Allocating Ideas: Horizontal Competition in Tournaments

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We develop a stylized model of horizontal and vertical competition in tournaments with two competing firms. The sponsor cares not only about the quality of the design but also about the design location. A priori not even the sponsor knows his preferred design location, which is only discovered once he has seen the actual proposals. We show that the more efficient firm is more likely to be conservative when choosing the design location. Also, to get some differentiation in design locations, the cost difference between contestants can be neither too small nor too big. Therefore, if the sponsor mainly cares about the design location, participation in the tournaments by the two lowest-cost contestants cannot be optimal for the sponsor.

1. Introduction

Tournaments are games in which players spend resources to win a prize and are extensively used as allocation mechanisms, because they are easy to implement. Only the relative performance of the participants

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has to be evaluated, which is usually less demanding than measuring the absolute performance. For example, tournaments are used in sports competitions and in procurement processes.[1] Promotion in labor markets, R&D races, and lobbying are often disguised tournaments. So-called contests of ideas are also tournaments that are used to promote the generation of new ideas, in particular in architecture; mechanical, civil, transport, and plant engineering; animation; and freeform/artistic expression.

The economic literature on tournaments is vast and has studied many relevant aspects, like one prize versus multiple prizes[2] and complete and incomplete information scenarios.[3] Tournaments are usually modeled as games in which players maximize their expected profit by choosing a costly effort level, which positively influences their probability of winning the prize. However, there are tournaments in which the decision of participants has several dimensions: the *amount* of effort (or design quality) as well as the *type* of effort (or design location). The following examples illustrate what we mean by these dimensions.

Example 1: The tournament used by the Florentine Republic to choose the design of the second doors of the baptistery of the Duomo.

This competition was announced in 1401, and artists were required to design a panel representing the sacrifice of Isaac. Some of the finest sculptors in Florence took part in the contest. They were seven in total, among them the two finalists Filippo Brunelleschi and Lorenzo Ghiberti. The panels of the two finalists were considered equally good. Although the interpretation of Ghiberti was still partly Gothic in style and hence fairly conservative for his time, Brunelleschi presented a more modern neoclassical design. Ghiberti's design won, not on the basis of quality but simply because it was easier to understand. Ghiberti's victory was a pure matter of taste.[4] The two designs were horizontally differentiated, and it was this horizontal competition that was decisive in choosing the winner.

1. Fullerton and McAfee (1999) provide several examples of procurement processes that have been carried out using tournaments. See Fullerton and McAfee (1999) for details.

2. Most papers study tournaments with a single prize (e.g., Tullock, 1980; Wright, 1983; Dixit, 1987; Baye et al., 1993; Amann and Leiniger, 1996; Baye et al., 1996; Fullerton and McAfee, 1999; Lizzeri and Perisco, 2000). Glazer and Hassin (1988) and Moldovanu and Sela (2001) study a contest with multiple prizes.

3. Complete information scenaries are among others: Tullock (1980), Dixit (1987), Glazer and Hassin (1988) and Baye et al. (1993, 1996). Amann and Leiniger (1996) and Lizzeri and Perisco (2000) study all-pay auctions with incomplete information.

4. The two panels they presented for the competition are now exhibited beside each other in the Museum of the Bargello. (See http//www.mega.it/eng/egui/monu/bo.htm). We thank Ignacio Conde Ruiz for sharing his knowledge in history of arts with us.

While working on their respective designs, the artists had to take two decisions, namely, (i) how to represent the sacrifice of Isaac and (ii) how much effort to put into this representation. Compared to the decision which interpretation to adopt, the effort decision is costly and determines the final quality of the design. We therefore refer to the first decision as *horizontal competition* (the type of effort chosen) and to the competition in effort levels (quality) as *vertical competition* (the amount of effort).

Example 1 also illustrates another important aspect of many contests of ideas. These contests are normally used to generate ideas, that is, in situations where the sponsor does not really know what he wants. In the example the Florentine Republic obviously wanted good quality—the vertical dimension was clear—but there was uncertainty about the horizontal dimension: a priori the sponsor had no clear idea which type of design he would like most, simply because he could not even imagine all possible types of design. He needed the actual design proposals to learn his *ex post* preferences. The horizontal dimension comes into play because of this uncertainty. Therefore, uncertainty will be a necessary element of our model. Notice that uncertainty is not equivalent to symmetry: although the sponsor's preferences were uncertain when the competition for the design of the sacrifice of Isaac was announced, they were definitely not symmetric: it was clearly understood that the Gothic style was predominant at that time and therefore any Gothic proposal had an *ex ante* advantage over any other design. There was an *ex ante* bias toward one particular design.

Example 2. The dream of the blue laser diode.

For two decades, researchers working for the biggest players in the electronics industry, like RCA, Hewlett-Packard, Matsushita, and Sony, tried their hands at the blue laser diode (LED) but failed. The blue laser was the ultimate dream in laser technology; blue light has the shortest wavelength of visible light and could quadruple the amount of data on a compact disc, CD or DVD. In 1989 there were two materials for making blue LEDs: zinc selenide and gallium nitride. A priori zinc selenide seemed superior to gallium nitride because of its better crystal quality. Zinc selenide has by far less defects, which is crucial for developing a reliable LED. Therefore, all big firms used this material in their race for the LED. Nakumara, a self-described country boy working for a small firm, Nichia Chemical Industries, tried to develop LEDs with gallium nitride and was the one to succeed.

Example 2 is of a different nature than the former example: the sponsor knows what he wants: the blue laser diode, but nobody knows how to get there, although everybody knows that one path has a higher *ex ante* probability of being successful. The competitors in the contest had to choose what *type* of research to perform as well as the effort level. In this case, the chosen path (type of effort) was decisive. Horizontal differentiation mattered.

