Behavioral Decisions and Welfare*

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Abstract

What are the normative implications of behavioral economics? This paper proposes a general and simple theoretical framework to answer this question. Although we show that revealed preferences cannot, in general, underpin welfare, we offer conditions validating such a link. We assess the scope of the existing normative criteria used elsewhere and propose a normative criterion based on individual autonomy as a refinement. We use our autonomy criterion to justify a class of public policy interventions we label soft-libertarian, which offers theoretical grounds for empowerment policies. Our theory is falsifiable and the testable implications of behavioral and standard decisions are different. Our approach unifies a variety of seemingly disconnected positive behavioral models, and it allows for preferences to be not necessarily complete or transitive and for action sets to be not necessarily convex. We use our model to study the interaction between the decision maker’s initial disadvantages and her capacity to aspire. (D01, D62, C61, I30)

Keywords: Behavioral Decisions, Indistinguishability, Revealed Preferences, Normative Preferences, Welfare, Paternalism, Autonomy, Existence.

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1 Introduction

Standard normative economics employs the revealed preference approach to extract welfare measures from behavior. The preferences revealed from the individual’s choices are assumed to be identical to the normative preferences representing the individual’s true interest. People are assumed to choose what is best for them. Vast empirical evidence, however, has identified an array of situations in which individuals often do not appear to do what is best for themselves, which establishes a wedge between normative and revealed preferences\(^1\).

This situation raises a fundamental problem for economic welfare analysis: how can an appropriate criterion for analysing welfare be identified if the individual’s choices fail to provide clear guidance on the individual’s best interest?

A body of research has recently emerged as a serious attempt to provide an answer to this key question. However, a consensus regarding the appropriate criteria for behavioral welfare analysis has not yet been reached. One approach is to continue basing judgments of welfare on choice (Bernheim and Rangel, 2008). Another approach instead rejects choice as a foundation for normative analysis and proposes alternative measures of individual welfare based on individual’s happiness (Kahneman et. al., 1997), opportunities (Sugden, 2004) or capabilities (Sen, 1985).

This paper proposes an analytic framework that contributes to this discussion in a simple and tractable way. Our framework is general enough to encompass some of the most widely used normative approaches, to assess their scope and relevance for guiding welfare analysis and to unify a variety of seemingly disconnected positive behavioral models. Despite its generality, our framework yields non-trivial testable implications.

The intuition behind our setting is the following. We propose a model of endogenous reference-dependent preferences. The decisions a person makes, consciously or unconsciously, affect the psychological states\(^2\) (or reference points) with which she will analyze future decisions. The psychological states should be interpreted as any pay-off relevant preference parameter that can eventually be affected by the choice of the individual, for example, moods, beliefs, aspirations, attitudes, emotions or values. The way in which the person makes the decision will define two types of decision problems. The person may

\(^1\)Loewenstein and Ubel (2008) point out that in the "heat of the moment," people often take actions that they would not have intended to take and they soon come to regret (Loewenstein, 1996). Koszegi and Ruben (2008) give examples of people making systematic mistakes (gamblers’ fallacy, projection bias, etc). Bernheim and Rangel (2005) record situations in which it is clear that people act against themselves: an anorexic refusal to eat; some people save less than what they would like; fail to take advantage of low interest loans available through life insurance policies; unsuccessfully attempt to quit smoking; maintain substantial balances on high-interest credit cards; etc.

\(^2\)Throughout the paper, we use "psychological states" or "preference parameters" interchangeably.
internalize the effect of what she does on her psychological states, or she may not. If she does internalize it, she chooses an action and, as a consequence, a psychological state, that maximize her true best interest. If she does not internalize it, she chooses an action taking as given her psychological state at the moment she decides, and in equilibrium, psychological states and actions are required to be mutually consistent. The former situation is a Standard Decision Problem (SDP), and the latter is a Behavioural Decision Problem (BDP).

Our model can be considered as a reduced-form representation of an intrapersonal game between two selves. A self-1, who chooses actions to maximize his utility, and a "psychological" self-2, who chooses psychological states to minimize some deeper psychological needs like anxiety, frustration or fear. The Nash equilibria of this intrapersonal game are equivalent to the outcomes of a BDP, while the Stackelberg equilibrium is equivalent to the outcome of a SDP. A person who solves her intrapersonal game a la Stackelberg is autonomous\(^3\), that is, she is in control of her own psychological states. A non-autonomous person, in contrast, is the person who solves her own intrapersonal game a la Nash, choosing an action taking the psychological states of self-2 as given\(^4\). Since a Stackelberg outcome (weakly) welfare dominates a Nash outcome, we advocate the degree of autonomy as a natural normative principle. This implies that the decision process itself has welfare implications. We assume that the person has a "true" interest, which is represented by her preferences over actions and psychological states when she optimally chooses both. Thus, in our model, normative preferences are the preferences of self-1 over outcomes when the person plays the intrapersonal game a la Stackelberg. At this stage, it is important to highlight two important features of our model. First, as in Harsanyi (1954), we assume that the intrapersonal comparison of utility is possible. Second, what makes a person non-autonomous is not related to her being uninformed or unaware of the consequences of her actions, but to the way she plays her intrapersonal game. Think of a doctor who is a smoker, who is fully aware of the risk of smoking and who has revealed her preferences to quit smoking by paying for costly treatments. This doctor knows exactly her self-2 best reply and she solves her intrapersonal game a la Nash. She will quit smoking the day she manages to play her intrapersonal game a la Stackelberg. Changing the way an intrapersonal game is played may require the enhancement of non-cognitive abilities such us emotional intelligence, capacity to aspire or to exert self-control. This issue will be addressed in the paper when we discuss the policy implications of our model.

\(^3\)Autonomy is the regulation by the self (Ryan and Deci, 2006). In Greek: "Auto-Nomos" - nomos meaning "law": one who gives oneself his/her own law.

\(^4\)For the rest of the paper, we refer to the person not having control of her psychological states as equivalent to the person taking her psychological states as given.
This paper contributes to the literature in several ways. The first contribution is to show that a BDP unifies seemingly disconnected positive behavioural economics models. This makes BDP a natural framework to study general properties of behavioural economics models. Second, it offers proof of existence for both SDP and BDP, which requires neither completeness or transitivity or convexity of preferences and actions. Third, it shows that in almost all the cases, behavioural and standard decisions are distinguishable from each other. Fourth, it demonstrates that BDP has non-trivial testable implications, and that the testable implications of BDP and SDP are different. Fifth, it shows that revealed preferences cannot, in general, underpin welfare, though we do offer conditions validating such a link. Sixth, it assess the scope of the existing non-choice base normative criteria such as opportunities or happiness, and proposes a normative criterion based on individual autonomy as a refinement. Although the concept of autonomy has been extensively studied in the literature of philosophy and psychology, this paper is, to our knowledge, the first to formally introduce autonomy into an economic decision-theoretic framework. Seventh, it uses the autonomy criterion to justify a class of public policy interventions labelled soft-libertarian, which directly address the preference formation mechanism. Finally, it proposes an application that links individual external constraints with the individual internal constraints.

The remainder of the paper is organized as follows. Section 2 illustrates our model with an example of aspirations that motivates the rest of the paper. Section 3 introduces the general framework, states the existence result and study the testable restrictions of our theory. Section 4 shows that our approach unifies seemingly disconnected models. Section 5 is devoted to an analysis of indistinguishability. Section 6 studies the welfare implications of our model. Section 7 discusses policy implications. Section 8 reviews relevant literature in philosophy and psychology, and the last section concludes and discusses directions for further research. The Appendix of the paper proposes an existence proof for both SDP and BDP, which requires neither completeness or transitivity or convexity of preferences and action sets. The Appendix also collects all the proofs not in the text.

2 A Motivating Example: Aspiration Traps

Appadurai (2004) and Ray (2006) discuss the way an individual can fail to aspire. Based on their insights, Heifetz and Minelli (2007) study a model of aspiration traps where an individual in period $t = 0$ makes a choice which will affect her attitude for the rest of her life. Here we introduce an example that can be considered as a reduced representation of their work. This example has two aims. First, it highlights the key aspects of our general
framework in a simple and intuitive way, and second, it prefigures what we do in the rest of the paper. In Section 7, we extend this example to a model that shows that initial relative disadvantage can negatively affect the way aspirations affect individual choices and makes explicit the key role that aspirations play for the poor to alter the conditions of their own poverty.

Consider an individual whose decision-making problem involves the following payoff-relevant variables:

(i) a set of actions \( A = \{ \underline{a}, \overline{a} \} \), \( \underline{a} < \overline{a} \), where \( \underline{a} \) represents maintaining the existing status quo and \( \overline{a} \) represents changing the existing status quo by undertaking higher effort (going to College, working harder at school, undertaking additional training, embarking on a new project, etc.). and

(ii) a set of utility parameters \( P = [p, \overline{p}] \) where \( p \in P \) represents the aspirations of the individual (or the intrinsic motivation or level of confidence).

The preferences of the individual are represented by a utility function \( u(a, p) = b(a) - c(a, p) \), where \( b(a) \) is the benefit the individual obtains from her new social status and \( c(a, p) \) is the perceived cost of effort, which is decreasing in \( p \) but increasing in \( a \). For simplicity, assume that \( u(\underline{a}, p) \), the individual’s utility from preserving the status quo, is normalized to zero for all values of \( p \) and for each \( p \), \( u(\overline{a}, p, \theta) \) is the perceived net gain (or loss) to the individual in deviating from the status quo. Then, under the assumptions made so far, \( u(\overline{a}, p') > u(\overline{a}, p) \), for \( p' > p \). For example, if \( \overline{a} \) is interpreted as going to College, and \( \underline{a} \) as staying at home, this inequality implies that the higher the person’s aspirations, the more she enjoys College. In addition, assume that \( u(\underline{a}, p) \) is continuous in \( p \).

For each \( p \), the individual solves the maximization problem

\[
\max_{a \in A} u(a, p)
\]

This generates an optimal action correspondence \( \alpha(p) = \arg \max_{a \in A} u(a, p) \). Under our assumptions there is a unique solution \( \hat{p} \) to the equation \( u(\overline{a}, p) = 0 \). Given \( p \), the optimal action correspondence of the individual is determined as follows:

(i) whenever \( p < \hat{p} \), \( \underline{a} = \alpha(p) \);

(ii) whenever \( p > \hat{p} \), \( \overline{a} = \alpha(p) \);

(iii) whenever \( p = \hat{p} \), \( \{\underline{a}, \overline{a}\} = \alpha(p) \).

So far, we have treated \( p \) as exogenous so there is nothing new in this setting: an individual with sufficiently low aspirations will prefer to remain in status-quo, whereas an individual with sufficiently high aspirations will see it as convenient to exert effort to change her status-quo. In our world, however, we allow preference parameters (psychological states) to be endogenous. In this example, the individual can, consciously or unconsciously, affect
her aspirations level. The way aspirations are affected is captured by an increasing function $\pi : \{a, \overline{a}\} \rightarrow P$, that assigns aspirations levels to each action. For example, the fact that the person goes to College makes her choose higher aspirations. Let $\underline{p} = \pi (a)$ and $\overline{p} = \pi (\overline{a})$, $\underline{p} < \overline{p}$, be the lowest and highest aspiration levels consistent with the actions available. That is, going to College is consistent with endorsing high aspirations, and staying at home is consistent with endorsing low aspirations.

This problem can be solved in two distinctive ways:

(a) we can assume that the person controls the effect of her actions on her aspirations and then chooses $a \in \arg\max_{a \in A} u(a, \pi(a))$, or alternatively,

(b) we can assume that the person does not control (takes as given) the effect of her actions on her aspirations and then chooses $a \in \arg\max_{a \in A} u(a, p)$ given $p \in P$. In equilibrium, both aspirations and actions are required to be mutually consistent.

A decision problem that is solved in the former way is a Standard Decision Problem (SDP), while a decision problem solved in the latter way is a Behavioural Decision Problem (BDP). An outcome of a SDP is a pair $(a, \underline{p})$ such that $a \in \arg\max_{a \in A} u(a, \pi(a))$. An outcome of a BDP is a pair $(a, p)$ such that (i) given $p$, $a \in \alpha(p)$ and (ii) $p = \pi (a)$.

In the context of the present example, there are three type of outcomes:

(i) if $\underline{p} < \hat{p}$, there exists a unique standard and behavioural equilibrium: $(a^*, p^*) = (a, \underline{p})$;

(ii) if $\overline{p} > \hat{p}$, there exists a unique standard and behavioural equilibrium: $(a^*, p^*) = (\overline{a}, \overline{p})$

(iii) if $\underline{p} \leq \hat{p} \leq \overline{p}$, there is a unique standard equilibrium $(a^*, p^*) = (\overline{a}, \overline{p})$, but two behavioural equilibria: $(a^*, p^*) = \{(a, \underline{p}), (\overline{a}, \overline{p})\}$

Call $(a, \underline{p})$ a type I equilibrium and $(\overline{a}, \overline{p})$ a type II equilibrium. In a type I equilibrium, there is no change in the status quo and in a type II equilibrium there is a change in the status quo. For an individual in the middle, $\underline{p} \leq \hat{p} \leq \overline{p}$, there are multiple welfare ranked behavioural equilibria. We come back to the insights we get from this example at the end of the section, but first let us introduce you with another way of understanding our model.

