Economic Theory **18**, 661–681 (2001)

Economic Theory

Secret information acquisition in Cournot markets*

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Received: August 9, 1999; revised version: May 31, 2000

Summary. Two-stage game models of information acquisition in stochastic oligopolies require the assumption that firms observe the precision of information chosen by their competitors before determining quantities. This paper analyzes secret information acquisition as a one-stage game. Relative to the two-stage game firms are shown to acquire less information. Policy implications based on the two-stage game yield, therefore, too high taxes or too low subsidies for research activities. For the case of heterogeneous duopoly we briefly discuss comparative statics results.

Keywords and Phrases: Information acquisition, Oligopoly, Uncertainty, Bayesian equilibrium.

JEL Classification Numbers: C72, D43, D82.

1 Introduction

It seems a commonplace that information is valuable. Good information facilitates good decisions. Since information can be ignored it seems obvious that the value of information must be positive. While this reasoning is valid in a one-person decision problem, it is not in the context of multi-person decision problems. Akerlof's (1970) seminal paper on the market for lemons has shown that information may hurt. A seller who has private information about the quality of his second-hand car may be unable to find a buyer, because buyers will

* We thank Antonio Cabrales, Andreu Mas-Colell, Fabrizio Zilibotti and an anonymous referee for helpful comments. Esther Hauk gratefully acknowledges financial support from DGES grant PB96-0118. Sjaak Hurkens gratefully acknowledges financial support from EC grant ERBCHGCT 93-0462, DGES grant PB96–0302, and CIRIT, Generalitat de Catalunya, grant 1997SGR 00138.
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believe that only bad quality cars will be sold. In this case information seems to have negative value. However, it is not the information about the car itself that hurts the seller, it is the fact that buyers *know* that sellers are privately informed and react accordingly by not buying. Hence, the information of the buyers about the informedness of the sellers is doing the damage. We propose to distinguish between the *informational value* and the *strategic value* of information. The informational value should be interpreted as the direct gain obtained from taking a better decision due to superior information, taking the decisions of all other agents as given. The strategic value refers to the indirect gain (or loss) caused by the reaction of other agents to the presence of private information. As suggested by the terminology the informational value is always positive, while the strategic value can either be positive or negative. The lemons market is a clear example where the strategic value of information is negative and even outweighs the informational value.

In Akerlof's story private information is exogenously given. In other situations agents can decide whether or not to acquire information. One could think of several scenarios: (i) firms engage in market research to estimate demand; (ii) stock brokers read financial reports of firms; (iii) bidders in auctions go to viewing days in order to get some idea of the value of the items on sale; (iv) potential entrants investigate the profitability of new markets. With endogenous information acquisition agents will have to trade off the cost of information against its benefit. Should an agent include the strategic value of information in this trade-off, or should he only take into account the informational value? He should include the strategic value only if the decision to get information affects the behavior of the other agents. This is only the case when the other agents observe the information acquisition decision. If, on the other hand, the other agents do not observe this decision, then his decision cannot affect their behavior, and, therefore, he should ignore the strategic value of information.

In the existing literature endogenous information acquisition has mostly been modeled by adding a stage before the "true" game (Cournot competition, investment game, bidding, entry game) is played. This modeling technique implies that the information acquisition decisions are *perfectly observable* and become *common knowledge*. In our opinion, this observability assumption is rather strong: at least in some circumstances it is not clear how a firm would be able to perfectly observe the information acquisition decisions of its competitors. Disregarding espionage[1], one possibility is that all firms publicly announce their information acquisition decisions. However, why should they announce the truth? If the strategic value of information is positive a firm will want to exaggerate its informedness while it would like to hide its informedness if it has negative strategic value. The structure of the widely used two-stage game implicitly assumes that firms will tell the truth. It seems, therefore, that the two-stage game is not always the most appropriate way to model information acquisition. We suggest to

[1] Recall that information acquisition decisions become public. In particular, each firm knows that its competitors know how well informed it is. This would imply that the presence of spies is also public knowledge.

use a one-stage game where information is gathered secretly. Given that in such a game the information acquisition decision is not observable, it cannot affect the behavior of other agents. Hence, unlike in the two-stage game, the trade-off between the cost and the benefit of information does not include the strategic value of information.

In this paper we compare the two-stage and one-stage game of information acquisition in the context of a Cournot market with stochastic demand. The two-stage game has been extensively studied by Chang and Lee (1992), Daughety and Reinganum (1994), Hwang (1993, 1995), Li et al. (1987), Ockenfels (1989), Ponssard (1979) and Vives (1988). To our knowledge this paper is the first to consider secret information acquisition in Cournot markets.[2] We show that in the two-stage game firms acquire more information than in the one-stage game. This means that the strategic value of information is positive in this context. The underlying intuition is as follows: firms gather information because they want to estimate residual demand. Raising the precision of information has two direct benefits, which are present in both models of information acquisition. It reduces the prediction errors of the intercept of demand and of the signals received by the competitors. Hence, both the demand curve and the production levels of the competitors can be estimated more accurately. This is the informational value of information. In the two-stage model there is the additional strategic value. If one firm raises its precision of information (in the first stage) other firms will react less aggressively to their own signal (in the second stage). This implies that the competitors' quantities will be predicted more precisely.

To illustrate this point, consider a Cournot duopoly model similar to Ponssard's (1979) where information is either learned perfectly or not at all at cost c and demand d is either high (h) or low (l) with equal probility, i.e. average demand is $a = (h + l)/2$. The price is $p = d - q$, where q is the aggregate production. Consider the situation in which one firm is informed and the other firm is uninformed. We will show that for some information costs this is an equilibrium outcome in the one-stage game but not in the two-stage game. In this situation the uninformed firm produces $a/3$ (making profits $a^2/9$) while the informed firm produces $(d - a/3)/2 = a/3 + \frac{1}{2}(d - a)$ which reveals that its responsiveness to its own signal is $\frac{1}{2}$. If the uninformed firm secretly acquires information, it would produce $\frac{1}{2}(d - (d - a/3)/2)$ (as the other firm does not adjust its production) increasing its profits to $a^2/9 + Var(d)/16$On the other hand, if this deviation is observed, the other firm will adjust its production to $d/3 = a/3 + \frac{1}{3}(d - a)$. Since $\frac{1}{3} < \frac{1}{2}$ the other firm responds now less aggressively with respect to its signal. The deviator then optimally produces $d/3$ and receives profits equal to $a^2/9 + Var(d)/9$. For intermediate costs of information, namely $c \in (Var(d)/16, Var(d)/9)$, the uninformed firm would deviate in the two-stage game, but not in the one-stage game.