The aim of this paper is to study a tournament in which the contest success function depends both on the effort exerted by the participants (vertical competition) and on the type of design/effort chosen by the participants (horizontal competition). We present a very stylized model in which contest participants face some uncertainty concerning the type of effort/design that matches the sponsor's preferences. This uncertainty can be made consistent with the above examples: on the one hand, the sponsor might not exactly know what he wants—as in our example 1; on the other hand, the uncertainty does not have to be about the final product, but about which research path to take as in our example about the blue laser diode. We assume that *this uncertainty is asymmetric*: not everything is equally likely to match the sponsor's preferences. Society/arquitects have some idea what is considered to be appropriate; different research paths usually have different probabilities to be successful. We model this asymmetric uncertainty by assuming that there are two possible designs, one of which has a higher *ex ante* probability of being the sponsor's preferred design. We call this design the conservative design and label the other design as radical.

We do not attempt to build a general model of vertical and horizontal competition in tournaments, but use the simplest possible model with only two possible design locations and two competing firms to show that some degree of horizontal competition in a classical model of tournaments can lead to qualitatively different results. In particular, it can change known results about optimal entry.

A well-established result in tournaments is that limiting entry can be an optimal strategy for two reasons: on the one hand, limiting entry can rise the effort level of contestants, because it increases their probability of winning. On the other hand, it reduces costs; fewer offers have to be evaluated. Nalebuff and Stiglitz (1983) show that the overall effort in a labor contract can be decreasing in the number of workers participating in the contest. Taylor (1995) proves a similar result for research tournaments with homogenous contestants. Fullerton and McAfee (1999) study a research tournaments with heterogenous contestants and conclude that the optimal number of contestants is two, and that these two contestants have to be the lowest, cost contestants. In this paper, we show that Fullerton and McAfee's (1999) result may not be robust to the introduction of horizontal competition with *ex ante* biased preferences of the sponsor. In our model, in which the number of contestants is restricted to two, it might be beneficial for the sponsor if the contestants are differentiated in costs. If the two participants have similar costs (e.g., if they were the lowest-cost firms in the industry), they choose similar designs too. But if the sponsor mainly cares about the type of design, it is worthy for him to get the lowest cost firm and another firm with higher costs to compete in the contest. In this way, the less efficient contestant is willing to choose a different design location than that chosen by the more efficient firm, and the sponsor has a higher probability to get a good match between one of the actual design proposals with his preferred design location. This happens because of the bias in the sponsor's preferences: if cost differences are sufficiently big, the less efficient firm is willing to choose the *ex ante* less attractive design to avoid competition. This strategic alternative does not arise if the sponsor's preferences are completely symmetric. The present paper shows that cost differentiation plays a role in promoting diversity as long as there is some asymmetry in the sponsor's *ex ante* preferences. This is the main result of the paper.

While limiting the number of possible designs to two is obviously a restriction of our model, we are confident that a more general model would also modify standard results about optimal entry in tournaments. There is a very intuitive reason for this, which the present model does not capture. Increasing the number of participants, increases the number of proposals and diversity can have some value in a horizontal competition framework, especially in situations where design proposals help the sponsor to learn his *ex post* preferences or where there are several possible research paths to be taken.

Another result of the paper is that the more efficient contestant is more likely to choose the conservative design, which has the higher *ex ante* probability for being the sponsor's preferred design. This result nicely fits our motivating examples. In the contest held by the Florentine Republic for the second doors of the baptistery Brunelleschi, the younger and less-known artist, chose a more innovative design. In the race for the blue laser diode, the winner, Nakamura, who worked for a small firm, chose to work with an a priori worse material. His justification of this choice is perfectly consistent with our model. In his own words,

> "At that time, in 1989, there were two materials for making blue LEDs: zinc selenide and gallium nitride. These had the right band gap energy for blue lasers. But everybody was working on zinc selenide because that was supposed to be much better. I thought about my past experience: if there is a lot of competition, I cannot win."

This result is also very much in line with those derived by Prendergast and Stole (1996) and Cabral (1999) in very different (and *dynamic*) models. Prendergast and Stole (1996) show that youngsters who have no reputation to defend are more impetuous than old timers who have to worry about the information that their new decision reveals concerning their past decisions. Cabral (1999) studies the important issue in which situation to choose an R&D project with a high variance versus a project with a low variance. He shows that the laggard has nothing to loose, that is, the follower chooses a riskier project than the leader. In Cabral's paper, R&D competitors have only one choice variable—the variance of the R&D project—whereas in our model, competitors have to make two choices: the type of R&D project and the amount of effort they will dedicate to the project. Moreover, in Cabral's model, the expected payoff of both projects is the same; they only differ in their degree of riskiness, the variance. In our model, one R&D path is a priori clearly superior to the other because the probability of success is higher. Nevertheless, the inefficient competitor might choose this dominated R&D path to avoid too fierce competition in effort levels. In addition, unlike in Cabral (1999), in our model it is not always the inefficient firm that is impetuous/radical. If firms are not too different and the sponsor is not very likely to be conservative, multiple equilibria exist: it might be the efficient firm that chooses the radical design, whereas the inefficient firm is more conservative.

The remainder of the paper is organized as follows. In Section 2, the model is introduced, whereas Section 3 solves the second stage of the model: the effort decision. In Section 4, we solve the first stage of the model and present the main results of the paper. Section 5 discusses the scope and implications of the model and presents conclusions. It also discusses the robustness of the results. All proofs as well as the robustness results are relegated to a technical appendix.

2. The Model

Consider a sponsor (administration) who wants to undertake a public project but does not have a clear idea about the design of the project. To learn about possible designs, the sponsor organizes a contest of ideas. Two risk neutral firms, firm 1 and firm 2, compete in the contest. The rules of the contest are simple: first, the sponsor announces the prize P for the winner of the contest. Then, participants submit design proposals, and finally, the sponsor selects a design and thereby the winner of the contest.

We assume that the design competition has two dimensions: location of the design d and effort in developing the design e. We restrict the space of design locations to only two possible design locations: conservative (C), and radical (R). Location captures the type of design. A conservative design is a design that is more likely to be the preferred design of the sponsor, because, for example, it is close to one that won in a previous contest or because it is the current fashion; by radical designs we mean "vanguard" designs that are less likely close to the sponsor's preferences. The effort is a variable related to the quality of the design. The bigger the effort of the firm, the higher the expected quality of the design, where quality is an index whether or not a given design is well done: some conservative (radical) designs might be better than others.