### 2.1 Individual Decision Problem as an Intrapersonal Game

Behind our model of endogenous reference-dependent preferences, there is an intrapersonal game between two different selves, self-1 and self-2. Self-1 chooses $a \in A$ and self-2 chooses $p \in P$. Self-1, can be thought to be the most active part of our selves, the one that a third person observes making choices. Self-2, can be interpreted as the psychological part of the person, the one that aims to satisfy deeper inner psychological needs such as anxiety, fear or peace of mind. For instance, self-2 may prefer to set aspirations low to minimize frustration, to believe that her job is safe to reduce fear, or to be an optimist to minimize anxiety. Each self has preferences over actions and psychological states, which may or may
not conflict with each other. In the context of our example on aspirations, preferences of self-1 are the same preferences of the individual:

\[ u_1(a, p) = b(a) - c(a, p) \]

and the preferences of self-2 can be thought of as a cost function, that self-2 aims to minimize choosing a psychological state \( p \in P \). For example:

\[ c_2(a, p) = \frac{1}{2}p^2 - p.a \]

One can interpret self-2’s "preferences" as if she looks for a balance between aspirations and actions. On the one hand, self-2 does not like feeling frustration, so when \( p > a \), she wants to decrease her aspirations. On the other hand, self-2 does not enjoy being unmotivated either, so when \( p < a \), she wants to aspire more. Thus, this psychological self will want to aspire neither more nor less than what self-1 chooses. Formally, self-2’s best reply is \( \pi(a) = \arg \min_{p \in P} c_2(a, p') \). In this case, her best reply is the identity map \( \pi(a) = p \), which is one possible "microfoundation" consistent with the complementarity assumption between actions and aspirations imposed as a feedback effect in the example above.

Therefore, our example can be modeled as an intrapersonal game in which the best reply function of self-1 is

\[
\alpha(p) = \begin{cases} 
  a & \text{whenever } p < \hat{p}, \\
  \bar{a} & \text{whenever } p > \hat{p}, \\
  \{a, \bar{a}\} & \text{whenever } p = \hat{p},
\end{cases}
\]

and the best reply function of self-2 is \( \pi(a) = p \) for each \( a \in A \).

If the two selves play this intrapersonal game as a simultaneous move game, the Nash equilibrium is a decision state \((a^*, p^*)\) for which \( a^* \in \alpha(p^*) \) and \( p^* \in \pi(a^*) \). Note that this is the definition of a behavioural equilibrium. However, if one of the selves is allowed to control the other self’s decisions, then the appropriate solution concept is Subgame Perfect Equilibrium. Suppose self-1 is the "Stackelberg leader," she chooses her action \( a \in A \) first, and self-2 observes \( a \) before choosing \( p \). In our example, self-1 will anticipate that if she plays \( \bar{a} \), self-2 plays \( \bar{p} \), and since \((\bar{a}, \bar{p}) = \arg\max_{a \in A} u(a, \pi(a))\), \((\bar{a}, \bar{p})\) is sub-game perfect equilibrium of the intrapersonal game or the "Stackelberg equilibrium." Note that \((\bar{a}, \bar{p})\) is also the unique outcome of the Standard Decision Problem in our example.

\footnote{Self-2’s preferences are in line with Ray’s (2006) idea of an individual who chooses a level of effort to minimize her aspirations gap, which is the relative distance between what she aspires and what she has. Here self-2’s best response aims bringing her aspirations gap equal to zero.}
2.2 Remarks from the Example

The above simple example of aspirations traps highlights several important features that motivate the rest of the paper.

First, our example and the vast range of models that can be reduced to our framework, can be reduced to an intrapersonal game between two selves with different preferences, one choosing actions and the other choosing preference parameters. Each positive behavioural model that we review in Section 4 has (often implicit) some $A, P, \pi, u$ and a decision-maker solving her "hidden intrapersonal game" a la Nash or a la Stackelberg.

Second, the way in which the person solves her intrapersonal game, that is, the decision making process itself, has positive and normative implications. On the positive front, we learned that under some range of $p \in P, p \leq \hat{p} \leq \bar{p}$, there is an outcome of a BDP, $(a^*, p^*) = (a, \bar{p})$, that would never be an outcome of a SDP. In such case, the two decision problems are distinguishable from each other. However, when $\bar{p} < \hat{p}$ and $\bar{p} > \hat{p}$, the two decision problems are indistinguishable from each other. In section 5, we offer the general conditions under which any two decision problems with the same $A, P, u$ and $\pi$ are distinguishable from each other. On the normative front, when both type I and type II equilibria exist, since the type II equilibrium dominates the status quo, a type I equilibrium can be interpreted as an aspirations failure, a low motivation trap for the individual. An aspirations failure would not be a possible outcome of a model in which preference parameters are exogenous, or even of a model in which preference parameters are endogenous, but the decision-maker internalizes the feedback effect (SDP). Therefore, the assumption on how the decision-maker plays her intrapersonal game is crucial for normative analysis. More importantly, the revealed preference for staying at home over changing status-quo when the person is trapped in an aspirations failure is not an appropriate indicator for welfare. In section 6, we address this issue and characterize the general conditions under which revealed preferences can be recovered as an appropriate welfare indicator.

Third, our framework opens new doors for policies that are currently understudied in the literature. In the context of our example, in the case where there is an aspirations failure, a policy can break the aspirations trap in two ways: either by affecting $\hat{p}$, or by affecting the aspirations formation itself via $F(p)$. In this example, $\hat{p}$ is exogenous, but in Section 7, we present an extension of this example in which $\hat{p}$ negatively depends on the initial disadvantages of the individual. So, a policy that improves individual initial disadvantages will help to break an aspirations failure. But more importantly, policies that directly affect the decision process enhancing the non-cognitive abilities of the individual such as the capacity to aspire (Appadurai, 2004) can be welfare improving. In economic terms, gaining the capacity to aspire means learning how to play the intrapersonal aspirations game as a
Stackelberg leader. In philosophical and psychological terms, it implies gaining individual autonomy.

Fourth, it opens new questions regarding identification of the decision process. Suppose we observe a person that chooses not to go to College, so she is in type I equilibrium. How do we know, whether that equilibrium correspond to an aspirations trap, or is simply a unique type I equilibrium. In other words, can we test whether the person is autonomous or not? We leave the complete answer to this ambitious question for further research. For the moment, in Section 3, we offer a partial answer to this question, by showing that the testable implications of BDP and SDP are different.

Fifth, although it is not explicit in the example, it can be easily shown that a behavioural equilibrium or a Nash equilibrium of an intrapersonal game is the steady state of an adaptive dynamics over actions and psychological states. A behavioural equilibrium is reached after people adapt their psychological states to their actions. In other words, in a behavioural equilibrium, people end up wanting what they do, but may not necessarily choose to do what they actually want. In the context of our example, suppose the following adaptive dynamics over $p$ and $a$. First, the initial psychological state of an individual, $p_0$, is picked at random from $[p, \bar{p}]$ according to some continuous pdf $f(p)$ (with associated cdf $F(p)$). Second, given $p_0$, the individual chooses $\alpha(p_0) \subset \{\bar{a}_1, \bar{p}_1\}$ which, in turn, generates a new $p_1 = \pi(a_1)$, for some $a_1 \in \alpha(p_0)$. This dynamics always converge to either a type I or a type II equilibrium. If $p_0 < \hat{p}$, then $\bar{a}_1 \in \alpha(p_0)$ which implies $\bar{p}_1 = \pi(\bar{a}_1)$ and the behavioural equilibrium is $(\bar{a}_1, \bar{p}_1)$. If $p_0 > \hat{p}$ then $\bar{p}_1 \in \alpha(p_0)$ which implies $\bar{p}_1 = \pi(\bar{a}_1)$ and the behavioural equilibrium is $(\bar{a}_1, \bar{p}_1)$.

The rest of the paper, which is more technical than this first part, introduces the general model and studies in a general way the insights highlighted with our example of aspirations.

3 The General Framework

A decision scenario $D = (A, P, \pi)$ consists of a set $A \subset \mathbb{R}^k$ of actions, a set $P \subset \mathbb{R}^n$ of psychological states and a map $\pi : A \rightarrow P$ modelling the feedback effect from actions to psychological states. It is assumed that $\pi(a)$ is single-valued and non-empty for each $a \in A$, and $\mathbb{R}^k$ and $\mathbb{R}^n$ are finite dimensional Euclidian spaces.

A decision state is a pair of action and psychological state $(a, p)$ where $a \in A$ and $p \in P$.

The preferences of the decision-maker are denoted by $\succeq$, a binary relation ranking pairs of decision states in $(A \times P) \times (A \times P)$. The expression $\{(a, p), (a', p')\} \in \succeq$ is written as $(a, p) \succeq (a', p')$ and is to be read as "$(a, p)$ is weakly preferred to (equivalently, weakly welfare dominates) $(a', p')$ by the decision-maker".
A *consistent state* is a decision state $(a, p)$ such that $p = \pi(a)$. Let

$$\pi(A) = \{p \in P : \exists a \in A \text{ s.t. } p = \pi(a)\}$$

be the set of consistent psychological states, and

$$\Omega = \{(a, p) \in (A \times P) : p = \pi(a) \text{ for all } a \in A\}$$

be the set of consistent decision states.

There are two types of decision problems studied here:

1. A *standard decision problem (SDP)* is one where the decision-maker chooses a pair $(a, p)$ within the set of consistent decision states. The outcomes of a SDP are denoted by $M$ where

   $$M = \{(a, p) \in \Omega : (a, p) \succeq (a', p') \text{ for all } (a', p') \in \Omega\}. $$

2. A *behavioral decision problem (BDP)* is one where the decision maker takes as given the psychological state $p$ when choosing $a$. Define a preference relation $\succeq_p$ over $A$ as follows:

   $$a \succeq_p a' \iff (a, p) \succeq (a', p) \text{ for } p \in P.$$  

The outcomes of a BDP are denoted by $E$ where

$$E = \{(a, p) \in \Omega : a \succeq_p a' \text{ for all } a' \in A\}. $$

Suppose $P = A$ and $a = \pi(a)$. In this case, the decision problems studied here offer a way of modelling situations where "the reference state usually corresponds to the decision maker’s current state." (Tversky and Kahneman, 1991, p. 1046). The following examples show that whether the decision-maker correctly anticipates the feedback effect from actions to the preference parameter or not, will have an impact on the decision outcomes.

**Example 1** ($M \subset E$)

Consider a decision problem where $A = P = \{a_1, a_2\}$, $\pi(a_i) = \{a_i\}, i = 1, 2$, and $(a_i, a_i) \succ (a_j, a_i), j \neq i$ and $(a_1, a_1) \succ (a_2, a_2)$. Then, $M = \{(a_1, a_1)\}$ but $E = \{(a_1, a_1), (a_2, a_2)\}$.

**Example 2** ($M \neq \emptyset, E \neq \emptyset, M \cap E = \emptyset$)

Consider a decision problem where $A = P = \{a_1, a_2\}$, $\pi(a_i) = \{a_i\}, i = 1, 2$, and $(a_2, a_j) \succ (a_1, a_j), j = 1, 2$, and $(a_1, a_1) \succ (a_2, a_2)$. Then, $M = \{(a_1, a_1)\}$ but $E = \{(a_2, a_2)\}$.

**Example 3** ($M \neq \emptyset, E = \emptyset$)

Consider a decision problem where $A = P = \{a_1, a_2\}$, $\pi(a_i) = \{a_i\}, i = 1, 2$, and $(a_j, a_i) \succ (a_i, a_i), i \neq j$, and $(a_1, a_1) \succ (a_2, a_2)$. Then, $M = \{(a_1, a_1)\}$ but $E$ is empty.
3.1 Existence

It is not hard to check that as long as both $A$ and $P$ are finite and $\pi(a)$ is single-valued for each $a \in A$, a random equilibrium exists, i.e. $E$ is not empty. Instead, this paper studies existence in situations where the underlying preferences are not necessarily complete or transitive and underlying action sets are not necessarily convex. Mandler (2005) shows that incomplete preferences and intransitivity is required for "status quo maintenance" (encompassing endowment effects, loss aversion and willingness to pay-willingness to accept diversity) to be outcome rational. Tversky and Kahneman (1979, 1991) argue that reference dependent preferences may not be convex. So we allow preferences to be incomplete, non-convex and acyclic (and not necessarily transitive) and we show existence of a behavioral equilibrium in pure actions extending Ghosal's (2007) result for normal form games to behavioral decision problems\(^6\).

**Theorem 1.** Suppose for each $a$, $\pi(a)$ is a compact sublattice of $P$ and the map $\pi : A \to P$ is increasing in $A$. Under assumptions of single-crossing, quasi-supermodularity and monotone closure, a pure action behavioral equilibrium exists.

**Proof.** See Appendix

3.2 Testability

Our model is about two distinctive theories of individual behaviour: one characterized as a Standard Decision Problem (SDP) and the other as a Behavioral Decision Problem (BDP). Are these theories falsifiable? If so, are the testable implications of each theory different from each other?\(^7\) Below we show that the answer to these questions is yes, they are falsifiable and have different testable implications.

A theory is falsifiable if there exists some outcome that cannot be rationalized as an equilibrium of that theory. For example, standard choice theory is falsifiable if Arrow's (1959) choice axiom holds. Arrow's choice axiom states that when the set of feasible alternatives shrinks, the choice from the smaller set consists precisely of those alternatives that were selected from the larger set and remain feasible, if there are any. What can we say about our two theories?