[2] Hwang (1995) stated that it would be desirable to analyse the one-stage game, while Ponssard (1979) mistakenly claimed that it did not matter. Hurkens and Vulkan (1996) considered secret information acquisition in an entry game, while Matthews (1984) and Persico (1997) studied secret information acquisition in auctions.

The extra information gathering in the two-stage game is due to the positive strategic value of being known to be informed. Nevertheless, the positive strategic value does not imply higher payoffs in the two-stage game than in the one-stage game. The additional benefit of information acquisition enjoyed by a firm in the two-stage game induces it to acquire more information which imposes a negative externality on other firms and lowers their profits. In fact, as will be shown in section 3, equilibrium payoffs of the two-stage game are strictly lower than those of the one-stage game.

This provides an argument why firms would not like to play the two-stage game even if they could choose to do so. To illustrate this point, suppose it were possible for firms to credibly announce their level of information acquisition.[3] Because of the positive strategic value they would acquire more information and announce it. This would lead to the two-stage game outcome. The precision announcement game is a type of prisoner's dilemma: each firm prefers to gather more information and announce it, but when all firms do that, they are all worse off. In a repeated game framework the Pareto dominant equilibrium of no-announcement corresponding to the one-stage game payoffs can be supported as a subgame perfect equilibrium. Given the higher equilibrium payoffs of the one-stage game firms will consciously choose not to reveal the precision of their information.

Information acquisition does not only affect firms' profits, it is also important for social welfare, because it allows demand and supply to be matched better. On the other hand, too much duplication of costly research is socially undesirable. A government may want to implement the socially efficient level of market research. However, policy measures based on the two-stage game will be biased in the direction of lower subsidies or higher taxes, compared to policy based on the (in our view) more appropriate one-stage model. In fact, we show that even the direction of policy can be overturned: For a certain range of parameters the one-stage game will advocate subsidies while the two-stage game will support taxes.

The rest of the paper is organized as follows. In Section 2 we present our general model in which firms can choose their information acquisition from a continuum and where firms may have different cost functions. We present our main result that firms acquire less information when information acquisition is not observed. Section 3 analyzes the special case of homogeneous oligopoly and reconsiders the models of Vives (1988) and Li et al. (1987). We derive the explicit expression for the equilibrium amount of information gathering. It is shown that equilibrium payoffs in the one-stage game are strictly higher than in the two-stage game. We reinforce Vives'(1988) result, that competitive markets, seen as the limit of finite Cournot markets when the number of firms goes to infinity, are second best efficient. In finite oligopolistic markets, however, firms

[3] Austen-Smith (1994) discusses the possibility of announcing informedness. In his model informed agents can credibly announce that they are informed but uninformed agents cannot prove that they are uninformed. In our model it would be necessary to credibly announce one's quality of information which seems highly problematic.

may over- or underinvest in information acquisition. Conditions are stated under which firms underinvest relative to the social optimum. Finally, it is shown that policy implications derived from the different models of information acquisition may give qualitatively different results. In section 4 we briefly discuss the special case of heterogeneous duopoly. Section 5 concludes and discusses some of the other models of information acquisition that have employed a two-stage model.

2 The model

We will set up a general model of information acquisition in Cournot markets such that the models of Vives (1988), Li et al. (1987) (symmetric firms) and Hwang (1993) (asymmetric duopoly) can be considered as special cases.

There are $n \geq 2$ firms. The inverse demand function is given by $p = \theta - \beta_n \sum_{j=1}^{n} x_j$, where x_j is the output of firm j, $\beta_n > 0$ is a constant and θ is a random parameter with mean μ and variance σ^2. Firm i has a cost function $C_i(x_i) = c_i x_i + \lambda_i x_i^2$, where $c_i \geq 0$, $\lambda_i \geq 0$. Each firm i can buy information of certain precision $1/\upsilon_i$ about demand. This means that firm i will receive a signal $s_i = \theta + \varepsilon_i$ where ε_i is a noise term with zero mean, variance υ_i and with $Cov(\theta, \varepsilon_i) = 0$. The signals received by the firms are independent conditional on θ and furthermore it is assumed that $E(\theta \mid s_i)$ is affine in s_i. These assumptions imply that $E(\theta \mid s_i) = E(s_j \mid s_i) = \mu + t_i(s_i - \mu)$, where $t_i = \sigma^2/(\sigma^2 + \upsilon_i)$. Note that as υ_i ranges from 0 to infinity, t_i ranges from 1 to 0. Instead of working with υ_i, we shall work with t_i and refer to t_i as the precision of information. We assume that the cost of information acquisition is linear in $1/\upsilon_i$, or equivalently, information of precision t_i costs

$$C(t_i) = \frac{ct_i}{\sigma^2(1 - t_i)}.$$

A typical example of such an affine information structure is for θ and ε_i to be Normal. s_i could be the average of n_i observations from a Normal distribution with mean θ and fixed variance. The precision of information $1/\upsilon_i$ is proportional to the number of observations. When the marginal cost of an extra observation c is constant, the cost of information will be linear in $1/\upsilon_i$. [See Vives (1988) for a description of a number of other distributions that define an affine information structure.]

Let Γ_1 denote the one-stage game of information acquisition. A strategy for firm i in this game is a pair $(t_i, x_i(\cdot))$, where $t_i \in [0, 1)$ is the choice of precision and where $x_i(\cdot)$ maps private signals into quantities. Let Γ_2 denote the two-stage game. In this game a strategy for firm i is a pair $(t_i, y_i(\cdot, \cdot))$, where t_i again denotes the precision of information and where $y_i(t, s_i)$ denotes the quantity produced by i in case it receives signal s_i while firms have chosen to acquire information according to the tuple $t = (t_1, \cdots, t_n)$. Let $\Gamma_2(t)$ denote the second stage continuation game of Γ_2 where precision tuple t was chosen in the first stage. It is quite straightforward to solve for the Perfect Bayesian Nash equilibrium of Γ_2. First one solves for the (unique) Bayesian Nash equilibrium of $\Gamma_2(t)$, for all t. Substitution of the equilibrium payoffs $\pi_i(t) - C(t_i)$ reduces the two-stage game to a game where only precision levels have to be chosen. This gives rise to reaction functions, and the equilibrium level of information is found by computing the intersection of these reaction functions, or, if the solution is interior, by equating the marginal value of information with its marginal cost:

$$\frac{\partial \pi_i}{\partial t_i} = C'(t_i) \text{ (all } i\text{)}.$$

At first sight it seems that solving Γ_1 will be more complicated. The precision of information t_i and the action function $x_i(\cdot)$ have to be chosen simultaneously. Hence, it is not possible to work with reaction functions. However, a first order condition approach can be used. In fact, it will turn out that this is easier for the one-stage game than for the two-stage game. In particular, an explicit solution for Γ_1 will be derived whereas the solution for Γ_2 can be given only implicitly. The key observation is that if (t, x) is a pure Nash equilibrium of Γ_1, it must be the case that x is an equilibrium of $\Gamma_2(t)$. This reduces the number of candidate solutions of Γ_1 considerably. Only these candidate solutions have to be checked against unilateral deviations (t_i', x_i').