Each firm has to choose first the design location d_i and then the amount of effort e_i it puts into developing the chosen type of design. The quality of the chosen design is linked to the firm's effort, but the relation between effort and actual produced quality is not deterministic. The bigger the effort of the firm is, the higher is the expected but not necessarily the actual quality of the design produced by this firm. There is no cost associated to choosing the design location. The cost of effort for firm i is $c_i e_i$. Without loss of generality, we assume that firm 1 is more efficient than firm 2, that is, $c_2 \geq c_1$.

The sponsor cares about the type of design and its quality. On the one hand, he wants to maximize the quality of the design location. On the other hand, he wants to minimize the distance between the project design and his preferred design location. A priori the sponsor and the firms face some uncertainty about this preferred design location. With probability $\alpha > 0.5$, the sponsor prefers the conservative design, whereas with probability $1 - \alpha$, he prefers the radical design.[5] Once the sponsor sees the actual design proposals, this uncertainty is resolved and the sponsor learns his preferred design location.

We do not state the exact form of the preferences of the sponsor but use the following contest success function instead, which can be seen as a reduced form of the sponsor's maximization problem. The contest success function tells us the probability that firm i wins the contest given that it had submitted a design (d_i, e_i).

$$p_i(d_i, e_i, d_j, e_j, d_p, \lambda) = \begin{cases} \dfrac{e_i}{e_i + e_j} & \text{if } d_i = d_j \\ (1 - \lambda)\dfrac{e_i}{e_i + e_j} + \lambda h_i(d_i, d_j, d_p) & \text{if } d_i \neq d_j, \end{cases}$$

5. We assume that firms know the parameter α. However, the results of our model would not change if we assumed that firms do not know the parameter α but have a prior distribution $F(\alpha)$ over it. As we will see later, payoff functions are linear in α, and therefore contestants only care about the expected value $E\{\alpha\}$.

where h_i is the comparative advantage of firm i due to horizontal competition

$$h_i(d_i, d_j, d_p) = \begin{cases} 1 & \text{if } d_i \neq d_j \quad \text{and} \quad d_i = d_p \\ 0 & \text{Otherwise,} \end{cases}$$

and $\lambda \in [0, 1]$ is a measure of the transportation cost. We can interpret λ as the relative weight given to the design location with respect to quality in the sponsor's preferences. This contest success function captures both aims of the sponsor (to maximize quality and to get as close as possible to his preferred design location) and his initial uncertainty about his preferred design location.[6] Notice that for $\lambda = 0$ this contest success function coincides with the standard contest success function introduced by Tullock (1980).[7]

The timing of the model is the following:

1. Nature choose the distribution of the preferences of the sponsor defined by α and the marginal cost c_i of effort for each firm i. This becomes common knowledge.
2. The sponsor announces the contest and the prize P for the winning firm.
3. The competing firms choose simultaneously the design location d_i.
4. The firms choose simultaneously the effort level e_i to develop the chosen type of design.
5. The sponsor's preferred design is determined by nature.

6. We have interpreted the link between the effort exerted by firms and the contest success function as stochastic production of quality. We can also regard our contest success function as a reduced form of a complex comparison between the two proposals that may include some learning about the sponsor's preferences and some lobbying. Let e be a combination of both types of effort (producing quality and lobbying). The parameter α describes the sponsor's initial prior about the optimal design, which cannot be affected by firms' lobbying effort, and is learned when he sees the two proposals. Our parameter λ can now be interpreted as the relative weight given by the sponsor to his initial priors with respect to lobbying in the sponsor's preferences. In other words, λ is a measure of how the sponsor can be influenced by lobbying effort. Low λ means an easily influenceable sponsor. With this alternative interpretation, firms compete in producing quality if they locate at the same design, but they also use the lobbying process to convince the sponsor about the superiority of their quality in developing the design because quality is difficult to measure. If firms locate at different designs, lobbying is limited because the a priori preferences enter into the contest success function. This captures the idea that it should be more difficult to convince the sponsor that he prefers something which a priori he likes less than to convince him to choose one of two identical designs. We thank an anonymous referee for pointing this out to us.

7. Fullerton and McAfee (1999) have shown that Tullock's contest success function can be derived from the following model of quality production: the choice of e_i determines the number of identical and independent draws from some arbitrary distribution function over the interval [0, 1]. The resulting quality of these draws is the maximum of these random draws.

6. The winning firm is determined by nature according to the design proposals and the contest success function.

   The game is solved by backward induction. All the proofs are relegated to the appendix.

3. The Effort Decision

When choosing how much effort to put into developing their design, firms already know the design locations that were chosen. The different situations that the firms can face, can be summarized by the following two main cases:

1. If both firms chose the same design, namely (C, C) or (R, R), the effort in developing the design will be decisive, because no firm has a locational advantage with respect to the other, and the contest success function only depends on effort levels. Each firm's problem becomes

$$\begin{aligned}&\max_{e_i}\left[E_{d_p}\{p_i(d_i, e_i, d_j, e_j, d_p, \lambda)P - c_i e_i \mid d_i = d_j\}\right]\\&= \max_{e_i}\left[\frac{e_i}{e_i + e_j}P - c_i e_i \mid d_i = d_j\right].\end{aligned}$$

   It is now easy to show that the solution to this Nash game is characterized by the following first-order conditions:

$$\begin{aligned}&\frac{Pe_2}{(e_1 + e_2)^2} - c_1 = 0\\&\frac{Pe_1}{(e_1 + e_2)^2} - c_2 = 0.\end{aligned}$$

   Therefore firms' optimal effort levels are

$$\begin{aligned}e_1 &= \frac{Pc_2}{(c_1 + c_2)^2}\\e_2 &= \frac{Pc_1}{(c_1 + c_2)^2}.\end{aligned}$$

   In this case, the firm with lower costs makes the higher effort and has a bigger chance to be the winner of the contest. The expected profits of the firms are

$$\pi_1(C,C) = \pi_1(R,R) = \frac{Pc_2}{(c_1+c_2)} - \frac{Pc_2c_1}{(c_1+c_2)^2} = \frac{Pc_2^2}{(c_1+c_2)^2} = P\delta^2$$

and

$$\pi_2(C,C) = \pi_2(R,R) = \frac{Pc_1}{(c_1+c_2)} - \frac{Pc_1c_2}{(c_1+c_2)^2} = \frac{Pc_1^2}{(c_1+c_2)^2} = P(1-\delta)^2,$$

where $\delta = \frac{c_2}{c_1+c_2}$ represents the probability that firm 1 wins the contest if no firm has a comparative advantage in design location. Notice that $\delta \in [0.5, 1]$ because $c_2 \geq c_1$. Therefore, the profits of the more efficient firm are higher.