Take $u : A \times P \to \mathbb{R}$, $\pi : A \to P$ and a family $\mathcal{B}$ of subsets of $A$. Define two correspon-

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\(^6\)The seminal proof for existence of equilibria with incomplete preferences in Shafer and Sonnenschein (1975) requires convexity both for showing the existence of an optimal choice and using Kakutani’s fix-point theorem.

\(^7\)The discussion and example we present in this subsection is owed to Andres Carvajal.
dences, $F$ and $G$, from $B$ to $A$ as

$$F(B) = \arg\max_{a \in B} u(a, \pi(a))$$

and

$$G(B) = \{ a^* : a^* \in \arg\max_{a \in B} u(a, \pi(a)) \},$$

so, the choices of the autonomous and non autonomous person, respectively.

Assume that $A$, $P$ and all $B \in \mathcal{B}$ are convex subsets in $\mathbb{R}$. If we assume that $v(a) = u(a, \pi(a))$ is strictly concave, then $F$ is single-valued. On the other hand, if $A = P = [0, 1]$, $\pi(a) = a$ and

$$u(a, p) = -(a - p)^2 - (1/2 - p)^2,$$

then $v$ is strictly concave and $F(A) = \{1/2\}$, but $G(A) = A$.

Then, the concern is that the explanatory power of $G$ is null. Suppose that we observe a correspondence $H$ from $B$ to $A$ such that $H(B) \subseteq B$. One may want to test two type of null hypothesis, one weak and the other strong. A weak null hypothesis for SDP would be that there exist $P$, $\pi$ and $u$ such that $v$ is strictly concave and $H(B) \subseteq F(B)$. Under this null, it must be that $H$ is single-valued and satisfy the strong axiom of revealed preferences, i.e. SDPs have strong restrictions. However, if we want to test the weak null hypothesis for BDP that there exist $P$, $\pi$ and $u$ such that $v$ is strictly concave and $H(B) \subseteq G(B)$, then a BDP does not seem to be falsifiable.

This latter observation does not apply if we impose a stronger hypothesis and require that $H(B) = G(B)$\footnote{The cost of testing this stronger null hypothesis is that we have to assume that we observe in our data all the points in $G(B)$ when we apply the test. Note that this assumption is not needed in the first case, since the null hypothesis guarantee that $F(B)$ is single-valued, so if we observe one point of $F(B)$, under the null, we are observing all the points.}. So suppose that we want to test this strong hypothesis and suppose that $H$ captures all the choices of the person. We know that under the null that there exist $P$, $\pi$ and $u$ such that $H(B) = F(B)$, we have that

**C.3.** If $B' \subseteq B$ anda $\in H(B) \cap B'$ then $a \in H(B')$, and

**C.4.** If $\{a,a'\} \subseteq H(B') \cap B$ anda $\in H(B))$ then $a' \in H(B)$

Under the null that there exist $P$, $\pi$ and $u$ such that $H(B) = G(B)$, we have that **C.3** holds but **C.4** doesn’t hold. **C.3** holds because if $a \in \arg\max_{a' \in B'} u(a', \pi(a))$ and $a \in B' \subseteq B$, then $a \in \arg\max_{a' \in B'} u(a', \pi(a))$. To show that **C.4** doesn’t hold if the null is strong, consider the following example: $A = \{a_1, a_2, a_3\}$, $P = \{p_1, p_2\}$, $\pi(a_1) = \pi(a_3) = p_1$, $\pi(a_2) = p_2$, and $G(B) = \{p_1, p_2\}$. Under the null that $H(B) = F(B)$, we have that $\arg\max_{a \in B} u(a, \pi(a)) = a_2$, but $a_2 \notin H(B)$, a contradiction.
\( \pi(a_2) = p_2 \), and \( u(a, p) \) is:

<table>
<thead>
<tr>
<th>( u(a, p) )</th>
<th>( p_1 )</th>
<th>( p_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_1 )</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( a_3 )</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

In this case, \( G(A) = \{a_1\} \) but \( G(\{a_1, a_2\}) = \{a_1, a_2\} \).

We can summarize this discussion in the following proposition.

**Proposition 1.** If the null hypothesis is weak, then SDPs have strong testable restrictions and BDPs do not. However, if the null hypothesis is strong, there are data that are behavioural-rationalizable and not standard-rationalizable, every outcome that is standard-rationalizable is also behavioral-rationalizable, and there are outcomes that are not behavioural-rationalizable.

### 4 Reduced form Representation

The example of Section 2 aimed to illustrate in a simple way the positive and normative implications of the two key premises of our model: (a) preferences change with actions and (b) people may not fully internalize this change. But how representative are these two premises of the existing literature? It can be shown that these two premises are enough to unify seemingly disconnected models in the literature, from situations where the preference parameter corresponds to the decision maker’s current state (Tversky and Kahneman, 1991, Shalev, 2000), beliefs (Geanakoplos, Pearce and Stacchetti, 1989; Akerlof and Dickens, 1982), emotions (Bracha and Brown, 2007), expected consumption (Koszegi, 2005; Koszegi and Rabin, 2006, 2007) or aspirations (Ray, 2006 and Heifetz and Minelli, 2007), to models of adaptive preferences like von Weizsacker (1975), Hammond (1976) and Pollak (1978).

In this section, we review some of these models and show how our model can be obtained as a reduced form representation of them.

#### 4.1 Psychological games with a single active player

Geanakoplos, Pearce and Stacchetti (1989) (hereafter, GPS) study psychological games where the payoffs of each player depend not only on the actions chosen by all other players but also on what other players believe, on what she thinks they believe others believe and so on. Each player takes beliefs and actions of other players as given when choosing her own action. In equilibrium, beliefs are assumed to correspond to actions actually chosen.

In the special case where there is a single active player, the payoffs of this single active
player can depend on his own actions and the beliefs of other players over his own actions.

Consider a two player psychological game. Player 1 is the active player with a set of pure actions $S$ and mixed actions $\Sigma = \Delta (S)$. A belief for player 2 is denoted by $\tilde{b} \in \tilde{B} = \Sigma$. The payoffs of player 1 over pure actions is given by a utility function $u : A \times \tilde{B} \rightarrow \mathbb{R}$ with $v (\sigma, b) = \sum_{s \in S} \sigma (s) u (s, b)$ being the corresponding payoffs over mixed actions. A psychological equilibrium is a pair $\left( \tilde{\sigma}, \tilde{b} \right) \in \Sigma \times \tilde{B}$ s.t. (i) $\tilde{b} = \tilde{\sigma}$, (ii) for each $\sigma \in \Sigma$, $u (\tilde{\sigma}, \tilde{b}) \geq u (\sigma, \tilde{b})$. Clearly, by setting $A = P = \Sigma$ and $\pi$ as the identity map, a behavioral decision problem is a psychological game with one active player. GPS show that there are robust examples where the two sets $M$ and $E$ differ.

### 4.1.1 Example of self-image

Suppose Pat is concerned with what Jane will think about him. He can take a bold decision, which exposes him to the possibility of danger, or a timid, safe decision. Thus, his action space is $A = \{ \text{bold, timid} \}$. Pat’s payoffs not only depends on what he does but also on what he thinks Jane thinks about his character. In other words, Pat cares about what he thinks Jane thinks he will do. Suppose that Pat himself is a timid person, so he would prefer to think that Jane thinks he is timid rather than bold. But he also doesn’t want to disappoint Jane, so if he thinks Jane expects him to be bold, he’d rather be bold than timid. Pat chooses bold with probability $b$ and timid with probability $1 - b$. Let $q$ represent Jane’s expectations of $b$ and $\bar{q}$ represents Pat’s expectations of $q$. The game and payoffs are described in the following Fig. 6

<table>
<thead>
<tr>
<th></th>
<th>$\bar{q}$</th>
<th>$1 - \bar{q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>bold</td>
<td>1</td>
</tr>
<tr>
<td>$1 - b$</td>
<td>timid</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\bar{q}$</th>
<th>$1 - \bar{q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>bold</td>
<td>1</td>
</tr>
<tr>
<td>$1 - b$</td>
<td>timid</td>
<td>3</td>
</tr>
</tbody>
</table>

FIG. 6.

The consistency requirement between actions and preference parameters of a behavioral equilibrium implies that we must have $b = q = \bar{q}$. Beliefs must correspond to equilibrium play, yet they can still exercise a decisive influence on Pat’s behavior. In this game, there are two welfare-ranked behavioral equilibria in pure strategies\(^9\). An optimal equilibrium in which $b = q = \bar{q} = 0$ with payoff 3, and a sub-optimal equilibrium $b = q = \bar{q} = 1$ with payoff 1.

It would be optimal for Pat to believe that Jane believes he is timid ($\bar{q} = 0$), but if she does not believe so ($q = 1$), Pat will meet her beliefs and play bold ($b = 1$) reinforcing

---
\(^9\)There is one equilibrium in mixed strategies, but we leave it aside for simplicity.
Jane’s beliefs. In such case, Pat will end up doing something that he wouldn’t do, was he able to internalize that playing timid would make Jane believe that he is indeed timid. If Pat reflected in that way, he would solve a standard decision problem in which $b = 0$ is the unique (Stackelberg) equilibrium. But in this behavioral decision problem, Pat is playing a la Nash in his intrapersonal game against his self-2 who set the beliefs about what Jane believes. Self-2 "best reply" to Pat is to assign a probability equal to 1 to what Pat does.

4.2 Loss aversion games with a single player

Shalev (2000) considers a class of games where players have reference dependent utilities and the reference utility depends on the action profile chosen by all players. Shalev defines two notions of equilibrium, a myopic loss aversion equilibrium and a non-myopic loss aversion equilibrium. In either equilibrium notion, each player takes as given the actions of others when choosing her actions. In a myopic loss aversion equilibrium, a player also takes as given the reference utility when choosing her actions (even though changing her actions might change the reference utility). In a non-myopic loss aversion equilibrium, a player takes into account the feedback effect from her actions to the reference utility when choosing her actions. A single player version of Shalev’s model has the player choosing a mixed action $\sigma \in \Sigma$ with payoffs $w(\sigma, r) = \sum_{s \in S} \sigma(s) v(u(s), r)$ where

$$v(u(s), r) = \begin{cases} u(s) & \text{if } u(s) \geq r \\ u(s) - \lambda (r - u(s)) & \text{if } u(s) < r \end{cases}$$

$r$ is the reference utility and $u : S \rightarrow \mathbb{R}$ is a standard utility function. A consistent reference point $r$ satisfies the equation $r = w(\sigma, r)$. Let $R(\sigma) = \{ r \in \mathbb{R} | r = w(\sigma, r) \}$. Shalev proves that $R(\sigma)$ is single valued and its values are contained in the closed interval $[\underline{r}, \bar{r}]$. Clearly, setting $A = \Sigma$, $P = [\underline{r}, \bar{r}]$ and $\pi(a) = R(\sigma)$, a behavioral decision problem is a myopic loss aversion decision problem while a non-myopic loss aversion decision problem corresponds to a standard decision problem. Shalev shows that in the static version of his model $M = E$ although the two sets differ in dynamic settings.

4.3 Reference dependent consumption and personal equilibrium

In Kozsegi and Rabin (2006), a person’s utility depends not only on her $K$-dimensional consumption bundle, $c$, but also on a reference bundle, $r$. She has an intrinsic “consumption utility” $m(c)$ that corresponds to the standard outcome-based utility. Overall utility is given by $u(c|r) = m(c) + n(c|r)$, where $n(c|r)$ is “gain-loss utility.” In their paper, both consumption utility and gain-loss utility are separable across dimensions, so that $m(c) = \sum_{k} m_{k}(c_{k})$ and $n(c|r) = \sum_{k} n_{k}(c_{k}|r_{k})$. They assume that $n_{k}(c_{k}|r_{k}) = \mu(m_{k}(c_{k}) - m_{k}(r_{k}))$, where $\mu(.)$
satisfies the properties of Kahneman and Tversky’s [1979] value function. Following Kozseg i (2005) they define a personal equilibrium as a situation where the optimal $c$ computed conditional on forecasts of $r$ coincides with $r$. Clearly, by setting $A$ and $P$ to be the set of feasible consumption bundles and $\pi$ to be the identity map, a personal equilibrium can be represented by a behavioral decision problem\textsuperscript{10}. Under the assumptions made in their paper, Koszegi and Rabin (2006) show that in deterministic settings $M = E$ while the two sets differ in stochastic settings.

### 4.4 Aspiration traps

Heifetz and Minelli (2007) study a model of aspiration traps where an individual in period $t = 0$ makes a choice $e \in E'$, at a cost $c(e)$. For a given choice $e$, the decision problem of the individual at $t = 1$ is described by the tuple $G_e = (X, u_e, \bar{B})$ where the strategy set of the individual is $X$, her payoff function is $u_e : X \times \bar{B} \to \mathbb{R}$, and the utility of the individual depends on her attitude (beliefs, aspirations) $b \in \bar{B}$. When choosing a strategy $x(e, b)$ at $t = 1$ to maximize $u_e$, the individual takes as given both $b$ and $e$. However, given $e$, $b$ is determined by some preferenceformation mechanism $\beta : E' \to \bar{B}$. At $t = 0$, Heifetz and Minelli consider two modes of choice. When choice is "transparent", the individual would "see through" the preference formation mechanism. At $t = 0$, she would then choose $e$ to maximize $u_e(x(e, \beta(e))) - c(e)$. When the individual choice is "self-justifying", her choice of $e$ satisfies a no-regret condition

$$u_e(x(e, \beta(e))) - c(e) \geq u_e(x(e', \beta(e))) - c(e') \quad \text{for all } e' \in E'.$$

By setting $A = E'$, $P = \bar{B}$ and $\pi(a) = \beta(e)$, it is easily checked that a transparent choice problem corresponds to a standard decision problem while a self-justifying choice problem corresponds to a behavioral decision problem. Along the lines of example 1, they show that $M \subset E$.