The continuation game

As outlined above, the first step in the analysis of both Γ_1 and Γ_2, focuses on the continuation games $\Gamma_2(t)$. From the previous literature it is known that the equilibrium strategies in each continuation game are affine with respect to the signal. Instead of proving this here, we will impose that all quantity choice functions are affine.

Let $t = (t_1, \cdots, t_n)$ be a tuple of information precisions and consider the continuation game $\Gamma_2(t)$. Fix strategies $x_j(s_j) = a_j(s_j - \mu) + b_j$ for all $j \neq i$. The best reply for firm i is that function $x_i(\cdot)$ that maximizes conditional expected profit

$$E\left(x_i(s_i)(\theta - \beta_n \sum_{j \neq i} x_j(s_j) - c_i - (\lambda_i + \beta_n) x_i(s_i)) \mid s_i\right).$$

Hence,

$$\begin{aligned} x_i(s_i) &= \frac{E(\theta \mid s_i) - \beta_n \sum_{j \neq i} E(x_j(s_j) \mid s_i) - c_i}{2(\lambda_i + \beta_n)} \\ &= b_i + \bar{a}_i t_i (s_i - \mu), \end{aligned} \tag{1}$$

where

$$b_i = \frac{\mu - c_i - \beta_n \sum_{j \neq i} b_j}{2(\lambda_i + \beta_n)} \quad \text{and} \quad \bar{a}_i = \frac{1 - \beta_n \sum_{j \neq i} a_j}{2(\lambda_i + \beta_n)}. \tag{2}$$

The conditional expected payoff (gross of information cost) from responding in this optimal way equals $(\lambda_i + \beta_n)(x_i(s_i))^2$. In the appendix the unconditional expected profit is computed to be

$$\pi_i(t) = (\lambda_i + \beta_n)(\overline{b}_i^2 + \overline{a}_i^2 t_i \sigma^2). \tag{3}$$

Lemma 1 *The continuation game $\Gamma_2(t)$ has a unique equilibrium in which player i plays $x_i(s_i) = b_i^e + \overline{a}_i^e[t] t_i (s_i - \mu)$, where:*

$$b_i^e = \left(\mu - c_i - \beta_n \left(\sum_j \frac{\mu - c_j}{2\lambda_j + \beta_n} \right) \frac{\sum_j (2\lambda_j + \beta_n)}{\beta_n + \sum_j (2\lambda_j + \beta_n)} \right) (2\lambda_i + \beta_n)^{-1}$$

$$\overline{a}_i^e[t] = (2(\lambda_i + \beta_n) - \beta_n t_i)^{-1} \left(1 + \beta_n \sum_j \frac{t_j}{2(\lambda_j + \beta_n) - \beta_n t_j} \right)^{-1}$$

Proof. See Appendix.

Note that b_i^e is independent of t and the responsiveness factor $\overline{a}_i^e[t]$ does not depend on any c_k for $k = 1, \ldots, n$.

Endogenous information acquisition

The information acquisition games are now easily solved. We consider first the two-stage game. The Perfect Bayesian equilibrium $(t^*, y^*(\cdot,\cdot))$ of Γ_2 needs to be such that $y^*(t,\cdot)$ is the unique Nash equilibrium of $\Gamma_2(t)$ as computed above, for all t. Hence, $y_i^*(t, s_i) = b_i^e + \overline{a}_i^e[t] t_i (s_i - \mu)$ for all t. Furthermore, no firm must have an incentive to gather any different amount of information. Assuming an interior solution this amounts to demanding that $(\partial \pi_i / \partial t_i)_{t^*} = C'(t_i^*)$ or

$$(\lambda_i + \beta_n)\sigma^2 \left((\overline{a}_i^e[t^*])^2 + 2 t_i^* \overline{a}_i^e[t^*] (\frac{\partial \overline{a}_i^e}{\partial t_i})_{|t^*} \right) = C'(t_i^*). \tag{4}$$

It is not so easily verified that the second order condition is satisfied, but it can be done. (Vives (1988) showed it for homogeneous firms. His proof is straightforwardly extended to heterogeneous firms.)

Consider now the one-stage game. In order for $(\overline{t}, \overline{x}(\cdot))$ to be a pure equilibrium of Γ_1, $\overline{x}(\cdot)$ needs to be the equilibrium of $\Gamma_2(\overline{t})$. As before, this strategy profile can be computed and written as $x_i(s_i) = b_i^e + \overline{a}_i^e[\overline{t}] \overline{t}_i (s_i - \mu)$. The additional condition is that no firm must have an incentive to deviate from this profile. Note that firms can deviate from the information precision and the quantity decision function at the same time. However, given a deviation from $\overline{t}_i$ to t_i, the optimal deviation from $\overline{x}_i(\cdot)$ is easily seen to be $x_i(s_i) = b_i^e + \overline{a}_i^e[\overline{t}] t_i (s_i - \mu)$. This follows from (1) and (2). (Recall that the opponents do not observe the deviation and stick therefore to their strategies.) Note that $\overline{a}_i^e[\overline{t}]$ depends on $\overline{t}$, but not on t_i.

Assuming an interior solution it follows from (3) that this amounts to demanding that

$$(\lambda_i + \beta_n)\sigma^2(\overline{a}_i^e[\overline{t}\,])^2 = C'(\overline{t_i}). \tag{5}$$

Here it is easily verified that the second order condition is satisfied, since $C''(t_i) > 0$ and the left-hand side does not depend on t_i.

Comparing (4) and (5) it becomes obvious that the solutions of the two different information acquisition games do not coincide, as long as they are interior. In fact, whenever $\partial\overline{a}_i^e[t]/\partial t_i > 0$ the solution of the two-stage game will yield higher levels of precision than the one-stage game. It follows immediately from Lemma 1 that indeed $\partial\overline{a}_i^e[t]/\partial t_i > 0$. This establishes our main result:

Theorem 2 *The equilibrium precisions of information in Γ_1 are strictly smaller than those in Γ_2, unless they are zero in both.*

The theorem says that firms choose more information in the two-stage game because the marginal value is higher in that case. Each firm i realizes that if it increases its level of information the other firms $j \neq i$ will use their information less (since $\partial\overline{a}_j/\partial t_i < 0$ by Lemma 1). This makes the behavior of others easier to predict and firm i's payoff increases. However, since in equilibrium every firm acquires more information, it is not clear whether equilibrium payoffs of the two-stage game are higher than those of the one-stage game. In fact, we will show in the next section that in the case of homogeneous firms the equilibrium payoffs of the two-stage game are strictly lower. By continuity this result continues to hold when firms' production costs are not too far apart.