2. If one firm chooses the conservative location and the other firm a radical design, that is, (C, R) or (R, C), effort levels will depend on the relative importance of design location and quality as captured by our parameter λ. Assuming that firm i chooses the conservative location C, its problem becomes:

$$\max_{e_i} \left[E_{d_p}\{p_i(d_i, e_i, d_j, e_j, d_p, \lambda)P - c_ie_i \mid d_i = C \text{ and } d_j = R\}\right]$$
$$= \max_{e_i} \left[\left((1-\lambda)\frac{e_i}{e_i+e_j} + \lambda\alpha\right) P - c_ie_i \mid d_i = C \text{ and } d_j = R\right],$$

whereas firm j's problem becomes:

$$\max_{e_j} \left[\left((1-\lambda)\frac{e_j}{e_i+e_j} + \lambda(1-\alpha)\right) P - c_ie_j \mid d_i = C \text{ and } d_j = R\right].$$

Using the same arguments as in case 1, the optimal efforts for firm i and j are

$$e_i = \frac{(1-\lambda)c_jP}{(c_1+c_2)^2}$$
$$e_j = \frac{(1-\lambda)c_iP}{(c_1+c_2)^2}.$$

Notice that the effort level is lower than under case 1, and it is decreasing in λ. The more weight the sponsor puts on design location, the lower is the competition in effort levels. The expected profits of the firms, when firm 1 chooses the conservative design and firm 2 chooses the radical location, are

$$\pi_1(C,R) = \frac{(1-\lambda)c_2^2P}{(c_1+c_2)^2} + \lambda\alpha P = (1-\lambda)\delta^2P + \lambda\alpha P$$
$$\pi_2(R,C) = \frac{(1-\lambda)c_1^2P}{(c_1+c_2)^2} + \lambda(1-\alpha)P = (1-\lambda)(1-\delta)^2P + \lambda(1-\alpha)P.$$

Finally, the expected profits of the firms, when firm 1 chooses the radical design and firm 2 chooses the conservative location, are

$$\pi_1(R, C) = \frac{(1-\lambda)c_2^2 P}{(c_1+c_2)^2} + \lambda(1-\alpha)P = (1-\lambda)\delta^2 P + \lambda(1-\alpha)P$$

and

$$\pi_2(C, R) = \frac{(1-\lambda)c_1^2 P}{(c_1+c_2)^2} + \lambda\alpha P = (1-\lambda)(1-\delta)^2 P + \lambda\alpha P.$$

4. The Location Decision

Now, we can define the payoff matrix of the first-stage game as follows taking the second-stage effort levels into account.

F2

		C	R
F1	C	$\pi_1(C, C) = \delta^2 P$ $\pi_2(C, C) = (1-\delta)^2 P$	$\pi_1(C, R) = (1-\lambda)\delta^2 P + \lambda\alpha P$ $\pi_2(R, C) = (1-\lambda)(1-\delta)^2 P + \lambda(1-\alpha)P$
	R	$\pi_1(R, C) = (1-\lambda)\delta^2 P + \lambda(1-\alpha)P$ $\pi_2(C, R) = (1-\lambda)(1-\delta)^2 P + \lambda\alpha P$	$\pi_1(R, R) = \delta^2 P$ $\pi_2(R, R) = (1-\delta)^2 P$

Player 1 is the row player and player 2 is the column player. Player 1's payoffs are represented in the upper corner of each cell.

4.1 The Equilibrium Outcome

Proposition 1 characterizes the equilibrium outcome of the first stage.

Proposition 1: *The equilibrium in the first stage is*

1. *If* $(1-\delta)^2 > 1-\alpha$ *firms locate at* (C, C).
2. *If* $(1-\delta)^2 < 1-\alpha$ *and* $\delta^2 > \alpha$*, both firms use a mixed strategy. Firm 1 locates at C with probability* β^* *and firm 2 locates at C with probability* σ^**, where* β^* *and* σ^* *are*

$$\beta^* = \frac{\alpha-(1-\delta)^2}{1-2(1-\delta)^2}$$

and

$$\sigma^* = \frac{\delta^2-\alpha}{2\delta^2-1}.$$

3. *If* $(1-\delta)^2 < 1-\alpha$ *and* $\alpha > \delta^2 > 1-\alpha$*, firms locate at* (C, R).

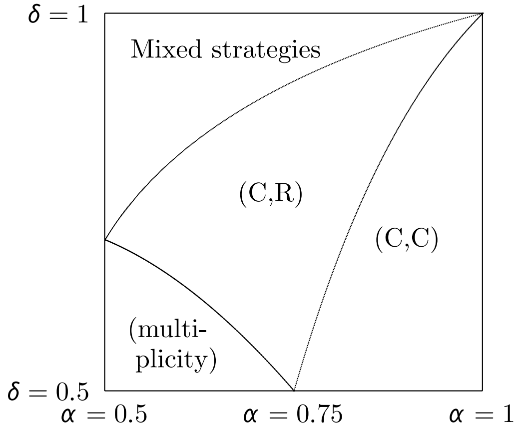

FIGURE 1. EQUILIBRIUM OUTCOMES AS FUNCTIONS OF α AND δ

4. *If* $(1-\delta)^2 < 1-\alpha$ *and* $\delta^2 < 1-\alpha$, *there is multiplicity of equilibria. The two pure equilibria are (i) firm 1 locates at C whereas firm 2 chooses location R, (ii) firm 1 locates at R whereas firm 2 chooses location C. In the mixed-strategy equilibrium, firm 1 locates at C with probability* β^{**} *and firm 2 locates at C with probability* σ^{**}, *where* β^{**} *and* σ^{**} *are*

$$\beta^{**} = \frac{\alpha - (1-\delta)^2}{1 - 2(1-\delta)^2}$$

and

$$\sigma^{**} = \frac{\alpha - \delta^2}{1 - 2\delta^2}.$$

The equilibrium outcomes for all possible values of α and δ are illustrated in Figure 1.