### 4.5 Cognitive Dissonance

Akerlof and Dickens (1982) models cognitive dissonance in the following way. Consider a worker in the hazardous industry. The probability of an accident is $q$ and the cost of an accident for him is $c_a$. The worker can purchase a new safety device which eliminates the possibility of an accident at a cost $c_s$ and it is economically convenient, i.e. $qc_a > c_s$. In

\textsuperscript{10}An analogous statement can be made for Kozseg i and Rabin (2007), since the solution concepts they use (i.e. unacclimating personal equilibrium, UPE and preferred personal equilibrium, PPE) are examples of a "personal equilibrium" defined in Koszegi (2005). The major feature of these solution concepts is that the decision maker does not internalize the effect of her choice on her expectations (or reference point).
addition, each worker has a psychological cost of fear, equal to $c_f f$, where $c_f$ is the unit cost of fear and $f$ is the level of the worker’s fear. It is assumed that $f = \tilde{q}$, with $\tilde{q}$ representing the worker’s subjective assessment of the probability of an accident. The worker is allowed to choose either $\tilde{q} = 0$ or $\tilde{q} = q$. Once that choice is made, the worker must behave as if the new value of $\tilde{q}$ is the true probability of an accident.

Formally, the worker solves:

$$
\min_{\tilde{q} \in \{0, q\}} c(\tilde{q}, q) = c_f \left( \frac{\tilde{q}}{q} \right) + qc_a + c_s, \quad q = \pi(\tilde{q})
$$

The map $\pi$ is the identity map in this case, and it represents the requirement that in equilibrium, the beliefs used to judge whether it is convenient to purchase a safety equipment are exactly the beliefs chosen to reduce the fear. The decision problem and payoffs can be summarized in the following matrix:

<table>
<thead>
<tr>
<th></th>
<th>$q = 0$</th>
<th>$q &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{q} = 0$</td>
<td>$qc_a$</td>
<td>$qc_a + c_f$</td>
</tr>
<tr>
<td>$\tilde{q} = q$</td>
<td>$c_s$</td>
<td>$c_s$</td>
</tr>
</tbody>
</table>

Suppose the worker solves this problem as a behavioral decision problem. If he chooses $\tilde{q} = 0$, i.e. to believe that his job is safe, then, by a self-justifying mechanism implied by $q = \pi(\tilde{q})$, he will evaluate the costs and benefits of purchasing a safety equipment using $q = 0$ as the subjective probability of an accident. Given $q = 0$, his cost minimizing action is not buying the equipment since $c_s > qc_a = 0$. On the contrary, if he chooses $\tilde{q} = q$, i.e. to believe that his job is hazardous, then, since $q = \pi(\tilde{q})$, he will evaluate the costs and benefits of purchasing a safety equipment using the true $q > 0$ as the subjective probability of an accident. Given $q > 0$, his cost minimizing action will be to purchase the equipment, since the cost of purchasing it $c_s$ is less than the cost of not purchasing it, $qc_a + c_f$.

A worker that solves this problem as a standard decision problem, will internalize the effect that his choice of beliefs has on his willingness to pay an equipment, and since it is assumed that buying an equipment is cost-effective, $qc_a > c_s$, then his unique (optimal) choice will be to belief that his job is hazardous.

The way workers decide (e.g. the decision process) affects their welfare. A worker that does not control the effect of his choice of beliefs on his decision to buy safety equipment will have $qc_a - c_s$ less welfare than one that internalizes that effect.
4.6 Adaptive Preferences

Pollak (1978) defines the concepts of short run and long run demand functions associated with an adaptive preference mechanism. He models habit formation by assuming that a household’s preference depend on its past consumption. Household’s preference ordering in period $t$ is denoted by $\succ_{c_{t-1}}$, where $c_t$ is the consumption vector for period $t$. The statement $c_t \succeq c_{t-1}$ means that the household finds $c_t$ at least as good as $c'_{t-1}$, given the consumption history $c_{t-1}$. The short-run demand functions are denoted by $c_t = h(P_t, m_t, c_{t-1})$, where $P_t$ is the vector of prices in period $t$ and $m_t$ is the total expenditure in period $t$. The long-run demand functions $c = H(P, m)$ are defined to be the steady-state solution to the short-run demand functions: $H(P, m) = h[P, m, H(P, m)]$. What this older literature of the 70’s including Von Weizsacker (1971), Hammond (1976) and Pollak (1978) called long run preferences are equivalent to the preferences over actions given $p^*$, when $p^* = \pi(a^*)$.

5 Indistinguishability

How relevant is the distinction between a BDP and a SDP? In this section, we state the conditions under which BDP and SDP are indistinguishable from each other. In those cases, BDP and SDP are outcome-equivalent and it is not worth making the distinction, although we show that this happens in very rare circumstances. In fact, in smooth settings, both decision problems are generically distinguishable. We then explore some peculiar outcomes of distinguishable decision problems.

5.1 Indistinguishability

A BDP is indistinguishable from a SDP if and only if $M = E$. Note that indistinguishability is, from a normative viewpoint, a compelling property. What matters for welfare purposes is the ranking of consistent decision states, which is the preference relation that a fully autonomous decision maker will use to make a decision. When $M = E$, the outcomes (consistent decision states) of a SDP coincide with that of a BDP, and in that case, there is no reason to distinguish between normative and revealed preferences.

If $\pi(a) = \pi(a')$ for all $a, a' \in A$, a BDP is, by construction, indistinguishable from a SDP\footnote{In this case, all possible decision states $(a, p)$ are consistent and therefore, the procedure of choice (or the autonomy of the person) does not affect the outcome of the decision. Both, non-autonomous and fully autonomous decision makers will rank the outcomes in the same way and will choose the maximum outcome of the ranking.}. So suppose the map $\pi$ has at least two distinct elements in its range. Next, consider the following conditions:
C1: No sub-optimal behavioral equilibria. For \( a, a' \in A \) such that \( a \succeq_p a' \) for some \( p = \pi(a) \), \( (a, p) \succeq (a', p') \) for each \( p = \pi(a) \) and \( p' = \pi(a') \);

C2: A standard equilibrium is also a behavioral equilibrium. For \( (a, p), (a', p') \in \Omega \) such that \( (a, p) \succeq (a', p'), (a, p) \succeq (a', p) \) for some \( p = \pi(a) \).

Condition (C1) states that if there is a behavioral equilibrium, then there is not other behavioral equilibrium that (strictly) welfare dominates it. In that way, (C1) rules out sub-optimal behavioral equilibria, which is one of the properties that an indistinguishable decision problem should not have. Condition (C2) ensures that there is no standard equilibrium that is not also a behavioral equilibrium.

Note that preferences in Example 1 violate (C1) but satisfy (C2) while the preferences in Example 2 violate both (C1) and (C2). Shalev (2000) shows (in Theorem 1 of his paper) that in the static case his loss averse preferences satisfy both (C1) and (C2). Rabin and Koszegi (2006) show that their reference dependent preferences also satisfy both (C1) and (C2). GPS construct examples where, with one active player, both (C1) and (C2) are violated. Heifetz and Minelli (2007) construct examples where (C1) is violated.

In the following Theorem, we state that (C1) and (C2) are the necessary and sufficient conditions for indistinguishability.

Theorem 2. Suppose that both \( E \) and \( M \) are non-empty. Then, (i) \( E \subseteq M \) if and only if (C1) holds. (ii) \( M \subseteq E \) if and only if (C2) holds.

Proof: See Appendix

5.2 Smooth Decision Problems

To further understand the conditions under which indistinguishability occurs, it is convenient to look at smooth decision problems where decision outcomes are characterized by first-order conditions. We show that for the case of smooth decision problems, behavioral decisions are generically distinguishable from standard decisions.

A decision problem is smooth if (a) both \( A \) and \( P \) are convex, open sets in \( \mathbb{R}^k \) and \( \mathbb{R}^n \) respectively, (b) preferences over \( A \times P \) are represented by a smooth, concave utility function \( u : A \times P \to \mathbb{R} \) and (c) the feedback map \( \pi : A \to P \) is also smooth and concave.

A set of decision problems that satisfies the smoothness assumptions is diverse if and only if for each \( (a, p) \in A \times P \) it contains the decision problem with utility function and feedback effect defined, in a neighborhood of \( (a, p) \), by

\[ u + \lambda p \]
\[ \pi - \mu(a' - a) \]
for each \( a' \) in a neighborhood of \( a \) and for parameters \((\lambda, \mu)\) in a neighborhood of 0.

A property holds generically if and only if it holds for a set of decision problems of full Lebesgue measure within the set of diverse smooth decision problems.

**Theorem 3:** For a diverse set of smooth decision problems, a standard decision problem is generically distinguishable from a behavioral decision problem.

**Proof.**

Let \( v(a) = u(a, \pi(a)) \). The outcome \((\hat{a}, \hat{p})\) of a SDP satisfies the first-order condition
\[ \partial_a v(\hat{a}) = \partial_a u(\hat{a}, \pi(\hat{a})) + \partial_p u(\hat{a}, \pi(\hat{a})) \partial_a \pi(\hat{a}) = 0 \quad (1) \]
while the outcome \((a^*, p^*)\) of a BDP satisfies the first-order condition
\[ \partial_a u(a^*, p^*) = 0, p^* = \pi(a^*). \quad (2) \]

For \((a^*, p^*) = (\hat{a}, \hat{p})\), it must be the case that
\[ \partial_p u(a^*, p^*) \partial_a \pi(a^*) = 0. \quad (3) \]

It is easily checked that requiring both \((C1)\) and \((C2)\) to hold is equivalent to requiring that the preceding equation also holds. Consider a decision problem with \((a^*, p^*) = (\hat{a}, \hat{p})\). Perturbations of the utility function and the feedback effect do not affect \((2)\) and hence \((a^*, p^*)\) but they do affect \((3)\) and via \((1)\) affect \((\hat{a}, \hat{p})\). Therefore, \((a^*, p^*) \neq (\hat{a}, \hat{p})\) generically.

Eq. \((3)\) shows in a simple quick way that BDP and SDP are indistinguishable only in isolated cases (e.g. when \(\pi(a^*)\) or \(u(a^*, p^*)\) are just constants).

Now that we know that making a distinction between BDPs and SDPs is a relevant route to take, we will explore some interesting peculiarities of distinguishable decision problems which pin down important policy implications. Our theoretical illustrations can be empirically complemented with Beshears et. al (2008), who describe situations in which revealed preferences deviate from normative preferences, or in our words, situations in which decision problems are distinguishable. They identify factors that increase the likelihood of having distinguishable decision problems, and discuss approaches to the identification of normative preferences when decision problems are distinguishable.

### 5.3 Distinguishable Decision Problems

We present a few examples that illustrate behavior that would be impossible to rationalize in a standard individual decision framework. In all these examples we assume, for simplicity,
that $A = P$ are finite sets and $\pi(a)$ is the identity map. The preferences of the decision maker are represented by an utility function $u : A \times P \to \mathbb{R}$. We distinguish between pure and random behavioral decisions. Let $\beta(\hat{a}) = \arg\max_{a \in A} u(a, \hat{a})$. A **pure action behavioral equilibrium** is an action profile $a^*$ such that $a^* \in \beta(a^*)$. Let $\Delta(A)$ denote the set of probability distributions over the set of actions. A random strategy is $\sigma \in \Delta(A)$, where $\sigma(a)$ is the probability attached to action $a$. A random distribution over the set of psychological states is $\mu \in \Delta(A)$, where $\mu(\hat{a})$ is the probability attached to psychological state $\hat{a}$. A random decision state is a pair $(\sigma, \mu)$. Given a random decision state $(\sigma, \mu)$, the payoff to the decision maker is

$$w(\sigma, \mu) = \sum_{a \in A} \sum_{p \in P} \sigma(a) \mu(\hat{a}) u(a, \hat{a})$$

A consistent random decision state is a pair $(\sigma, \mu)$ where $\mu = \sigma$. A **random behavioral equilibrium** is a profile $\sigma^*$ such that $\sigma^* \in \arg\max_{\sigma \in \Delta(A)} w(\sigma, \sigma^*)$.

In each example, the decision problem is represented by a payoff table where rows are actions and columns are the psychological states. Under the assumptions made so far, consistent decision states are the diagonal of these payoff tables.

**Example 4. Unique inefficient behavioral equilibrium in dominant actions**

Consider the following payoff table:

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$a_2$</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

(Table 1)

Notice that $a_2$ is dominant action for both values of $p$. The unique behavioral equilibrium is $(a_2, a_2)$ with a payoff of 0. However, note that there is a consistent decision state $(a_1, a_1)$ with a payoff of 1 and therefore, $(a_2, a_2)$ isn’t efficient. Finally, note that the unique inefficient behavioral equilibrium in dominant actions is robust to arbitrary but small perturbations in payoffs.