3 Homogeneous firms

In this section the special case of homogeneous firms will be considered. Let $\lambda_i \equiv \lambda$ and $c_i \equiv 0$ for all i.[4] Let $\hat{t}$ denote a tuple of information precisions where $\hat{t}_j \equiv t^*$ for all $j \neq i$. Consider the equilibrium $x(\cdot)$ of $\Gamma_2(\hat{t})$. From Lemma 1 the equilibrium strategies can be computed. In particular, $x_i(s_i) = b_i^e + \overline{a}_i^e[\hat{t}\,]\hat{t}_i(s_i - \mu)$ where

$$b_i^e = \mu/(2\lambda + (n+1)\beta_n) \tag{6}$$

and

$$\overline{a}_i^e[\hat{t}\,] = \frac{2(\lambda+\beta_n) - \beta_n t^*}{2(\lambda+\beta_n)(2(\lambda+\beta_n) + \beta_n(n-2)t^*) - \beta_n^2(n-1)t^*\hat{t}_i}.$$

It follows that

$$a_i^e[\hat{t}\,]_{|\hat{t}_j = t^*} = 1/(2(\lambda+\beta_n) + (n-1)t^*\beta_n) \tag{7}$$

and that

$$\left(\frac{\partial\overline{a}_i^e[\hat{t}\,]}{\partial t_i}\right)_{|\hat{t}_i = t^*} = \frac{\beta_n^2(n-1)t^*}{(2(\lambda+\beta_n) - \beta_n t^*)(2(\lambda+\beta_n) + (n-1)\beta_n t^*)^2}. \tag{8}$$

[4] The marginal cost parameter c_i enters only in the constant term of the equilibrium strategies and does, therefore, not affect the results.

Substitution of (7) and (8) into (4) and some further manipulations yield that the symmetric equilibrium precision of information t^* is found by solving $MPV_2(t^*) = C'(t^*)$, where

$$MPV_2(t) = \sigma^2(\lambda + \beta_n)\frac{2(\lambda + \beta_n)(1 + (n-1)\gamma) + (n-1)\gamma t\beta_n}{(2(\lambda + \beta_n)(1 + (n-1)\gamma) - (n-1)\gamma t\beta_n)^3},$$

if this solution is nonnegative. In this expression

$$\gamma = \frac{t\beta_n}{2(\lambda + \beta_n) - t\beta_n}.$$

We use the notation $MPV_2(t)$ to denote the *marginal private value* to a firm of increasing its precision when all firms have acquired information of precision t. It is impossible to get an explicit solution for t^*. Only the limit solution for the case of infinitely many firms can be computed after taking the limit of MPV_2 as n goes to infinity. This limit case will be of interest in order to compare our results with Vives (1988) and Li et al. (1987) who focused on this case.

Consider now the one-stage model. Substitution of (7) into (5) yields that the symmetric equilibrium precision of information $\bar{t}$ is found by solving $MPV_1(\bar{t}) = C'(\bar{t})$, where

$$MPV_1(t) = \frac{\sigma^2(\lambda + \beta_n)}{(2(\lambda + \beta_n) + t(n-1)\beta_n)^2}, \tag{9}$$

as long as this solution is nonnegative. Here $MPV_1(t)$ denotes the *marginal private value* of information in the one-stage game. Using $C'(t) = c/(\sigma^2(1-t)^2)$, the above expression can be solved explicitly to obtain:

$$\bar{t} = \max\left\{0, \frac{\sigma^2 - 2\sqrt{c(\lambda + \beta_n)}}{\sigma^2 + (n-1)\beta_n\sqrt{c/(\lambda + \beta_n)}}\right\}.$$

It is easily checked that $MPV_2(t) - MPV_1(t) > 0$ for all $t > 0$. To be precise,

$$\frac{MPV_2(t) - MPV_1(t)}{\sigma^2(\lambda + \beta_n)} = \frac{2(n-1)(t\beta_n)^2}{(2(\lambda + \beta_n) - t\beta_n)(2(\lambda + \beta_n) + (n-1)t\beta_n)^3}$$

The difference is proportional to (and of the same sign as) $\partial\bar{a}_i/\partial t_i$. (Compare (4) and (5).) Hence, $t^* > \bar{t}$, unless $t^* = \bar{t} = 0$.

Note, however, that when n tends to infinity $MPV_2(\cdot)$ and $MPV_1(\cdot)$ converge to the same function. Therefore, in the limit the difference between the outcomes of the two different information acquisition games disappears. This is independent of whether the market is replicated à la Vives (1988) or à la Li et al. (1987). In the model of Li et al. (1987) this result is not surprising at all. Since in their model $\beta_n = \beta$ is independent of n, demand is not replicated when the number of firms grows. When n goes to infinity, the gross profits per firm go to zero. Therefore, the amount of money spent on research has to go to zero as well. In the model of Vives (1988) where demand is replicated since $\beta_n = \beta/n$ the result is not that obvious. In this case private information acquisition has an additional benefit. If one firm raises its precision of information other firms will react less aggressively to their own signal. Therefore competitors' quantities will be predicted more precisely. This implies that in each finite Cournot market firms' quantity decisions can be manipulated by one single firm changing its information acquisition. When the number of firms grows, the influence per opponent diminishes. However, it is not obvious that the aggregate of these small influences is not substantial.

Note also that $MPV_2(0) = MPV_1(0)$ and that $MPV_2'(0) = MPV_1'(0)$. This implies that when the equilibrium amounts of information acquisition are close to zero (because information gathering is very costly or because initial uncertainty is quite small), then the two models predict approximately the same levels of information acquisition. For low information cost and high initial uncertainty the models will, however, predict very different levels of information gathering.

Since firms acquire different levels of information in the two games, their payoffs will also differ. We obtain:

Lemma 3 *The equilibrium payoffs of Γ_2 are strictly lower than those of Γ_1, unless no information is gathered in any of the games.*

Proof. See Appendix.