Figure 1 shows how the equilibrium outcomes vary with α and the cost differences (comparative (dis)advantage) of the two firms. First, notice that the solution of the location stage does not depend on the parameter λ, although effort decisions and profits do.

The solution of the game is quite intuitive. Consider α larger than 0.75. In this case, the conservative design has a big advantage over the radical design. Therefore, if the comparative cost advantage of firm 1 is not very large, none of the firms wants to give up the privilege of being located at the conservative design: the equilibrium is (C, C); firms only compete in effort levels. For intermediate levels of comparative advantage, the inefficient firm will give up the conservative design and locates at the radical design because the competition in effort levels is too costly for this firm. In this case, there is maximal differentiation of design locations, and the total effort exerted by firms is reduced. Finally, if the comparative advantage is large, the efficient firm has a very high probability of winning the competition in effort levels. This firm therefore tries to force this competition by choosing the same design as the inefficient firm, whereas the latter tries to avoid the competition in effort levels by choosing a different location than the efficient firm.[8] In other words, the inefficient firm wants to achieve differentiated designs, whereas the efficient firm wants to achieve identical designs. This is a special case of the game called "matching pennies." The equilibrium in this case is in mixed strategies.

If α is lower than 0.75, the analysis of the equilibria is the same except for small values of δ. If the difference in competitive advantage is small, the equilibrium changes from (C, C) to multiplicity of equilibria. Now the sponsor's preferences for the conservative design are not very strong. Therefore, if the inefficient firm locates at the conservative design, the efficient firm prefers to choose the radical design to avoid competition in effort levels and vice versa. This is a special version of the game called "chicken game." Both firms prefer to be located at the conservative design, but the first objective is to avoid competition in effort levels.

To finalize the characterization of the equilibrium, we want to highlight the possible equilibrium configurations with symmetric duopolists $\delta = 0.5$. In this location, game in which the comparative advantage plays no role, there are two main effects that determine firms' choices: (i) both firms want to be located in the *ex ante* more profitable design and (ii) they prefer to avoid competition by choosing different design locations.

When $\alpha < 0.75$, the second effect dominates and the pure strategy equilibria exhibits maximum design differentiation ((C, R) or (R, C)). However, a closer look at the mixed-strategy equilibrium reveals that things are more continuous than they look like. The mixed-strategy equilibrium has four possible final outcomes, $\{(C, C), (R, R), (C, R),$ and $(R, C)\}$, and firms locate at the conservative design with probability $2\alpha - \frac{1}{2}$. Only the probability of (C, C) is increasing in α and converges to 1, at $\alpha = 0.75$, whereas the probability of the other outcomes is decreasing in α and converges to 0 at $\alpha = 0.75$. The intuition of this result is clear: when α increases the profitablility, the conservative design increases. Hence, firms are more eager to locate at this design, and more reluctant to avoid competition by choosing the less profitable design. When

8. This result is similar to Cabral (1999), where the laggard wants to differentiate from the leader, whereas the leader wants to follow the follower.

$\alpha > 0.75$, no firm is willing to lose the opportunity of being located at the conservative design.

The next corollaries provide additional characterizations of the equilibrium.

Corollary 1: *Let* $0.5 < \alpha < 1$. *(i) Maximal differentiation in design locations is obtained for intermediate levels of comparative advantage. (ii) The effort level is non-monotonic in the comparative advantage of firm 1.*

Corollary 1 has important implications for a more general model with optimal entry into the tournament. It shows why well established results of optimal entry into tournaments might be modified by the introduction of horizontal competition, in particular, the results that the sponsor wants to induce the participation of the two lowest-cost contestants and that any technological improvement that implies some cost reduction of the participating firms always increases the overall effort exerted by firms. Part (i) of Corollary 1 tells us that whether or not entry by the lowest-cost contestants is optimal depending on the relative weight the sponsor gives to the design location. In particular, if λ is large, that is, the sponsor mainly cares about design, this sponsor would like to induce participation in the contest of firms that have not too different and equal costs to achieve maximal differentiation in design locations, which guarantees a better match of one of the design proposals with the sponsor's preferred design. Part (ii) of Corollary 1 states how differences in efficiency will determine the level of effort competition, in particular, total effort is decreasing in δ in first place, when the equilibrium changes from Multiplicity or (C, C) to (C, R). When δ is large enough, total effort is increasing in δ because the equilibrium changes from (C, R) to the mixed-strategy equilibrium. The first part of this result—the larger the disadvantage between firms, the lower the competition—is found in many models of IO. However, the second part—for large enough disadvantages competition increases—is more surprising, and it is an insight that is related to the multi-dimensional competition in horizontal and vertical dimensions. The efficient firm wants to be as homogenous as possible to the inefficient firm in the horizontal dimension, because it is confident that the difference in efficiency will be decisive in the vertical competition. An interesting and unintuitive consequence of part (ii) of Corollary 1 is that a reduction in costs of the most efficient firm can lead to a reduction in effort of both firms and therefore in expected quality. To illustrate this point, assume that before the cost reduction both firms choose the central design. The resulting sum of effort levels is $e_1 + e_2 = \frac{P}{c_1 + c_2}$. A cost reduction of firm 1 to c_1' might imply that it is beneficial for firm 2 to move to the radical design location resulting in an overall effort level of $\frac{(1-\lambda)P}{c_1' + c_2}$. It is easy to see that there exist some parameter values of c_1' and λ such that the overall effort level is lower after the cost reduction of firm 1.