**Example 5. Unique random equilibrium**

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

(Table 2)

Notice that when the utility parameter is $a_1$, the decision-maker prefers $a_2$ to $a_1$ while when the utility parameter is $a_2$, the decision-maker prefers $a_1$ to $a_2$. Therefore, there is no behavioral equilibrium in pure strategies. However, there is a behavioral equilibrium in
mixed strategies. It follows that there is a unique random outcome in the payoff table 2, $(\frac{1}{2}a_1 + \frac{1}{2}a_2, \frac{1}{2}a_1 + \frac{1}{2}a_2)$.

**Example 6.** Equilibrium in weakly dominated actions and domination by random actions

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

(Table 3)

In this example, there are two behavioral equilibria, one in pure actions, $(a_1, a_1)$ and the other random, $(\frac{1}{2}a_2 + \frac{1}{2}a_3, \frac{1}{2}a_2 + \frac{1}{2}a_3)$. Note that in the pure action equilibrium $(a_1, a_1)$ the decision-maker is choosing a weakly dominated action and at the random equilibrium $(\frac{1}{2}a_2 + \frac{1}{2}a_3, \frac{1}{2}a_2 + \frac{1}{2}a_3)$, the decision-maker is strictly better off than at $(a_1, a_1)$. Note also that there is no pure action that (strictly) dominates $a_1$ although there are a continuum of random actions $qa_2 + (1 - q)a_3, 0 < q < 1$, that strictly dominates $a_1$.

**Example 7.** Multiple welfare ranked equilibria in undominated actions

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

(Table 4)

In this example, there are two behavioral equilibria in pure undominated actions $(a_1, a_1)$ and $(a_2, a_2)$. Note that the pure action equilibrium $(a_1, a_1)$ is dominated by the pure action equilibrium $(a_2, a_2)$. Note also that there is a random behavioral equilibrium $(\frac{2}{3}a_1 + \frac{1}{3}a_2, \frac{2}{3}a_1 + \frac{1}{3}a_2)$. However, in addition, sunspots (i.e. payoff-irrelevant events) may play a role in decision-making. Suppose there are two payoff irrelevant states of the world $\{s_1, s_2\}$ with an associated probability distribution $\{\alpha, 1 - \alpha\}$. Suppose the decision-maker observes the realization of the sunspot variable before choosing her action. Then, for example, the decision-maker could choose $(a_1, a_1)$ conditional on observing $s_1$ and $(a_2, a_2)$ conditional on observing $s_2$ thus obtaining, in expected utility, payoffs in the interval $[1, 2]$. Note that all these features are robust to arbitrary but small perturbations in payoffs.

**Example 8.** More information may make the decision-maker worse-off

Consider a decision problem with payoff relevant uncertainty, with two states of the
world \{\theta_1, \theta_2\} where the payoff tables are

\begin{align*}
\theta_1 & \rightarrow \\
 & \begin{array}{ccc}
  a_1 & a_2 & a_3 \\
  a_1 & -1 & 0 & 0 \\
  a_2 & 0 & 3 & \frac{1}{2} \\
  a_3 & 1 & 4 & 1 \\
\end{array} \\
\text{(Table 5 (A))}
\end{align*}

\begin{align*}
\theta_2 & \rightarrow \\
 & \begin{array}{ccc}
  a_1 & a_2 & a_3 \\
  a_1 & 1 & 4 & 1 \\
  a_2 & \frac{1}{2} & 3 & 0 \\
  a_3 & 0 & 0 & -1 \\
\end{array} \\
\text{(Table 5 (B))}
\end{align*}

Suppose, to begin with, the decision-maker has to choose before uncertainty is resolved. At the time when she makes the decision, the individual attaches a probability \(\frac{1}{2}\) to \(\theta_1\) and \(\frac{1}{2}\) to \(\theta_2\). In this case, expected payoff matrix is

\begin{align*}
 & \begin{array}{ccc}
  a_1 & a_2 & a_3 \\
  a_1 & 0 & 2 & \frac{1}{2} \\
  a_2 & \frac{1}{2} & 3 & \frac{1}{4} \\
  a_3 & \frac{1}{2} & 2 & 0 \\
\end{array} \\
\text{(Table 5 (C))}
\end{align*}

It follows that the unique behavioral equilibrium is \((a_2, a_2)\) with expected payoff 3.

Next, suppose that the decision-maker knows with probability one the true state of the world. Then, when the state of the world is \(\theta_1\), \(a_3\) strictly dominates all other actions and the unique behavioral equilibrium is \((a_3, a_3)\) with payoff 1 and when the state of the world is \(\theta_2\), \(a_1\) strictly dominates all other actions and the unique behavioral equilibrium is \((a_1, a_1)\) with payoff 1. Therefore, the decision-maker is worse-off with more information\(^{12}\).

**Example 9. Autonomy versus non-autonomy**

Consider the payoff table in Table 2. In that example, if the decision maker took into account the feedback effect from actions to the utility parameter and maximized the induced utility function \(v(.), v(a_1) = v(a_2) = 0\). Therefore, a fully autonomous decision-maker who takes into account all the consequences of her actions would obtain a payoff of 0. However when the decision-maker doesn’t take this feedback effect into account, we have

\(^{12}\)Note that in this example we are referring only to information that solves the uncertainty about exogenous states of the world. Our statement "the decision-maker is worse-off with more information" would not be right in the case in which additional information helps the decision-maker to control her own feedback effect, i.e. to gain autonomy.

\(^{13}\)This result is consistent with Carrillo and Mariotti’s (2000) results, although they use a dynamic model with time-inconsistent preferences.
already seen that there is a unique random outcome of the behavioral decision problem 
\((\frac{1}{2}a_1 + \frac{1}{2}a_2, \frac{1}{2}a_1 + \frac{1}{2}a_2)\) with an expected payoff of \(\frac{1}{2} > 0\). On the face of it, it would seem 
that a non-autonomous decision-maker will be better-off than a fully autonomous decision-
maker. But this interpretation isn’t strictly true. In fact, if a fully autonomous decision 
maker is also allowed to choose mixed strategies in the payoff matrix in Table 2, she will 
also randomize \(\{a_1, a_2\}\) by choosing the probability distribution \(\{\frac{1}{2}, \frac{1}{2}\}\).

6 Revealed Preferences and Welfare

We turn now to the central question of the paper: what is the appropriate criterion, if any, 
for behavioral welfare analysis?

As stressed in the Introduction, some scholars like Bernheim and Rangel (2008) propose 
to continue basing judgments of welfare on choice. Others, like Kahneman et. al (1997) and 
Sugden (2004), reject choice as a foundation for normative analysis and propose alternative 
measures of individual welfare. Here, we assume that the person has a "true" interest, which 
is represented by a fix preferences over actions and psychological states when the person 
optimally chooses both. Thus, in our model, normative preferences are the preferences 
of self-1 over outcomes when the person plays the intrapersonal game a la Stackelberg. 
Thus, the preferences that should be used for welfare analysis are those over consistent 
decision states. In this section, we show that welfare judgements based on choice alone may 
not represent the individual’s best interest when the individual is not fully autonomous. 
Moreover, in such cases, the use of "happiness" or "opportunity" criterion for measuring 
individual welfare may also provide inadequate measures of wellbeing. We explain these 
observations below.

6.1 Generalized choice-based Welfare Criterion

Bernheim and Rangel (2008) (hereafter BR) and Rubinstein and Salant (2007) (hereafter 
RS) study decision problems where there is a set of actions \(A\) and frames (RS) or ancillary 
conditions (BR) \(P\) which determine choices in \(A\). BR and RS argue that a standard choice 
situation corresponds to one where an individual chooses between elements in \(A\). Both 
papers interpret elements in \(P\) as additional observable information (like "the order of 
candidates in a ballot box" (RS) or "the point of time at which a decision is made" (BR) 
which affects choices in \(A\). Both papers make the point that, in practice, it is difficult to 
draw a distinction between characteristics of elements in \(A\) and variables in \(P\) which could 
also be viewed as characteristics of elements in \(A\). In any actual decision problem studied 
in their papers, an individual takes the frame or ancillary condition as given when choosing
an action.

BR define an action $a$ to be a *weak welfare optimum* if and only if for each $a' \in A$ (other than $a$), $a$ is chosen with $a'$ present ($a'$ may be chosen as well). They also define a *strict welfare optimum* as an action $a$ if and only if for each $a' \in A$ (other than $a$), either $a$ is chosen and $a'$ is not or it is never the case that $a'$ is chosen and $a$ is not with $a$ present. Their definitions make a clear link between revealed preferences and welfare.

Our framework suggests that what matters for welfare purposes is the ranking over consistent decision states, $\Omega$. The issue is whether revealed preferences over actions can be used to rank consistent decision states as well. Examples 2 and 4 in Section 3 and 4 show that this is not always the case. In those examples, where $\pi(a) = a$ for all $a \in A$, $a_2$ is always chosen and $a_1$ is never chosen. Therefore, $a_2$ is a strict (and hence, weak) welfare optimum as defined by BR. However, the decision state $(a_2, a_2)$ is dominated by $(a_1, a_1)$ and so the individual’s revealed preferences over actions cannot be used to rank consistent decision states in these examples, and it is this latter ranking that matters for welfare assessments.

The following Proposition states a necessary and sufficient condition for revealed preferences to rank consistent decision states.

**Proposition 2.** Let $a \in A$ be a weak welfare optimum. Then, any consistent decision state containing $a$, weakly welfare dominates any other decision state containing $a' \neq a$, $a' \in A$ if only if (C1) holds.

**Proof:** See Appendix

From Proposition 2 we learn that BR’s welfare criterion only requires Condition (C1) to be recovered as an appropriate welfare criterion in our framework. The strength of this requirement will depend on the specific positive theory to which BR’s approach is applied. For example, C1 holds in positive models with reference dependent preferences like Rabin and Koszegi (2006) or the static case of Shalev’s loss averse preferences (Shalev, 2000). However, C1 does not always hold in psychological games with one active player (Geanakopolos et. al, 1989) or in models of aspirations traps (Heifetz and Minelli, 2007).

### 6.2 Non-choice-based Welfare Criteria

As shown above, if C1 does not hold, preferences revealed from choice may not represent a clear guidance for making welfare assessments in our model, even if we applied BR’s generalized criterion. Alternative criteria that base welfare on the individual’s happiness or opportunity rather than choice avoid dealing with revealed preferences’ problems. However, as it will be shown below, these alternative criteria does not seem to provide unambiguous
welfare guidance when the individuals are not fully autonomous.

6.2.1 Criterion based on Happiness or Experienced Utility

Kahneman et. al. (1997) distinguish between *experienced utility*, which is a revival of the original hedonic conception of cardinal utility proposed by Bentham in 1879, and *decision utility*, which reflects the attractiveness of options as inferred from the individual’s decisions. They claim that experienced utility is both measurable and empirically distinct from decision utility, and they propose to use the former as a relevant criterion for evaluating outcomes. Kahneman (2000) argues that experienced utility is best measured by moment-based methods that assess the experience of the present, such as self-reports of current well-being. The question that is generally asked to measure subjective well-being is: “how happy are you, overall?” The question we ask here is whether this subjective measure may differ with the type of decision problems the individual faces. We conjecture that the answer is Yes.

Take Example 4 in Section 5, with the following pay-off matrix:

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$a_2$</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Suppose the decision-maker solves this decision problem as a BDP because she does not control the feedback effect, and chooses $a_2$. If she is asked how happy she is, she may well assess her level of happiness relative to what she thinks she can achieve given a fixed reference point $a_2$, and since $0 > -1$, she may report a high subjective wellbeing. After all, she is as good as she can be, given that she is solving the game *a la* Nash. However, if the decision maker, for some reason, understands that she can solve this game *a la* Stackelberg, but she cannot, then she might report lower levels of happiness than before. This is because she would be able to compare the happiness she could get if she changed, 1, with what she is currently getting, 0. Finally, a decision maker that fully controls $\pi$ should report a higher level of happiness ($1 > 0$). This example illustrates our first observation.

**Observation 1.** *Subjective well-being measures may provide ambiguous welfare guidance in the context of a BDP. If one still wanted to use such a measure, it should then be necessary to identify how autonomous the individual is at the moment to provide an answer to the subjective well-being question.*

---

This version of the model, however, does not allow a situation in which the decision maker is aware that she could play the game in a different way but she doesn’t. This type of extension of the model will be considered for further research.
6.2.2 Criterion based on Opportunities

Sugden (2004) proposes an "opportunity criterion" to guide welfare assessments. The normative principle behind Sugden’s approach is that the size and richness of an opportunity set, that is, the set of options from which the individual is free to choose, has value. The following example illustrates that this may not be the case when a decision maker is not fully autonomous.

Example 10.