Welfare

Firms gather information in order to estimate residual demand and make higher profits. Consumers also benefit from the fact that demand and supply are matched better. When firms receive imprecise signals, some firms will overestimate demand while others will underestimate it. As a result firms will produce different quantities and, since production costs are convex, they will produce at different marginal costs, which clearly indicates an inefficiency. Better information reduces this inefficiency. On the other hand, a firm gathering information imposes a negative externality on its rivals. It raises its profits at the expense of the other firms. (Cf. Lemma 2.) At high levels of information acquisition this lowers total industry profit. The duplication of market research by many firms also has a negative effect on social welfare.

The welfare aspects of information acquisition are therefore not clear and need to be examined. We need to define the efficient level of information and examine which policy measures are needed in order to obtain this optimal level. Since firms acquire more information in the two-stage game than in the one-stage game, policy implications are likely to differ with the model we use. Moreover, if our claim that the one-stage game is more appropriate is true, it is important to understand how wrong policies based on the two-stage game would be. Will policy implications be reversed, i.e. will the two-stage model recommend to tax (subsidize) information acquisition when it ought to be subsidized (taxed)? Or will it advocate a different magnitude of the same policy direction?

To address this issue three different definitions for the best (efficient) level of information will be examined that are characterized by a trade-off between efficiency and feasibility.

Definition 1 *The* **first best (efficient) level of information** *is that level of information acquisition that maximizes welfare when firms use welfare maximizing quantities in production and the information of all firms can be pooled.*

Vives (1988) has shown that (with strictly convex cost functions) the competitive market cannot attain the first best level of information, unless the cost of information is zero. There are simply no strategies that could yield the first best outcome, since convex costs imply that firms will surely operate at different marginal costs if they are to rely on their own private signal. With constant marginal cost, however, first best efficiency is possible. This result is opposed to the one of Li et al. (1987). The difference of results is caused by the fact that Li et al. (1987) do not replicate the market appropriately. Therefore, from now on we will only consider the properly replicated market, that is $\beta_n = \beta/n$.

The assumption that the information of all firms can be pooled does not respect the decentralized decision structure of the economy. Efficiency of competitive markets is restored if the constraint of decentralized information acquisition is recognized.

Definition 2 *The* **second best (efficient) level of information** *is that level of information acquisition that maximizes welfare when firms use welfare maximizing quantity functions in production while information cannot be pooled.*

Vives (1988) has shown that the competitive market attains this second best level. Since in the limit case firms acquire the same amount of information in the one-stage game of information acquisition as in the two-stage game, we get the following corollary to Vives' result:

Corollary 4 *When the number of firms goes to infinity, the one-stage game model of information acquisition yields the second best efficient level of information.*

The second best efficient level of welfare is problematic because it is based on firms maximizing welfare in production. It thereby implicitly assumes either a policy measure in the form of subsidizing production that induces firms to do so, or perfect competition. A subsidy on production is hard to implement since the size of the correct subsidy depends on the pool of information. The alternative implicit assumption of perfect competition makes the criterion inapplicable to finite oligopolistic markets. In perfectly competitive markets the second best efficient level of information coincides with the following criterion:

Definition 3 *The* **third best (efficient) level of information** *is that level of information acquisition that maximizes welfare when firms use profit maximizing quantity functions in their production decision and information cannot be pooled.*

Given that the third best efficient level of information respects the market structure in both information acquisition and production it seems the appropriate criterion to be used for policy recommendations. The first best level is irrelevant since firms have no incentives to pool their information. (See Gal-Or (1985).) Moreover, like the second best level it assumes some policy measure that ensures welfare maximization in production. Only the third measure concentrates on the pure effects of information acquisition and will therefore be the basis for our welfare analysis.

For given precision of information t for each firm, each firm j will use the equilibrium strategy $x_j(s_j) = a(s_j - \mu) + b\mu$ where a and b are determined by equations (6) and (7), respectively. Total welfare (gross of information cost) for given t, θ and signals s_j equals

$$TW(t, \theta, s_1, ...s_n) = \frac{\theta^2}{2\beta_n} - \frac{1}{2\beta_n}\left(\theta - \beta_n \sum_j x_j(s_j)\right)^2 - \lambda \sum_j x_j(s_j)^2.$$

We can compute the expected total welfare, $ETW(t)$ by first taking the expectation over signals conditional on θ, and then taking the expectation over θ. The third best efficient level t_{e3} satisfies $ETW'(t_{e3}) = nC'(t_{e3})$, or equivalently,

$$MSV(t_{e3}) = C'(t_{e3}),$$

where $MSV(t) = ETW'(t)/n$ denotes the *per capita* marginal social value of information. This is equal to the marginal effect on total welfare when one firm increases its precision, when all firms have precision t. In the appendix we show that

$$MSV(t) = \sigma^2 \frac{2\lambda^2 + 3\beta_n^2 + 5\beta_n\lambda + \lambda(n-1)t\beta_n + (n-1)t\beta_n^2/2}{(2(\lambda + \beta_n) + (n-1)t\beta_n)^3}. \qquad (10)$$

We are now ready to compare the efficient level with the equilibrium level of information acquisition. Recall that at the equilibrium marginal private value equals marginal cost, while at the third best efficient level of information, the (per capita) marginal social value equals marginal cost. Whether under- or overinvestment takes place depends therefore on the relative positions of the curves C', MSV, and MPV_1 (for the one-stage game) and MPV_2 (for the two-stage game). We already know that MPV_1 lies below MPV_2 from Theorem 2. The following Lemma shows how the relative positions of the other curves exactly depend on the parameters of the model.

Lemma 5

(i) $MSV(t) > MPV_1(t)$ *if and only if* $t < \hat{t}_1$, *where* $\hat{t}_1 = \frac{2+2n\lambda/\beta}{n-1}$. $\hat{t}_1 < 1$ *if and only if* $n(1 - 2\lambda/\beta) > 3$.

(ii) $MSV(t) > MPV_2(t)$ *if and only if* $t < \hat{t}_2$, *where* $\hat{t}_2$ *is the positive root of*

$$[(n-1)(2n\lambda/\beta + 3/2)]t^2 + [n(n\lambda/\beta + 1)]t - 2(n\lambda/\beta + 1)^2 = 0.$$

$\hat{t}_2 < 1$ *if and only* $(4(\lambda/\beta)^2 - 6\lambda/\beta)n^2 + (12\lambda/\beta - 5)n + 7 \geq 0$.