The results of Corollary 1 depend on the model assumption that firms choose their design location simultaneously[9] and on the fact that we are considering α to be an interior point. For the limit cases of $\alpha = \frac{1}{2}$ and $\alpha = 1$, Corollary 1 does not hold. When $\alpha = 1$, there is no uncertainty about sponsor's preferences, and therefore both firms locate at the sponsor's preferred design. The horizontal dimension does not play any role. The case of $\alpha = \frac{1}{2}$ is more interesting. Our results depend on the fact, that when the differences in costs are not important, firms choose similar location strategies. However, when the differences in efficiency increase, the less efficient contestant is willing to choose the *ex ante* less attractive design to avoid competition. This strategic alternative does not arise for the inefficient firm when $\alpha = \frac{1}{2}$, because there does not exist an *ex ante* less attractive design. In other words, the inefficient firm cannot avoid competition by giving up the attractive design. Therefore, when the preferences of the sponsor are symmetric, cost differences are not expected to generate diversity.

Corollary 2: *For all parameter values with a unique equilibrium, the firm with the cost advantage makes higher (expected) profits. The profits of the less efficient firm are increasing in* λ.

The first part of the result is intuitive and does not need further explanations. When λ increases, the competition in effort levels decreases because the horizontal competition becomes more important. This reduction in competition in effort levels increases the profits of both firms: less effort is exerted, which reduces not only the firms' cost, but also the comparative advantage of the most efficient firm. As both effects reinforce each other in case of the less efficient firm, we can conclude that its profits are increasing in λ.

Corollary 3: *For all parameter values with a unique equilibrium, the disadvantaged firm is more likely to choose a radical design.*

The intuition of this result is straightforward: given its comparative disadvantage when competing in effort levels, the inefficient firm has

9. When $\alpha < 0.75$, bidders have incentives to diversify in designs; if they do not do so, it is due to a coordination failure given the simultaneous nature of the game. If firms chose the design location sequentially, they would not locate at the same design for low values of δ, and effort levels would be monotonic in δ. Hence for low values of α, cost differentiation is important in promoting design diversity, as long as a coordination failure can take place. We believe that this coordination failure is appealing: on the one hand, in many situations it is hard to assume that contestants can observably commit to their strategies; on the other hand, although both contestants want to avoid competition, both of them want to be located at the conservative design, and this makes the coordination failure more likely.

a bigger interest than the efficient firm in achieving an equilibrium with differentiated design locations. Because the radical design is the less attractive design, choosing the radical design is a way to get the differentiated equilibrium.

5. Conclusions

In this paper, we have developed a very stylized model of horizontal and vertical competition in tournaments. We show that introducing horizontal competition in a standard research tournament can change some well-known results. In particular, entry into the tournament by the lowest-cost contestants may not be optimal for a sponsor who mainly cares about design, because some cost difference between firms can induce them to choose different design locations. Furthermore, it is shown that the importance of cost differentiation in promoting diversity increases in the bias of the sponsor's preferences. In the limit case when the sponsor's preferences are completely symmetric, we do not expect efficiency differences to play a role in promoting diversity—and they do not in our model. We have argued that our model also fits R&D races where the design location should be interpreted as the R&D path taken. The race for the blue laser diode nicely illustrates that it might be in the sponsor's interest that also a priori less efficient research paths are exploited.[10] The blue laser was developed by the disadvantaged contestant working with the a priori worse material. This material was chosen to avoid competition, exactly as our model predicts.

Despite the relevance of the problem, there is scarce research on horizontal and vertical competition in the literature. The main reason for this is that models with competition in several dimensions tend to become easily intractable, and few specific results can be derived. Our aim in constructing the model was to make its structure as simple as possible, while still capturing the relevant aspects of the problem. The advantage of proceeding in this manner is that it enables us to obtain clear explicit results with only a limited amount of computations. The drawback of this tractability is that we have imposed several restricted assumptions, and we need to clarify to what extent the results can be generalized. We therefore discuss now in how far our results are driven

10. The US has recognized this need in military aeronautical engineering: in 1943 the so-called Skunk Works was created to design and develop the P-80 Shooting Star. Skunk Works was supposed to follow "crazy ideas" and most of the aeronautical military innovations came from this special research group, for example, the U-2, A-11, A-12, the F-104 Starfighter, the SR-71 Blackbird, and the Sea Shadow stealth ship. In the meantime, skunk work has become the general name for a special working environment for special projects involving a variety of special conditions.

by our specific modeling assumptions, in particular by (i) our contest success function and (ii) our design space.

Our contest success function, which converges to the standard contest success function when firms only compete in quality (effort levels), is very convenient because the results of the location game do not depend on the relative weight that the sponsor gives to the design location compared to the design quality (λ in our model). This effect, which is due to the mixture of having two possible designs only and the contest success function depending linearly on the locational advantage, clarifies that our results can be obtained in setups in which the sponsor mainly cares about the horizontal competition and environments in which competition in effort levels is the sponsor's main concern. If we use an arbitrary contest function, we expect the equilibrium location to depend on the relative weight of horizontal competition in the contest success function. Using our notation, the location equilibrium will depend in general on λ. Nevertheless, we are confident that this will not affect the qualitative nature of our results because our results are not due to the sponsor caring mainly about the design location.

We are also confident that our results extend to richer design spaces *as long as the design spaces are discrete*. Further research is needed to extend the present analysis to a continuous design space like the Hotelling line.[11] In a continuous design space, it is more difficult to characterize the equilibrium because there are many possible deviations in the location stage. We conjecture that we need to impose very restrictive assumptions about the sponsor's preferences concerning design location to get some analytical results. On top of that, there are many real-word situations that are better described by a discrete design space, for example, in the race for the blue LED the possible materials to develop a blue laser clearly belonged to a discrete choice set. In the case of the Florentine Republic, it is not clear that the convex combination of the designs would have been a realistic design space. In other words, our model describes economic situations in which the design space is discrete. A model with N possible designs that captures this idea nicely is an adaptation of the so-called spokes model (Chen and Riordan, 2003) to our setup.[12] In the

11. But for a discrete Hotelling line, our results would go through. To see this point, notice that the present model with two design locations could be translated into a model with a central design and two extreme designs that are equally likely to be the principal's preferred design. In a previous version of the present paper, we analyzed this alternative model, and the results are qualitatively the same.