Consider first a situation where the payoff table is

\[
\begin{array}{cc}
  a_1 & a_2 \\
  a_1 & -1 & 0 \\
  a_2 & 0 & 3 \\
\end{array}
\]

(Table 6 (A))

In this case, the decision-maker has a unique efficient undominated action, \(a_2\) and there exists a corresponding outcome of the behavioral decision problem \((a_2, a_2)\) with payoff 3. Now, expand the set of choices so that the following payoff table represents the decision problem

\[
\begin{array}{ccc}
  & a_1 & a_2 & a_3 \\
  a_1 & -1 & 0 & 0 \\
  a_2 & 0 & 3 & 1 \\
  a_3 & 1 & 4 & 2 \\
\end{array}
\]

(Table 6 (B))

Note that \(a_2\) continues to strictly dominate \(a_1\) although now \(a_3\) strictly dominates both \(a_1\) and \(a_2\). The unique behavioral equilibrium is \((a_3, a_3)\) with payoff 2 < 3. This means that although the action set of the decision-maker has been expanded so that (a) the ranking of existing actions is unaffected and (b) the new action strictly dominates all existing actions, the individual is made worse-off. Note that all these features are robust to arbitrary but small perturbations in payoffs. This example illustrates our second observation:

Observation 2. Larger action sets may make the decision-maker worse-off.

Intuitively, what matters for the welfare of a non-autonomous decision maker is how she manages to take advantage of and use the opportunities she has at hand. We regard this remark as being close to the essence of Amartya Sen’s capability approach. Sen’s (1985) capability approach is a framework for the evaluation of individual welfare. Sen proposes to assess people’s welfare in terms of their functionings and capabilities. A functioning is defined as an achievement of a person, i.e. what she manages to do (or be), whereas a capability reflects the various functionings an individual can potentially achieve. The space of functionings is the space of activities or state of being and the space of capabilities is
the space of potential activities or states of being. Functionings measure realized welfare, whereas capabilities measure potential or feasible welfare. In that sense, the set of outcomes of a BDP could be interpreted as the set of functionings, whereas the set of outcomes of a SDP can be thought to be the set of capabilities. Given a Decision Problem \( D \), an individual would be able to achieve her maximum potential welfare when \( E \cap M \neq \emptyset \) and \( E \subseteq M \). Therefore, we would get back to a result analogous to Theorem 3, which would state that the individual will achieve her potentiality if and only if \( C1 \) holds (i.e. \( E \subseteq M \)). However, these are only conjectures, and the link between Sen’s (1985) work and ours need further investigation.

The discussion in this section would suggest that the autonomy criterion refines some of the most widely used normative criteria, in the sense that the autonomy criterion implies the other criteria, but the reverse is not true.

**Observation 3.** If a choice has been made by a fully autonomous individual, then the observed choice, the individual’s happiness and her opportunities can be indistinguishably used as appropriate guidance for welfare judgements. However, the reverse of this statement is not true.

Finally, it may be of interest to compute the value of individual autonomy. Take any decision problem \( D \). Let \((\hat{a}, \hat{p})\) be the outcome of the SDP, \((a^*, p^*)\) the outcome of the BDP and \( v(a, p) \) the indirect utility function. The value of the individual’s autonomy, \( \alpha \in \mathbb{R}_+ \), is defined as:

\[
\alpha = v(\hat{a}, \hat{p}) - v(a^*, p^*)
\]

When the individual is fully autonomous, \( \alpha = 0 \). A social planner aiming to maximize individual welfare should choose a policy to minimize \( \alpha \). The next section discusses some appropriate policies to achieve that aim.

### 7 Behavioral Public Policy

What are the policy implications of behavioral decision making, i.e. assuming that people don’t necessarily follow their own best interests?

The goal of any public policy ought to be to maximize people’s well-being. The route a social planner chooses to take in order to achieve that goal will depend on the social planner presumption on the way individual chooses. In this section we (a) discuss the optimal extent of public policy, (b) claim that the existing public policy approaches do not necessarily maximize people’s well-being and (c) introduce an alternative type of public policy based on empowerment.
7.1 Optimal extent of Public Policy

Before discussing what the appropriate policy interventions are, we wonder when a government intervention is necessary. In particular, is there an optimal extent of public intervention?

Take any Decision Scenario $D = (A, P, \pi)$. Let $v(a) = u(a, \pi(a))$. Now think of an individual maximizing:

$$\max_{a \in A} \mu v(a) + (1 - \mu) u(a, p)$$

where with some probability $\mu$ she takes the feedback effect from actions to preference parameters into account, and with some probability $(1 - \mu)$ she does not. Given this situation, let $\beta(p)$ denote the set of solutions to this maximization problem. An equilibrium of this problem is a decision state $(a^*, p^*)$ such that (i) given $p^*$, $a^* \in \beta(p^*)$ and (ii) $p^* = \pi(a^*)$.

The social planner’s goal is to maximize $v(a)$ choosing an action $a \in A$. Suppose that the social planner has incomplete information about the procedure the decision maker is taken, i.e. whether she is taking (Stackelberg) or not (Nash) the feedback effect into account. In this context, the social planner maximizes:

$$\max_{a \in A} \mu' v(a) + (1 - \mu') \tilde{v}(a)$$

where with some probability $\mu'$ the social planner indeed chooses the $a \in A$ that maximizes the true interest or preference of the individual, and with some probability $(1 - \mu')$ the social planner uses a completely wrong set of preferences $\tilde{v}(\cdot) \neq v(\cdot)$.

**Observation 4.** The extent of paternalism is limited by the trade-off between $\mu$ and $\mu'$. If the individual is internalizing the feedback effect with very high probability (high $\mu$) and the social planner has very few information about the individual (low $\mu'$), then there is no scope for intervention. On the other hand, if $\mu$ is low and $\mu'$ is high, then the social planner should intervene.

How this trade-off can be identified is a relevant empirical question, although it is out of the scope of this paper.

Note that, here, we have only considered the case of "hard" paternalism, in which the social planner chooses an action instead of the individual or, what is equivalent, forces the individual to choose a particular action\textsuperscript{15}. However, the social planner can design soft-

\textsuperscript{15}Examples of paternalistic policies include banning narcotics, warnings on cigarettes, public health advertising, safety regulations such as the use of helmet or seatbelts, etc.
libertarian policies that allow the individual to internalize the feedback effect. We devote the next sub-section to explore the type of policy interventions implied by our framework.

7.2 Type of Policy Interventions

Once we defined when the government ought to intervene, we turn to study how such interventions should be implemented.

Intermediate forms of soft paternalism have recently emerged as a compromise between fully libertarian and pure paternalistic views\(^{16}\). The goal of soft paternalistic policies is to guide individual’s behavior in directions that will promote individual’s welfare while minimizing coercion. Bernheim and Rangel (2008) propose the softest version of such approaches. In fact, they suggest to use a libertarian approach under most circumstances, and use non-choice data to discern which circumstances are most relevant for policy\(^{17}\).

Thaler and Sustein (2003) recommend other type of soft paternalism labelled libertarian paternalism. They claim that, in the cases in which the choice is reference-dependent (e.g. status quo bias or default option bias), the social planner should choose the reference point or default option in order to steer people’s choices in desirable directions. In that way, the social planner would achieve her goal of maximizing people’s welfare without forcing anybody to do anything they wouldn’t do. As illustrated by example 4, Section 5, this type of intervention does not necessarily maximizes people welfare.

We propose a soft-libertarian approach which stands in between a fully libertarian and a libertarian paternalistic approach. A soft-libertarian policy intervention for efficiency purposes has the following features:

(a) It is only justifiable when

(i) condition $C1$ does not hold, i.e., choices cannot be used to guide welfare judgments,

(ii) the probability that the person is fully autonomous, $\mu$, is low,

(iii) the probability that the social planer knows $v(a)$, $\mu'$, is high.

(b) the intervention should not be coercive. On the contrary, it should aim to enhance the non-cognitive abilities needed to change the way a person solves her different intrapersonal games: from Nash to Stackelberg. If the intrapersonal game is one of aspirations traps, then a soft libertarian policy should affect the capacity to aspire. Empowering the person is one example of such policy. Stern et. al. (2005) defines empowerment policies as those that help

\(^{16}\)See Loewenstein and Haisley (2008) for a review of methodological issues that arise in designing, implementing and evaluating the efficacy of "soft" paternalism.

\(^{17}\)They don’t advovate a libertarian approach when there is evidence of a malfunction of sensory, informational or computational brain processess at the time of the choice.
the person to "gain control over her own life." In the same line, Mullainathan (2006) argues that "good institutions also help to reduce problems that arises within a person." Likewise, Duflo (2006) claims that "what is needed is a theory of how poverty influences the decision making, not only by affecting the constraints, but also by changing the decision making process itself." If the intrapersonal game is one of temptation, then a soft-libertarian policy should facilitate the capacity to exert self-control. Some examples of such soft-libertarian policies can be found in Bernheim and Rangel (2005). For example, behavioural therapies that teach cue-avoidance to addicts have shown to be successful. In our lens, by encouraging the adoption of new life-styles and the development of new interests, these therapies show the addict the way to become a Stackelberg leader. As pointed out by Bernheim and Rangel (2005), "these therapeutic strategies affect addict’s choices without providing new information."

One conjecture is that this type of policies may have a more permanent impact on the individual well-being than the other existing "soft paternalistic" policies that propose changing the reference point exogenously, without helping the individual to do it herself. In fact, psychological studies show that autonomy support leads to greater program involvement, adherence and maintained change for behaviors such as smoking cessation, weight loss, glucose control and exercise (see Williams, 2002).

Finally, it is important to highlight that some standard policies that have always thought to be (at least weakly) welfare improving, may fail in our framework. For instance, as it was illustrated in Example 8 and 10, a policy that provides more information or more opportunities to a "non-autonomous" decision maker may make her worse-off. Or coming back to our example of aspirations, as Atkinson (1998) argues, creating jobs is not necessarily an effective policy to solve an aspiration failure: "ending social exclusion will depend on the nature of these new jobs. Do they restore a sense of control?"

7.3 Aspiration Traps and Extrinsic Circumstances

Another type of policy recommendation, this time only consistent with scenarios of multiple behavioural equilibria, would be to affect exogenous variables associated with the process that defines the potentially endogenous psychological states. In many cases, psychological states do not only depend on decision makers’ own actions, as we modeled in Section 2, but also on her extrinsic circumstances (e.g. relative status, social exclusion, poverty, etc.). Appadurai (2004) and Ray (2006), for example, provide an analysis of the negative impact of persistent poverty on the "capacity to aspire" and the key role that aspirations play for the poor to alter the conditions of their own poverty. Stern et. al. (2005) refers to this issue arguing that “an individual can be constrained by their aspirations and perceptions
of their role, so that development depends on relaxing these constraints.” Then he adds “to understand path out of poverty, we have to focus not only on the growth of opportunity but also on [...] internal constraint on aspirations and behaviour [...] that limit poor people’s ability to participate.” In the remainder of this section, we extend the example in section 2 to one in which extrinsic circumstances of the individual interact with her aspirations and choices. This issue is clearly important for policy purposes: when should policy address the extrinsic circumstances of an individual (like initial wealth social status, health) and when should it address the process that defines her aspirations? We provide answers for these questions.

Consider the model in section 2. Now introduce a set of extrinsic circumstances \( \Theta \), where \( \theta \in \Theta \) represents the initial wealth or social status or state of health or location or level of nutrition of the individual. Assume that \( \Theta \) is an interval in \( \mathbb{R} \). For concreteness, assume that the extrinsic circumstances of the individual represent her social status. Higher values of \( \theta \) representing more favorable social status. The individual can potentially improve her initial level of social status by choosing effort. Her new social status, \( \tilde{\theta} \), is generated by the map \( s : A \times \Theta \rightarrow \Theta \). Assume that \( s(a, \theta) \) is increasing in \( a \) and \( \theta \), effort is costly and the individual derives benefit from \( \tilde{\theta} \). As before, higher values of \( p \) correspond to higher levels of aspirations and lowers effort costs. The preferences of the individual can be represented by a utility function \( v(a, p, \tilde{\theta}) = b(\tilde{\theta}) - c(a, p) = b(s(a, \theta)) - c(a, p) \) where \( b(\tilde{\theta}) \) is the benefit the individual obtains from her new social status and \( c(a, p) \), the cost of effort, which is decreasing in \( p \) but increasing in \( a \). Again, assume that \( u(a, p, \theta') = 0 \) and \( u(a, p, \theta) \) is continuous in \( p, \theta \). Since \( s(a, \theta) \) is increasing in \( \theta \), for each \( p \), \( u(\pi, p, \theta') > u(\pi, p, \theta) \), \( \theta' > \theta \). Thus, this is a model in which actions, aspirations and extrinsic circumstances are complementary.

The For each \( p, \theta \), the individual solves the maximization problem

\[
\max_{a \in A} u(a, p, \theta)
\]

This generates an optimal action correspondence \( \alpha(p, \theta) \) and given \( \theta \), \( (a^*, p^*) \) is a behavioural equilibrium if (i) given \( \theta, p^* \), \( a^* \in \alpha(p, \theta) \) and (ii) \( p^* \in \pi(a^*, \theta) \).

Under our assumptions there is a unique solution, \( \hat{\alpha}(\theta) \), to the equation \( u(\pi, p, \theta) = 0 \) with \( \hat{\alpha}(\theta) \) decreasing in \( \theta \). Given \( \theta \), \( p \), the optimal action correspondence of the individual is determined as follows:

(i) whenever \( p < \hat{\alpha}(\theta) \), \( a = \alpha(p, \theta) \);
(ii) whenever \( p > \hat{\alpha}(\theta) \), \( \pi = \alpha(p, \theta) \);
(iii) whenever \( p = \hat{\alpha}(\theta) \), \( \{a, \pi\} = \alpha(p, \theta) \).