Proof. See Appendix. □

Lemma 5 tells us that the MSV and MPV curve intersect in a point $\hat{t}$ which depends on λ/β and n. For $t < \hat{t}$, $MSV(t) > MPV(t)$ and for $t > \hat{t}$ the reverse holds. The reason is as follows. At low levels of information acquisition some

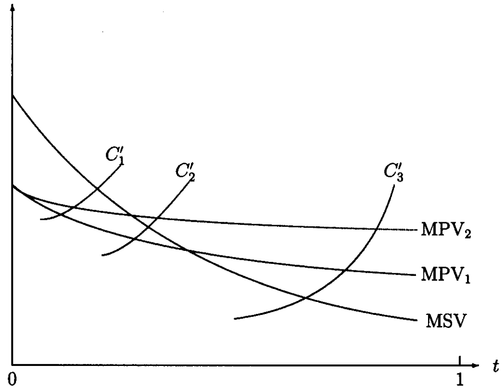

Figure 1. Comparing marginal cost and marginal values

firms under- and others overestimate demand considerably. This means that they will choose very different production levels, and since costs are convex, they will produce at different marginal costs, which indicates an inefficiency. (The inefficiency increases with λ/β.) Moreover, at low levels of information precision the negative externality that firms inflict on each other is smaller than at high levels. The marginal social value at low (high) levels of t is therefore relatively high (low) compared to the marginal private value.

Note that $\hat{t}_1 \geq 1$ when $\lambda/\beta \geq 1/2$ and that $\hat{t}_2 \geq 1$ when $\lambda/\beta \geq 3/2$. Hence, when the inefficiency caused by firms producing at different marginal costs is high, the marginal social value is larger than the marginal private value, and as a consequence firms underinvest. This is true, whatever the size of the market, the cost of information gathering and the initial uncertainty. In this case subsidies on information acquisition activities could improve welfare. Note that the one-stage game model advocates higher subsidies than the two-stage model.

When the inefficiency caused by firms producing at different marginal costs is not severe (λ/β is low), then for sufficiently large markets the intersection point of the marginal social value curve and the marginal private value curve lies within the interval $(0, 1)$. Whether firms over- or underinvest now depends on the initial uncertainty and the cost of information acquisition. To be precise, it depends on the ratio σ^4/c (see Lemma 6 in the Appendix). Figure 1 illustrates the three possible cases.

When information is cheap and initial uncertainty relatively large (see curve C_3' in Figure 1), firms will overinvest. Taxes on information acquisition activities could restore this. (Note that the one-stage game calls for lower taxes than the two-stage model.) When information is expensive and initial uncertainty small (curve C_1'), firms will underinvest relative to the optimum: subsidies are in order. (The one-stage game model calls for higher subsidies than the two-stage model.) Note that for intermediate values of the ratio c/σ^4 the one-stage game predicts underinvestment and calls for subsidies, while the two-stage game model predicts overinvestment and advocates taxes. (Curve C_2'.)

Note that σ^2 appears both in $MSV(t)$ and in $MPV(t)$ as a factor. Higher initial uncertainty amplifies the difference between the social and the private value, while it lowers and flattens $C'(t)$. This means that for high initial uncertainty overinvestment will occur and that the introduction of the right tax could make up for a substantial welfare improvement. In this case the two different models of information acquisition would advocate very different tax levels and it is therefore important to use the relevant model. When initial uncertainty is very small, on the other hand, a small subsidy would be needed. The welfare improvement would not be very substantial in this case, and also the two different models of information acquisition would not yield very different policy recommendations.

The above results show how the different parameters determine whether over- or underinvestment occurs. They also show that the two-stage model either advocates too low subsidies, too high taxes or a tax instead of a subsidy. In some circumstances the degree of over- or underinvestment is very small, in which case it does not really matter which model of information acquisition is used. This happens when $\beta \to 0$, $\sigma^2 \to 0$ or $n \to \infty$. For $\beta \to \infty$, $\lambda \to \infty$ or $c \to \infty$ it is optimal not to acquire information and no policy measure is needed. In all other cases introducing the right policy measure can account for a substantial welfare improvement. In those cases it is important to use the right model, especially when initial uncertainty is large.

4 Heterogeneous duopoly

For the case of heterogeneous duopoly ($\lambda_1 > \lambda_2$) the model of Section 2 reduces to the model of Hwang (1993). Hwang (1993) analyzed the two-stage game and performed comparative statics exercises in order to analyze how the parameters of the model affect the equilibrium precisions of information. In this section we briefly compare Hwang's results with the ones that can be obtained for the one-stage model. For details we refer the reader to our working paper Hauk and Hurkens (1997).

Let us first remark that in the two-stage model one cannot give explicit expressions for the equilibrium precisions of information as the relevant equations are of high order. Comparative statics exercises must be done by investigating how the reaction curves shift when some parameter changes. In the case of the one-stage game we can explicitly calculate the equilibrium precisions of information (as the relevant equations turn out to be quadratic), which makes the comparative statics exercises easier as well.

It turns out that all qualitative comparative statics results for the two-stage game hold true for the one-stage game. That is, (i) firm 1 (the firm with the steeper marginal cost curve) acquires less information than firm 2; (ii) a decrease of information cost or an increase in initial uncertainty implies that both firms will increase their precision of information, but firm 1 will increase its precision more than firm 2; (iii) when the slope of the marginal cost curve for one firm increases, this firm will decrease its precision while its rival will increase precision; (iv) a steepening of the demand curve leads firm 2 to acquire less information, while the effect for firm 1 is ambiguous.[5]

5 Conclusions

In situations of endogenous information acquisition agents trade off the benefits from having better information against the cost of acquiring information. This paper argued that the benefits depend on whether information is acquired privately or secretly. Under private information acquisition (two-stage game) the information acquisition decisions of agents become common knowledge and have therefore a strategic value. By acquiring more information an agent can manipulate the reactions of its opponents. Under secret information acquisition (one-stage game) this is impossible, since opponents will not observe how well informed their rivals are. This latter case seems to be the more appropriate set-up, given that agents have no incentives to reveal their level of informedness truthfully. We illustrated these ideas in an oligopolistic market with uncertain demand. It was shown that the strategic value of information is positive in this context. If one firm is known to have good information, other firms will act less aggressively towards their own private information and this makes their behavior easier to predict. In the two-stage model firms therefore overinvest in market research relative to the case of secret information acquisition. It was shown, that this overinvestment vanishes when the number of firms becomes very large. This implies that in a competitive market the second (and third) best efficient level of information is acquired. In smaller markets firms may under- or overinvest with respect to the efficient level of information acquisition. Policy implications depend always quantitatively on which model of information acquisition is considered. In some instances the policies advocated by the two models are even qualitatively different (tax versus subsidy). Using the appropriate model is therefore important.