12. The special feature of the spokes model is that the distance in terms of sponsor's utility between two designs is always the same. We do not believe that our results would be qualitatively different if the distance varied with the pairs of design under consideration, although the locational equilibrium would depend on λ, and the analysis would be very tedious.

working paper version of this article, we analyze this adjusted spokes model with two competing firms. We show that the introduction of N possible designs does not alter the equilibria of the game: the firms will ignore all possible designs but two, namely, the two designs that are *ex ante* most likely to be the sponsor's preferred designs. The equilibrium choice of these two designs coincides with the pure strategy equilibria of the game in which there are only two designs from the beginning.[13] In other words, our model is robust to the introduction of more designs.

To our knowledge, the present paper is the first to study vertical and horizontal competition in *tournaments*. On the other hand, there exists a literature focusing on multi-dimensional product differentiation. In how far is our analysis restricted to the tournament context and how do our results relate to this literature?

The fundamental difference of our tournament model to existing models with multidimensional product differentiation is that our firms do not compete in prices. Therefore, the analysis of our model is very different from the analysis of models in the industrial organization literature. However, we conjecture that our model can be applied to markets where the firms' differentiation decision is discrete. For example, our model can be useful to understand an episode of the browser war between Netscape and Microsoft. In that market, Microsoft started distributing the Internet Explorer for free, and Netscape was forced to match Microsoft's policy of giving away its Navigator. Once the prices are zero, firms are constrained to compete mainly in horizontal and vertical dimensions, as in our model. In January of 1998, Netscape decided to make the source code of its browser public. With this move, Netscape captured the market niche of sophisticated programmers who were now able to customize the Communicator browser to their own idiosyncratic needs. It was clear that Microsoft would not want to make its browser code public. Microsoft's success is based on uniformity[14] and indeed Microsoft kept this conservative strategy. In other words, Netscape chose a different design to capture a different, but smaller market niche, thereby avoiding competition in the main market niche where its chances of surviving were small. Notice that this fits well with the prediction of our model. According to our model the disadvantaged firm should move from the conservative strategy to the radical strategy to avoid competition when the comparative disadvantaged becomes large. It seems that the advantage of Microsoft

13. The interested reader can download this result from the following webpage: http:/www.econ.upf.edu/~hauk/allocating.

14. For more details, see Shapiro and Varian (1998).

was increasing at that time since it was increasing its market share continuously.[15]

In the Netscape–Microsoft example design competition boils down to competition for different market niches, where consumers' heterogeneity is clearly segmented discretely. Because we do not know how our model extends to continuous design locations, the model can only be applied to contexts where consumers' heterogeneity is discrete. This is a further difference to most models in the industrial organization literature where consumers' heterogeneity is usually assumed to be continuous (Neven and Thisse, 1990; Heeb, 2001). Exceptions are Adner and Zemsky (2001), Motta and Polo (1997), and Nilssen and Sorgard (1998). Adner and Zemsky (2001) is closest to our model. They study the phenomenon of disruptive technologies that initially serve isolated market niches and—as they mature—replace existing technologies in mainstream segments. The quality of each technology is assumed to follow an exogenously fixed trajectory. This is the main difference to our model where the vertical competition is endogenous.

In Motta and Polo (1997), TV channels are horizontally differentiated, but this is taken as an exogenous variable, and it is analyzed how this variety affects the quality decisions of firms. In Nilssen and Sorgard (1998), two TV channels compete with respect to time schedule and program profile. Time schedule is a continuous choice variable; there are only two possible program profiles. This is similar to our setup where effort levels are chosen continuously, whereas design location is a discrete choice. Notice, however, that in Nilssen and Sorgard (1998), both decisions are costless and horizontal in nature. Some viewers prefer one program over the other and all viewers like different time schedules. In contrast, in our model, the competition in effort levels is vertical (higher quality is always better) and costly.

There are two types of models with continuous consumers' heterogeneity: models in which the multidimensional product differentiation is only horizontal in nature (Irmen and Thisse, 1998) and models that are closer to ours in that they combine both vertical and differential competition (Neven and Thisse, 1990; Heeb, 2001). Irmen and Thisse (1998) extend Hotelling's analysis to an n-dimensional characteristic space and show that firms will only compete severely along one dimension (maximum differentiation) and locate at the same point in all other dimensions. Neven and Thisse (1990) show that for horizontal and vertical competition the principal of minimum differentiation holds in one dimension, whereas maximum differentiation holds in the other.

15. In 1995 Microsoft had virtually zero market share and Netscape had 90%. Four years later, in August of 1999, Microsoft's share was 76%, whereas Netscape had only 23% (see www.statmarket.com).

Heeb (2001) shows that this max–min principle crucially depends on the assumption of market coverage and taste symmetry. Both models are very different to ours because they work with continuous design spaces.