Given \( \theta \), let \( \underline{\alpha}(\theta) = \pi(a, \theta) \) and \( \overline{\alpha}(\theta) = \pi(\pi, \theta) \). Note that \( \underline{\alpha}(\theta) < \overline{\alpha}(\theta) \).
Let $\Theta = \{ \theta : p(\theta) < \hat{p}(\theta) \}, \Theta_0 = \{ \theta : \hat{p}(\theta) < p(\theta) \},$ and $\Theta_M = \{ \theta : p(\theta) \leq \hat{p}(\theta) \leq \bar{p}(\theta) \}$. Assume that all the three sets $\Theta, \Theta_0$ and $\Theta_M$ are non-null subsets of $\Theta$. By computation, it follows that

(i) when $\theta \in \Theta$, the unique behavioural equilibrium is $(a, p(\theta))$;
(ii) when $\theta \in \Theta_0$, the unique behavioural equilibrium is $(\bar{a}, \bar{p}(\theta))$;
(iii) when $\theta \in \Theta_M$, there are two behavioural equilibria, $(a, p(\theta))$ and $(\bar{a}, p(\theta))$.

Call $(a, p(\theta))$ a type I equilibrium and $(\bar{a}, p(\theta))$ a type II equilibrium. In a type I equilibrium, there is no change in the status quo while in a type II equilibrium there is a change in the status quo. When a type II equilibrium exists, the individual is always better off at the type II equilibrium decision state relative to the status quo. When both type I and type II equilibria exist, as the type II equilibrium dominates the status quo, a type I equilibrium can be interpreted as an aspirations failure, a low motivation trap for the individual.

The set of equilibria is "weakly increasing" in $\theta$. For an individual of low social status with low $\theta$, the unique equilibrium is type I while for an individual with high social status with high $\theta$ the unique equilibrium is type II. For an individual in the middle, with intermediate values of $\theta$, there are multiple welfare ranked equilibria. In order get round this difficulty, we use the equilibrium selection argument introduced in Section 2, that assigns a probability to each behavioural equilibrium as a function of $\theta$.

Fix $\theta \in \Theta_M$ and consider the following adaptive dynamics over $p$:

Step 1: The initial psychological state of an individual is picked at random from $[\underline{p}, \bar{p}]$ according to some continuous pdf $f(p)$ (with associated cdf $F(p)$)

Step 2: Given the $p$, the individual chooses $\alpha(p, \theta) \subset \{a, \bar{a}\}$ which, in turn, generates a new $p' = \pi(a, \theta)$, for some $a \in \alpha(p, \theta)$.

Note that the above adaptive dynamics will always converge to either a type I or a type II equilibrium. Further, note that the basin of attraction for a type I equilibrium is $[\underline{p}, \hat{p}(\theta))$ while the basin of attraction for a type II equilibrium is $(\hat{p}(\theta), \bar{p}]$. Therefore, the probability that the dynamics will converge to a type I equilibrium is $F(\hat{p}(\theta))$ while the probability that the dynamics will converge to a type II equilibrium is $1 - F(\hat{p}(\theta))$. As $\hat{p}(\theta)$ is decreasing in $\theta$, it follows that there exists a $\hat{\theta}$ such that whenever (a) $\theta < \hat{\theta}$, $F(\hat{p}(\theta)) > \frac{1}{2}$ and a type I equilibrium while will have a higher probability of emerging while (b) $\theta > \hat{\theta}$, $F(\hat{p}(\theta)) < \frac{1}{2}$ and a type II equilibrium while will have a higher probability of emerging.

The preceding discussion can be summarized in the following proposition:

**Proposition 3:** When multiple welfare ranked behavioural equilibria exist, both aspirations and choices, via equilibrium selection, can be determined as a (stochastic) function of the individual extrinsic circumstances.
From Proposition 3 we can present some important remarks for policy analysis.

First, the key point in the above equilibrium selection argument is the way the basins of attraction for each of the two equilibria change for different values of $\theta$: the size of the basin of attraction of the type I equilibrium becomes smaller relative to the size of the basin of attraction for a type II equilibrium. There is a critical value of $\theta$ below which (respectively, above which) the probability attached to the type I equilibrium is smaller (respectively, larger) than the probability attached to the type II equilibrium. Moreover, the equilibrium selection developed here is non-ergodic i.e. the initial aspiration level determines where the adaptive dynamics ends up. Therefore, the process by which the initial aspiration level is determined, $F(p)$, is of critical importance.

Second, both $\theta$ and/or $F(p)$ can interpreted as a characteristic of the individual being studied. For example an individual who has a low social status but has the right motivation could tend to do better than another low status individual with low motivation. From a policy perspective, the relevant instruments will be both $\theta$ and/or $F(p)$. For example changes in $\theta$ could correspond to things like changes in initial wealth (social status, health, location, nutrition, housing etc.) of an individual while changes in $F(p)$ could correspond to process by which the initial aspirations levels are generated. The formal analysis suggests that direct attempts to change the extrinsic circumstances (by, for example, enhancing the economic status via transfers of wealth) will be welfare enhancing for very poor individuals while for individuals with intermediate wealth levels, policy interventions that directly impact probability with initial aspirations are generated will also be welfare improving. In this sense, the argument presented here distinguishes between absolute and relative deprivation and makes a case for different policy interventions in the two cases.

8 Psychological and Philosophical grounds for our Premises

Our framework relies on three key conceptual ideas. First, there is a feedback effect from actions to preference parameters that may not be fully internalized by the decision maker. Second, the individual’s best interest is defined in the space of outcomes only when the feedback effect is internalized. Third, the individual always chooses what she judges best for her. In this section, we briefly review part of the literature in social psychology and the moral philosophy that supports these conceptual ideas.

On the social psychology front, there is extensive work led by Albert Bandura who views human functioning as the product of a dynamic interplay of personal, behavioral, and environmental influence. Bandura points out that the way in which people interpret the results of their own behaviour informs and alters their environments and personal factors.
which, in turn, inform and alter subsequent behaviour through an "environmental feedback effect." He labelled this view “reciprocal determinism” (see e.g. Bandura, 1986, 1997, 2001). In line with Bandura’s theory, there is a great deal of work favouring the hypothesis that the individual’s actions may affect preference parameters. For example, Lazarus and Folkman (1984) argue that people are able to cope with stress, anger or anxiety, by changing their response to a situation (emotion-focused problem) or by changing the environment (problem-focused coping). Baron (2008, pg. 68) shows that emotions are partly under our control and argues that individuals can "induce or suppress emotions in themselves almost on cue." Baron argues that some people may even reshape their character, so that their emotional responses change. In a similar vein, William James (1890/1981) used the term "self-esteem" to refer to the way individuals feel about themselves which in turn depends on the success they have to accomplish those things that they wish to accomplish (in Pajares and Schunk, 2001, 2002).

On the philosophical front, the state of acting against one’s better judgment has been studied since Plato and it has been labelled “Akrasia18". In the dialogue, Socrates sustains that “akrasia” is an illogical moral concept, claiming “No one goes willingly toward the bad” (358d). If a person examines a situation and decides to act in the way he determines to be best, he will actively pursue this action. In accordance to the normative principle advocated in this paper, Socrates postulated that an all-things-considered assessment of the situation will bring full knowledge of a decision’s outcome and worth linked to well-developed principles of the good. Donald Davidson (1980), a contemporary American philosopher, argued that when people act in “akrasia” they temporarily believe that the worse course of action is better, because they have not made an all-things-considered judgment, but only a judgment based on a subset of possible considerations.

The concept of personal autonomy has been subject of study specially in the literature of philosophy (Friedman, 2003) and psychology (Ryan and Deci, 2006). As Ryan and Deci (2006) point out, an act to be autonomous it must be endorsed by the self, fully identified with and owned. They also stress that "autonomy is not defined by the absence of external influences but rather by one’s assent to such influences or inputs. Autonomy is thus not equivalent to independence." (pg. 1561). When autonomously functioning, people are more deeply engaged and productive, generating human capital and welfare (Woo, 1984). Ekstrom (2005) and Kernis and Goldman (2005) stress that autonomous acts proceed from one’s core self, representing those preferences and values that are wholeheartedly endorsed. Dworkin (1988) maintains that people are autonomous only to the extent that their first order motives are endorsed at a higher order of reflection. As Ryan and Deci

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18In ancient Greek: Akrasia means “lacking command” (over oneself)
(2006) illustrate, a man who decides to "have another drink" would not be autonomous unless, in reflecting on this motive, he could fully endorse it. A lack of full endorsement would imply that the act is not autonomous. Moreover, Dworkin (1988) underscores that autonomy does not require behaving without or against constraints. For example, "although one might feel constrained in stopping for school bus, if one assents to the value of traffic laws for ensure children’s safety, one could willingly consent to the constraint and, in doing so, lose no autonomy."

As for the empirical part, self-determination theory (SDT) in psychology provides a comprehensive picture of the importance of autonomy for well-being. Autonomy is considered a basic psychological need.

9 Final Remarks

To summarize, this paper offers a simple and general framework that contributes to normative and positive behavioral economics.

It unifies seemingly disconnected models in the literature, from more recent positive behavioural economics models to older models of adaptive preferences. The generality of our framework makes it a natural one to study general (positive and normative) properties of behavioural economics models. Second, the paper proposes a new equilibrium existence result in pure actions without complete and transitive preferences. A result like that is important on its own, since incomplete and non-transitive preferences are a common token in behavioral economics models. Third, it shows that the behavioural decisions are falsifiable, which respond to some concern in the literature that behavioural models may not be falsifiable. In addition, we show that standard and behavioural decision problems have different testable implications. This is an important step towards the answer of a more ambitious question on whether we could test if a person is solving a decision problem in an autonomous or a non-autonomous way. Fourth, it shows that in almost all the cases, behavioural and standard decisions are distinguishable from each other. Fifth, it shows that revealed preferences cannot, in general, underpin welfare, though we do offer conditions validating such a link. When choices fail to reveal true preferences, the paper argues that normative evaluations should be based on the degree of autonomy of the person, defined as the extent into which the person controls her own psychological states. A policy aiming at improving individual welfare should then look for developing people’s non-cognitive abilities needed to make autonomous decisions. Examples of such non-cognitive abilities are capacities to aspire, to exert self-control or to correct (wrong) beliefs. Although the concept of autonomy has been extensively studied in the literature of philosophy and psychology,
this paper is, to our knowledge, the first to formally introduce autonomy into an economic decision-theoretic framework. Our hope is that this framework allows economists to engage on policy concerns like empowerment or emotional intelligence that are largely addressed in other disciplines but disregarded in the economics literature.

The paper leaves unanswered many questions about theory, data and policy. We hope, however, that this work has been sufficient to suggest that the line of enquiry advocated here is a promising one, and that will stimulate new undertakings.

We imagine several directions for fruitful future research. On the theoretical front, it would be interesting to explore if different behavioural decision problems, associated with the same underlying standard decision problem, have distinct testable implications. Also, it would be possible to introduce the notion of "empathy" into game theory by extending our framework to an N-person strategic context. The capacity to empathize, that is, to simulate others' internal decision process, should help players to predict and understand other's actions and intentions, with important economic implications. On the empirical front, the challenge is to identify the decision making process, and to distinguish when the person is being autonomous or non-autonomous. It would be important to identify policies that change the way people play their intrapersonal games. This definitively opens new routes to explore in experimental and Neuroeconomics.

References


Appendix

Theorem 1: Existence Result

Recall that the preferences of the decision-maker is denoted by \( \succeq \) a binary relation ranking pairs of decision states in \( (A \times P) \times (A \times P) \). As the focus is on incomplete preferences, in this section, instead of working with \( \succeq \), we find convenient to specify two other preference relations, \( \succcurlyeq \) and \( \sim \). The expression \( \{ (a, p), (a', p') \} \in \succcurlyeq \) is written as \( (a, p) \succcurlyeq (a', p') \) and is to be read as "(a, p) is strictly preferred to (a', p') by the decision-maker". The expression \( \{ (a, p), (a', p') \} \in \sim \) is written as \( (a, p) \sim (a', p') \) and is to be read as "(a, p) is indifferent to (a', p') by the decision-maker". Define

\[
(a, p) \succeq (a', p') \iff \text{either } (a, p) \succ (a', p') \text{ or } (a, p) \sim (a', p').
\]

Once \( \succeq \) is defined in this way, the results obtained in the preceding sections continue to apply.

Suppose \( \succ \) is

(i) acyclic i.e. there is no finite set \( \{ (a^t, p^t), ..., (a^1, p^1) \} \) such that \( (a^{t-1}, p^{t-1}) \succ (a^t, p^t) \), \( t = 2, ..., T \), and \( (a^T, p^T) \succ (a^1, p^1) \), and

(ii) \( \succ^{-1} \) \( (a, p) = \{ (a', p') \in A \times P : (a, p) \succ (a', p') \} \) is open relative to \( A \times P \) i.e. \( \succ \) has an open lower section\(^{19} \). Suppose both \( A \) and \( P \) are compact. Then, by Bergstrom (1975), it follows that \( M \) is non-empty.