In this paper it was argued that the one-stage game is the more relevant model since firms are not able to observe the information acquisition decisions of their opponents. The two-stage game would be appropriate if firms could credibly commit to (a lower bound on) the precision of information and would deliberately choose to do so. However, Lemma 3 showed that firms are better off not announcing the precision of their information.

This paper reconsidered the models of Ponssard (1979), Li et al. (1987), Vives (1988) and Hwang (1993) in detail. There are some other models of information acquisition that have not been discussed yet. Ockenfels (1989) considers a model very similar to the one of Ponssard (1979). The only difference is that in Ockenfels' model quantity choices are discrete (in fact binary). It is clear that his model exhibits the same problem as Ponssard (1979).

[5] Hwang (1993) claimed that a steepening of the demand curve leads both firms to decrease information precision. As is pointed out in Hauk and Hurkens (1997), however, he was mistaken.

Chang and Lee (1992) discuss a model of quantity competition in a differentiated duopoly which did not fit nicely in the model presented in Section 2, although the present model could be extended to include differentiated products as well. Again information acquisition is modeled as a two-stage game. It can be easily verified by computing the best reply against an affine strategy, as was done in Section 2, that also in their model firms overinvest in research relative to the case of secret information acquisition. The same is true for price competition with differentiated products. In this case, however, the information acquisition stage is of the strategic complements variety (see Chapter 8 in Vives, 2000). This, coupled with the fact that in the Bertrand game the strategic value of information is positive and that a firm likes its rivals to be informed (see Section 8.3.1 in Vives, 2000), means that firms' profits will be higher in the two-stage game than in the one-stage game.[6] The externality of information acquisition is thus positive.

Hwang (1995) considers a model of information acquisition that is designed to compare monopoly, duopoly, and competitive markets. There are only two players in the model. The second stage game is modeled using conjectural variations. By varying the conjectural variations the model can represent monopoly, duopoly or a competitive market. However, in the first stage there are no conjectural variations. Hence, the influence of raising the precision of information is more or less the same as in the ordinary duopoly game. This means that firms overinvest in research in Hwang's (1995) model even in the case of a competitive market. The peculiarity of this model is further illustrated by Lemma 4 in Hwang (1995). It says that the level of information precision that maximizes joint profit is smaller than the equilibrium precision. The Lemma is mathematically correct, but does not make any sense in the case the model is to represent a monopoly.

Further models of information acquisition have been studied for auctions. Milgrom (1981) considers a two-stage version whereas Matthews (1984) considers the one-stage version. Unfortunately Matthews (1984) was unable to get an explicit solution. Persico (2000) compares secret information acquisition in a first- and second-price auction. Further research has to be conducted for the case of auctions. One should note, though, that the main interest in the literature on information acquisition in auctions is when the number of bidders becomes very large. The question addressed is whether the winning bid will converge (with probability one) to the true value of the object. Because of the similarity with competitive markets one might conjecture that it does not matter whether information is acquired secretly or not. However, this needs to be examined carefully.

[6] We thank the referee for pointing this out.

6 Appendix

6.1 Existence of equilibrium

The unconditional expected profit is

$$\begin{aligned}\pi_i(t) &= E\left((\lambda_i + \beta_n)(x_i(s_i))^2\right)\\ &= (\lambda_i + \beta_n)([E(x_i(s_i))]^2 + Var(x_i(s_i)))\\ &= (\lambda_i + \beta_n)(\overline{b}_i^2 + \overline{a}_i^2 t_i^2(\sigma^2 + v_i))\\ &= (\lambda_i + \beta_n)(\overline{b}_i^2 + \overline{a}_i^2 t_i \sigma^2)\end{aligned}$$

Proof of Lemma 1. Using (1) and (2) the equilibrium strategies can be computed and written as $x_i(s_i) = b_i^e + \overline{a}_i^e[t]t_i(s_i - \mu)$, where

$$2(\lambda_i + \beta_n)b_i^e = \mu - c_i - \beta_n \sum_{j\neq i} b_j^e \qquad (i = 1, \dots, n) \tag{11}$$

and

$$2(\lambda_i + \beta_n)\overline{a}_i^e[t] = 1 - \beta_n \sum_{j\neq i} t_j \overline{a}_j^e[t]. \qquad (i = 1, \dots, n) \tag{12}$$

Note that the constants b_i^e $(i = 1, \cdots, n)$ do not depend on the precision of information in the market.

To find the solution to (11) we first subtract $\beta_n b_i^e$ from both sides:

$$(2\lambda_i + \beta_n)b_i^e = \mu - c_i - \beta_n \sum_j b_j^e \tag{13}$$

Summing over all i and rearranging yields

$$\sum_j b_j^e = \frac{\sum_i \frac{\mu - c_i}{2\lambda_i \beta_n}}{1 + \beta_n / \sum_i (2\lambda_i + \beta_n)} \tag{14}$$

Substituting (14) into (13) gives the result.

The solution of (12) is obtained in a similar way: now $\beta_n t_i \overline{a}_i^e[t]$ is subtracted in the first step. □

6.2 Equilibrium payoffs

Proof of Lemma 3. Recall that $\overline{t}$ (resp. t^*) denotes the equilibrium level of information acquisition in the one-stage game (resp. two-stage game), and that $\overline{t} < t^*$. Let $\overline{a}^e[t] = 1/(2(\lambda + \beta_n) + (n-1)t\beta_n)$ and $b^e = \mu/(2\lambda + (n+1)\beta_n)$, such that $x(s) = b^e + \overline{a}^e[t]t(s - \mu)$ is the equilibrium strategy of each firm in the second stage game $\Gamma(t, ..., t)$. Using (3) we know that the equilibrium payoff in this second stage game equals

$$\pi(t) = (\lambda + \beta_n)((b^e)^2 + (\overline{a}^e[t])^2 t\sigma^2.$$

The equilibrium payoffs for the one- and two-stage game are therefore $\pi(\overline{t}) - C(\overline{t})$ and $\pi(t^*) - C(t^*)$. Now

$$\begin{aligned} \pi'(t) - C'(t) &= \sigma^2(\lambda + \beta_n)\frac{2(\lambda + \beta_n) - (n-1)\beta_n t}{(2(\lambda + \beta_n) + (n-1)\beta_n t)^3} - C'(t) \\ &< MPV_1(t) - C'(t) \end{aligned} \tag{15}$$

where $MPV_1(t)$ is the marginal private value of information acquisition in the one-stage game defined by (9). The right-hand side of (15) is negative for $t > \overline{t}$. Hence, $\pi(\overline{t}) - C(\overline{t}) > \pi(t^*) - C(t^*)$. □

