl}(x^H + y^L - q_A^{hl} - q_B) + \mu^{lh}(x^L + y^H - q_A^{lh} - q_B))$, while $q_A^{hl} = 953/102$ maximizes $q_A^{hl}(x^H + y^L - q_B - q_A^{hl})$, and $q_A^{lh} = 647/102$ maximizes $q_A^{lh}(x^L + y^H - q_B - q_A^{lh})$. The strategy of firm A is indeed optimal, given the beliefs.

Firm B's strategy is also optimal along the equilibrium path. However, we have to consider whether a deviation by entering in the high–low or low–high state of demand is profitable. The best production level for firm B after entering in the high–low demand state is $q = (x^H + y^L - q_A^{hl})/2 = 1699/204$, which yields benefits of $q^2 - K \approx -0.64 < 0$. It is, therefore, not profitable to deviate and to enter in the case of high–low demand, given the expectations of firm A. Similarly, entering in the low–high state is not profitable since the optimal production level would be equal to $(x^L + y^H - q_A^{lh})/2 = 1393/204$, and the loss (≈ -23.37) would be even greater.

In this example firms learn demand perfectly in equilibrium. However, in the high–low state of demand only firm A enters. Were firm B to deviate and enter in this state, then firm A would be faced with an unexpected event and might believe that firm B made a mistake and entered without having gathered information. The uninformed firm B would then assign relatively high probability to the low–high state, and produce only $373/51 \approx 7.31$, which is less than the production level of a Cournot duopolist in the high–low state ($26/3 \approx 8.67$). This implies then that firm A would produce more than that level ($953/102 \approx 9.34$) which then makes the deviation by B unprofitable.

4. Conclusion

Our examples show that when potential entrants need to acquire information about the profitability of the market, the number of firms that in fact enter is not necessarily determined by the (almost) zero profit condition, even when information acquisition is almost free and firms do in fact acquire the information in equilibrium. This casts some doubt on the literature that uses the zero profit condition to derive certain results. For example, [Dixit and Stiglitz \(1977\)](#) use the zero profit condition to derive the degree of product diversity. Empirical papers as [Bresnahan and Reiss \(1988, 1990, 1991\)](#) estimate market size based on the entry decision of firms. As our examples show, however, the fact that only one firm enters does not necessarily mean that the market is too small for two firms.

We have assumed Cournot competition in the production stage, but our claim that beliefs may bar entry also holds for other oligopolistic competition forms like price competition with differentiated products or rent seeking contests.

Acknowledgements

Hurkens gratefully acknowledges financial support from CIRIT, Generalitat de Catalunya, grant 1997SGR 00138 and DGES grant PB96-0302. Vulkan gratefully acknowledges financial support from the EPSRC, grant no. M07052 and from the ESRC grant no. R000222642.

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