Define

\[
a \succ_p a' \iff (a, p) \succ (a', p).
\]

The preference relation \( \succ_p \) is a map, \( \succ : P \rightarrow A \times A \). If \( \succ \) is acyclic, then for \( p \in P \), \( \succ_p \) is also acyclic. If \( \succ \) has an open lower section, then \( \succ_p^{-1} \) \( (a') = \{ a' \in A : a \succ a' \} \) is also open

\(^{19}\)The continuity assumption, that \( \succ \) has an open lower section, is weaker than the continuity assumption made by Debreu (1959) (who requires that preferences have both open upper and lower sections), which in turn is weaker than the assumption by Shafer and Sonnenschein (1975) (who assume that preferences have open graphs). Note that assuming \( \succ \) has an open lower section is consistent with \( \succ \) being a lexicographic preference ordering over \( A \times P \).
relative to $A$ i.e. $\succ_p$ has an open lower section. In what follows, we write $a' \not\succ_p (a)$ as $a \not\succ_p a'$ and $a' \in\succ_p (a)$ as $a' \succ_p a$.

Define a map $\Psi : P \to A$, where $\Psi(p) = \{a' \in A : a' \not\succ_p (a') = \emptyset\}$: for each $p \in P$, $\Psi(p)$ is the set of maximal elements of the preference relation $\succ_p$.

We make the following additional assumptions:

(A1) $A$ is a compact lattice;

(A2) For each $p$, and $a, a'$, (i) if $\inf(a, a') \not\succ_p a$, then $a' \not\succ_p \sup(a, a')$ and (ii) if $\sup(a, a') \not\succ_p a$ then $a' \not\succ_p \inf(a, a')$ (quasi-supermodularity);

(A3) For each $a \geq a'$ and $p \geq p'$, (i) if $a' \not\succ_{p'} a$ then $a' \not\succ_p a$ and (ii) if $a \not\succ_{p'} a'$ then $a \not\succ_p a'$ (single-crossing property)$^{20}$

(A4) For each $p$ and $a \geq a'$, (i) if $\succ_p (a') = \emptyset$ and $a' \not\succ_p a$, then $\succ_p (a) = \emptyset$ and (ii) if $\succ_p (a) = \emptyset$ and $a \not\succ_p a'$, $\succ_p (a') = \emptyset$ (monotone closure).

Assumptions (A2)-(A3) restate, for the case of incomplete preferences, the assumptions of quasi-supermodularity and single-crossing property defined by Milgrom and Shannon (1994). Assumption (A4) is new.

The role played by assumption (A4) in obtaining the monotone comparative statics with incomplete preferences is clarified by the following examples. In all these examples, $P$ is single valued and $A$ is the four point lattice in $\mathbb{R}^2 \{((e, e), (f, e), (e, f), (f, f))\}$ where $f > e$.

**Example 12.** Suppose that $(f, f) \succ (e, e)$ but otherwise no other pair is ranked. Then, $\Psi$ consists of $\{(f, e), (e, f), (f, f)\}$ clearly not a lattice. Note that in this case, $\succ$ satisfies acyclicity (and transitivity) and quasi-supermodularity (and trivially, single-crossing property). However, $\succ$ doesn’t satisfy monotone closure: $(f, e) \succeq (e, e)$, with $\succ ((f, e)) = \emptyset$ and $(f, e) \not\succ_p (e, e)$, but $\succ ((e, e)) \neq \emptyset$.

**Example 13.** Suppose that $(f, f) \succ (e, e)$, $(f, e) \succ (e, e)$, $(e, f) \succ (e, e)$ but otherwise no other pair is ranked. Then, $\Psi$ again consists of $\{(f, e), (e, f), (f, f)\}$ clearly not a lattice. Note that in this case, $\succ$ satisfies acyclicity and monotone closure but not quasi-supermodularity.

**Example 14.** Suppose that $(f, f) \succ (e, e)$, $(f, e) \succ (e, e)$, $(e, f) \succ (e, e)$, $(f, f) \succ (f, e)$, $(f, e) \succ (e, f)$ but the pair $\{(f, f), (e, f)\}$ is not ranked. Note that $\succ$ satisfies acyclicity but not transitivity and also quasi-supermodularity, monotone closure. In this case, again $\Psi$ consists of the singleton $\{(f, f)\}$.

Example 12 demonstrates that with incomplete preferences, quasi-supermodularity on its own, is not sufficient to ensure that the set of maximal elements of $\succ$ is a sublattice

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20 For any two vectors $x, y \in \mathbb{R}^K$, the usual component-wise vector ordering is defined as follows: $x \geq y$ if and only if $x_i \geq y_i$ for each $i = 1, \ldots, K$, and $x > y$ if and only if both $x \geq y$ and $x \neq y$, and $x \gg y$ if and only if $x_i > y_i$ for each $i = 1, \ldots, K$. 

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of $A$ even when $>$ is acyclic (and transitive). Example 12 also demonstrates that $>$ can satisfy transitive but not monotone closure. Example 14 demonstrates that $>$ can satisfy monotone closure but not transitivity. Therefore, monotone closure and transitivity are two distinct conditions. Example 13 demonstrates that monotone closure without quasi-supermodularity cannot, on its own, ensure that the set of maximal elements of $>$ is a sublattice of $A$.

The following result shows that assumptions (A1)-(A4), taken together, are sufficient to ensure monotone comparative statics with incomplete preferences and ensure the non-emptiness of $E$.

**Theorem 1.** Under assumptions (A1)-(A4), each $p \in P$, $\Psi(p)$ is non-empty and a compact sublattice of $A$ where both the maximal and minimal elements, denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively, are increasing functions on $P$. Moreover, $E \neq \emptyset$.

**Proof:** By assumption, for each $p$, $\succ_p$ is acyclic, $\succ_p^{-1}(a)$ are open relative to $A$ and $A$ is compact. By Bergstrom (1975), it follows that $\Psi(p)$ is non-empty. As Bergstrom (1975) doesn’t contain an explicit proof that $\Psi(p)$ is compact, an explicit proof of this claim follows next. To this end, note that the complement of the set $\Psi(p)$ in $A$ is the set $\Psi^c(p) = \{a' \in A : \succ_p (a') \neq \emptyset\}$. If $\Psi^c(p) = \emptyset$, then $\Psi(p) = A$ is necessarily compact. So suppose $\Psi^c(p) \neq \emptyset$. For each $a' \in \Psi^c(p)$, there is $a'' \in A$ such that $a'' \succ_p a'$. By assumption, $\succ_p^{-1}(a'')$ is open relative to $A$. By definition of $\Psi(p)$, $\succ_p^{-1}(a'') \subset \Psi^c(p)$.

Therefore, $\succ_p^{-1}(a'')$ is a non-empty neighborhood of $a' \in \Psi^c(p)$ and it is clear that $\Psi^c(p)$ is open and therefore, $\Psi(p)$ is closed. As $A$ is compact, $\Psi(p)$ is also compact. Next, I show that for $p \geq p'$ if $a \in \Psi(p)$ and $a' \in \Psi(p')$, then $\sup(a, a') \in \Psi(p)$ and $\inf(a, a') \in \Psi(p')$. Note that as $a' \in \Psi(p')$, $\inf(a, a') \not\succ_{p'} a'$. By part (i) of quasi-supermodularity, it follows that $a \not\succ_{p'} \sup(a, a')$. By part (i) of single-crossing, it follows that $a \not\succ_p \sup(a, a')$. As $a \in \Psi(p)$, $\succ_p (a) \neq \emptyset$ and therefore, by part (i) of monotone closure, $\succ_p (\sup(a, a')) \neq \emptyset$ and therefore, $\sup(a, a') \in \Psi(p)$. Next, note that as $a \in \Psi(p)$, $\sup(a, a') \not\succ_{p'} a$. By part (ii) of single-crossing, it follows that $\sup(a, a') \not\succ_{p'} a$ and by part (ii) of quasi-supermodularity, $a' \not\succ_{p'} \inf(a, a')$. As $a' \in \Psi(p')$, $\succ_{p'} (a') \neq \emptyset$, and by part (ii) of monotone closure, as $a' \not\succ_{p'} \inf(a, a')$, $\succ_{p'} (\inf(a, a')) \neq \emptyset$ and therefore, $\inf(a, a') \in \Psi(p')$. Therefore, (i) $\Psi(p)$ is ordered, (ii) $\Psi(p)$ is a compact sublattice of $A$ and has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{a}(p)$ and $\underline{a}(p)$, and (iii) both $\bar{a}(p)$ and $\underline{a}(p)$ are increasing functions from $P$ to $A$.

Define a map $\Psi : A \times P \rightarrow A \times P$, $\Psi(a, p) = (\Psi_1(p), \Psi_2(a))$ as follows: for each $(a, p)$,

\[
\Psi_1(p) = \{a' \in A : \succ_p (a') = \emptyset\} \quad \text{and} \quad \Psi_2(a) = \pi(a).
\]

By Theorem 2, $\Psi_1(p)$ is non-empty and compact and for $p \geq p'$ if $a \in \Psi_1(p)$ and $a' \in \Psi_1(p')$, then $\sup(a, a') \in \Psi_1(p)$ and $\inf(a, a') \in \Psi_1(p')$. It follows that $\Psi_1(p)$ is ordered and hence a compact (and consequently,
complete) sublattice of $A$ and has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{a}(p)$ and $\underline{a}(p)$ respectively. By assumption 1, it also follows that for each $a$, $\pi(a)$ has a maximal and minimal element (in the usual component wise vector ordering) denoted by $\bar{\pi}(a)$ and $\underline{\pi}(a)$ respectively. Therefore, the map $(\bar{a}(p), \bar{\pi}(a))$ is an increasing function from $A \times P$ to itself and as $A \times P$ is a compact (and hence, complete) lattice, by applying Tarski's fix-point theorem, it follows that $(\bar{a}, \bar{\pi}) = (\bar{a}(p), \bar{\pi}(a))$ is a fix-point of $\Psi$ and by a symmetric argument, $(\underline{a}(p), \underline{\pi}(a))$ is an increasing function from $A \times P$ to itself and $(\underline{a}, \underline{\pi}) = (\underline{a}(p), \underline{\pi}(a))$ is also a fix-point of $\Psi$; moreover, $(\bar{a}, \bar{\pi})$ and $(\underline{a}, \underline{\pi})$ are respectively the largest and smallest fix-points of $\Psi$. ■

A different approach to the existence of equilibrium would be to deduce the existence result for games with incomplete preferences from the standard existence result for games with complete preferences as in Bade (2005). We refer the reader to Ghosal (2007) for details as to why this approach will not work, in general, with the incomplete preferences.

Schofield (1984) shows that if action sets are convex or are smooth manifolds with a special topological property, the (global) convexity assumption made by Shafer and Sonnenschein (1975) can be replaced by a "local" convexity restriction, which, in turn, is equivalent to a local version of acyclicity (and which guarantees the existence of a maximal element). However, here, as action sets are not necessarily convex and are allowed to be a collection of discrete points, Schofield's equivalence does not apply.

Proof of Theorem 2

(i) Suppose $(a, p) \in E$. By definition, for all $a' \in A, a \succeq_p a'$ for some $p = \pi(a)$. By (C1), for all $a' \in A$, $(a, p) \succeq (a', p')$ for each $p = \pi(a)$ and $p' = \pi(a')$. It follows that $(a, p) \in M$. Next, suppose, by contradiction, $(a, p) \in E \cap M$ but (C1) doesn't hold. As $(a, p) \in E$, for all $a' \in A, a \succeq_p a'$ for $p = \pi(a)$. As, by assumption, (C1) doesn't hold there exists $a' \in A$ such that $a \succeq_p a'$ for $p = \pi(a)$ but $(a, p) \prec (a', p')$ for $p = \pi(a)$ and $p' = \pi(a')$. But, then, $(a, p) \notin M$, a contradiction. (ii) Suppose $(a, p) \in M$. As $(a, p) \succeq (a', p')$ for all $(a', p') \in A \times \pi(A)$, by (C2), $(a, p) \succeq (a', p)$ for $p = \pi(a)$. It follows that $(a, p) \in E$. Next, suppose, by contradiction, $(a, p) \in M \cap E$ but (C2) doesn't hold. As $(a, p) \in M$, $(a, p) \succeq (a', p')$ for all $(a', p') \in A \times \pi(A)$. As, by assumption, (C2) doesn’t hold, there exists $a' \in A$ such that $a' \succ_p a$ for $p = \pi(a)$. But, then, $(a, p) \notin E$, a contradiction. ■

Proof of Proposition 2

Suppose for each $a' \in A$ (other than $a$), $a$ is chosen with $a'$ present ($a'$ may be chosen as well). By assumption, for all $a' \in A, a \succeq_p a'$ for $p = \pi(a)$. By (C1), for all $a' \in A$, $(a, p) \succeq (a', p')$ for each $p = \pi(a)$ and $p' = \pi(a')$. It follows that any consistent decision state containing $a$ weakly welfare dominates any other decision state containing $a' \neq a, a' \in A$. Next, suppose, by contradiction, for each $a' \in A$ (other than $a$), $a$ is chosen with $a'$ present
(a’ may be chosen as well), but (C1) doesn’t hold. By assumption, for all a’ ∈ A, a ≥_p a’ for p = π(a). As (C1) doesn’t hold, there exists a’ ∈ A such that a ≥_p a’ for p = π(a) but (a, p) ≺ (a’, p’) for p = π(a) and p’ = π(a’), a contradiction. ■