6.3 The third best level of information

We assume that the solution is symmetric. For given precision of information t for each firm, each firm j will use strategy $x_j(s_j) = a(s_j - \mu) + b\mu$ where a and b are the equilibrium strategies as computed in Section 4, i.e.

$$b = 1/(2\lambda + (n+1)\beta_n),$$

and

$$a = t/(2(\lambda + \beta_n) + (n-1)\beta_n t).$$

Total welfare, gross of information costs, equals for given precision t and fixed θ and fixed signals s_j

$$TW(t, \theta, s_1, ...s_n) = \frac{\theta^2}{2\beta_n} - \frac{1}{2\beta_n}\left(\theta - \beta_n \sum_j x_j(s_j)\right)^2 - \lambda \sum_j x_j(s_j)^2.$$

Expected total welfare, given θ equals

$$\begin{aligned} E(TW|\theta) &= \frac{\theta^2}{2\beta_n} - \frac{1}{2\beta_n}\left\{\left[E((\theta - \beta_n \sum a(s_j - \mu) + b\mu)|\theta)\right]^2 + \right. \\ &\quad \left. +Var\left(\left(\theta - \beta_n \sum a(s_j - \mu) + b\mu\right)|\theta\right)\right\} + \\ &\quad -\lambda \sum [E(a(s_j - \mu) + b\mu|\theta)^2 + Var(a(s_j - \mu) + b\mu|\theta)] = \\ &= \frac{\theta^2}{2\beta_n} - \frac{1}{2\beta_n}[\theta - \beta_n n(a(\theta - \mu) + b\mu)]^2 + \\ &\quad - \frac{1}{2\beta_n}\beta_n^2 n a^2 v - \lambda n(a(\theta - \mu) + b\mu)^2 - \lambda n a^2 v. \end{aligned}$$

Taking the expectation over θ gives unconditional expected welfare

$$\begin{aligned} ETW(t) &= \frac{\mu^2+\sigma^2}{2\beta_n} - \frac{1}{2\beta_n}(\mu - \beta_n n(a(\mu-\mu)+b\mu))^2 + \\ &\quad -\frac{1}{2\beta_n}(1-\beta_n na)^2\sigma^2 + \\ &\quad -\frac{1}{2}\beta_n na^2 v - \lambda na^2\sigma^2 - \lambda n(b\mu)^2 - \lambda na^2 v. \end{aligned}$$

At the optimal level of information acquisition, t_{e3}, we have $ETW'(t_{e3}) = nC'(t_{e3})$. Define the (per capita) marginal social value of information as

$$MSV(t) = ETW'(t)/n.$$

Now it is straightforward to check that

$$\begin{aligned} MSV(t) &= a'\sigma^2 - n\beta_n aa'\sigma^2 - \beta_n aa'v - \beta_n a^2 v'/2 \\ &\quad -2\lambda aa'\sigma^2 - 2\lambda aa'v - \lambda a^2 v' \\ &= \sigma^2 \frac{2\lambda^2 + 3\beta_n^2 + 5\beta_n\lambda + \lambda(n-1)t\beta_n + (n-1)t\beta_n^2/2}{(2(\lambda+\beta_n)+(n-1)t\beta_n)^3}. \end{aligned}$$

6.4 Welfare analysis

Proof of Lemma 5.

(i) It is easily verified that $MSV(t) > MPV_1(t)$ if and only if

$$\lambda\beta_n + \beta_n^2 - (n-1)t\frac{\beta_n^2}{2} > 0.$$

In particular, $MSV(0) > MPV_1(0)$. Furthermore, the two curves intersect at $\hat{t}_1 = 2(n\lambda/\beta + 1)/(n-1)$. $\hat{t}_1 \leq 1$ if and only if $n(1 - 2\lambda/\beta) \geq 3$. Obviously, when $2\lambda/\beta \geq 1$, no n exists for which the inequality holds. On the other hand, if $2\lambda/\beta < 1$, the inequality holds for large enough n.

(ii) It is easily verified that $MSV(t) > MPV_2(t)$ if and only if

$$(n-1)\left[t\beta_n(\lambda+\beta_n) + t^2\beta_n\left(2\lambda + \frac{3}{2}\beta_n\right)\right] - (\lambda+\beta_n)[(2(\lambda+\beta_n) - t\beta_n] < 0.$$

In particular, $MSV(0) > MPV_2(0)$. Furthermore, the two curves intersect only once in the halfline $[0, \infty)$, namely in $\hat{t}_2$, the positive root of the equation mentioned in the statement of the Lemma.

$$\hat{t}_2 = \frac{(n\lambda/\beta + 1)(-n + \sqrt{n^2 + 8(n-1)(2n\lambda/\beta + 3/2)})}{2(n-1)(2n\lambda/\beta + 3/2)}$$

This intersection point lies in the interval $[0, 1]$ only if the left-hand side of the above inequality, evaluated at $t = 1$, is positive, i.e. if

$$(n-1)(6n\lambda/\beta + 5) \geq 2(n\lambda/\beta + 1)(2n\lambda/\beta + 1).$$

This is equivalent to

$$n^2(6\lambda/\beta - 4(\lambda/\beta)^2) + n(5 - 12\lambda/\beta) - 7 \geq 0.$$

Obviously, when $\lambda/\beta \geq 3/2$, there exists no $n > 0$ for which the above inequality holds. If $\lambda/\beta < 3/2$, for large enough n the inequality is satisfied. □

Lemma 6
(i) Suppose that λ, β and n are such that $\hat{t}_1 < 1$, i.e. the marginal social value curve intersects the marginal private value curve in the interval (0,1). Then there exists some threshold x_1 (which depends on λ, β and n) such that overinvestment occurs if and only if $\sigma^4/c > x_1$.

(ii) Suppose that λ, β and n are such that $\hat{t}_2 < 1$, i.e. the marginal social value curve intersects the marginal private value curve in the interval (0,1). Then there exists some threshold x_2 (which depends on λ, β and n) such that overinvestment occurs if and only if $\sigma^4/c > x_2$.

Proof. (i) Suppose that MSV and MPV_1 intersect in $\hat{t}_1 < 1$. Overinvestment occurs when $\bar{t} > \hat{t}_1$ or, equivalently, when $MPV_1(\hat{t}_1) > C'(\hat{t}_1)$. Hence, overinvestment occurs if and only if

$$\frac{\sigma^4}{c} > \frac{MPV_1(\hat{t}_1)}{\sigma^2(1-\hat{t}_1)^2}.$$

The right-hand side depends only on n, β, and λ. Call it x_1. Now overinvestment occurs if and only if $\sigma^4/c > x_1$.
The proof of (ii) goes along the same lines. □

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