There are obviously many further elements that should be included in a more full-fleshed model of contests of ideas. In particular, we would like to work on the following extensions in the future:

1. In our model, firms' cost are independent of the design location. In many real-world situations, it might be the case that one firm has a comparative advantage in producing some specific design but a disadvantage on a different design location. Consider, for example, the contest for organizing the Olympic Games, which works as follows. To elect the host city, the International Olympic Committee (IOC) calls for applications for candidacy. Interested cities submit their official candidature files to the IOC. The IOC Executive Board evaluates the applications and selects the winning city. In this bidding process, cities compete in several dimensions: some cities offer better sport infrastructures, whereas other cities offer more accommodation capacity or a better transportation system. It is clear, that some cities have a initial advantage over some dimension, and other cities over other dimensions. In future research, we would like to extend our model to capture these cost discrepancies.
2. In our contest, there are only two competing firms. As we already said in the introduction, contests of ideas are usually called for the sponsor to get ideas and learn about his preferences. Therefore, it might be optimal to have more than two competing firms. An important question for future research is to extend the model to N bidders and to study optimal entry decisions. This might enable us to find the optimal contest form for this environment. Our conjecture is that the optimal contest must have two stages. The first stage serves to select the most interesting proposals. In the second stage, the selected firms compete vertically developing their proposals.
3. In our model, firms' beliefs about the sponsor's preferences are given. Hence, we do not consider that the sponsor can manipulate these beliefs. This is certainly true in the case of the blue laser; the superiority of one material over the other was public, nonmanipulable information. It is also true if sponsor's preferences are driven by current fashions and therefore a mean of distributions of preferences as in our motivating example of the Florentine republic; the neoclassical style was less known and less appreciated at this time. However, there are other situations in which the sponsor might be in a position to try to manipulate firms' beliefs about his preferences. From our

model, it is clear that the sponsor might have incentives to announce a preference parameter different than his real α. If, for example, the real parameter causes firms to choose the same design in equilibrium, the sponsor may prefer design differentiation to warranty a good matching between preferences and designs. One possible way to obtain that is to convince the contestants that the parameter α is lower than the real one. Of course this announcement may not be credible. In general, the sponsor may want to be ambiguous about his preferred design, because he wants to insure himself against a mismatching between preferences and designs. This "strategic ambiguity" is an interesting topic for future research.

APPENDIX

Proof of Proposition 1. Notice that for firm 2, the best strategy when firm 1 plays radical is to play C, because $(1-\delta)^2 < \frac{1}{4}$ and $\alpha > \frac{1}{4}$. To solve the first-stage game, we have to distinguish four cases.

Case 1: $(1-\delta)^2 > 1-\alpha$. In this case, it is a dominant strategy for firm 2 to choose C, because $(1-\delta)^2 > 1-\alpha \Rightarrow (1-\delta)^2 P > (1-\lambda)(1-\delta)^2 P + \lambda(1-\alpha)P$. If firm 2 plays C, the best response of firm 1 is to play C, because $(1-\delta)^2 > 1-\alpha \Rightarrow \delta^2 > 1-\alpha \Rightarrow \delta^2 P > (1-\lambda)\delta^2 P + \lambda(1-\alpha)P$. Hence, the only Nash equilibrium is that both firms choose the central design (C, C).

Case 2: $(1-\delta)^2 < 1-\alpha$ and $\delta^2 > \alpha$. In this case, there is no equilibrium is pure strategies, because firm 1 wants to choose firm 2's location and firm 2 wants to choose a different location than firm 1. To analyze the equilibrium in mixed-strategies, we compute the best-response function of the firms. Let $\beta(\sigma)$ be the optimal probability with which firm 1 chooses C if firm 2 plays C with probability σ. Straightforward calculations show that

$$\beta(\sigma) = \begin{cases} 1 \text{ if } \sigma > \dfrac{\delta^2-\alpha}{2\delta^2-1} \\ \in [0,1] \text{ if } \sigma = \dfrac{\delta^2-\alpha}{2\delta^2-1} \\ 0 \text{ if } \sigma < \dfrac{\delta^2-\alpha}{2\delta^2-1}. \end{cases}$$

Let $\sigma(\beta)$ be the optimal probability with which firm 2 chooses C if firm 1 plays C with probability β. Straightforward calculations show that

$$\sigma = \begin{cases} 0 \text{ if } \beta > \dfrac{\alpha - (1-\delta)^2}{1 - 2(1-\delta)^2} \\ \in [0, 1] \text{ if } \beta = \dfrac{\alpha - (1-\delta)^2}{1 - 2(1-\delta)^2} \\ 1 \text{ if } \beta < \dfrac{\alpha - (1-\delta)^2}{1 - 2(1-\delta)^2}. \end{cases}$$

Given these reaction functions, the equilibrium in mixed-strategies is that firm 1 plays C with probability $\beta^* = \frac{\alpha - (1-\delta)^2}{1 - 2(1-\delta)^2}$ and firm 2 plays C with probability $\sigma^* = \frac{\delta^2 - \alpha}{2\delta^2 - 1}$.

Case 3: $(1 - \delta)^2 < 1 - \alpha$ and $\alpha > \delta^2 > 1 - \alpha$. In this case, it is a dominant strategy for firm 1 to choose C, because $\delta^2 > 1 - \alpha \Rightarrow \delta^2 P > (1 - \lambda)\delta^2 P + \lambda(1 - \alpha)P$ and $\delta^2 < \alpha \Rightarrow \delta^2 P < (1 - \lambda)\delta^2 P + \lambda\alpha P$. If firm 1 plays C, the best response of firm 2 is to play R, because $(1 - \delta)^2 < 1 - \alpha \Rightarrow (1 - \delta)^2 P < (1 - \lambda)(1 - \delta)^2 P + \lambda(1 - \alpha)P$. Hence, the only Nash equilibrium is (C, R).

Case 4: $(1 - \delta)^2 < 1 - \alpha$ and $\delta^2 < 1 - \alpha$. On the one hand, the best response of firm 1 if firm 2 plays C is to play R, because $\delta^2 < 1 - \alpha \Rightarrow \delta^2 P < (1 - \lambda)\delta^2 P + \lambda(1 - \alpha)P$ and if firm 2 plays R is to play C, because $\delta^2 < 1 - \alpha \Rightarrow \delta^2 < \alpha \Rightarrow \delta^2 P < (1 - \lambda)\delta^2 P + \lambda\alpha P$. On the other hand, the best response of firm 2 if firm 1 plays C is to play R, because $(1 - \delta)^2 < 1 - \alpha \Rightarrow (1 - \delta)^2 P < (1 - \lambda)(1 - \delta)^2 P + \lambda(1 - \alpha)P$ and if firm 1 plays R is to play C, because $(1 - \delta)^2 < 1 - \alpha \Rightarrow (1 - \delta)^2 < \alpha \Rightarrow (1 - \delta)^2 P < (1 - \lambda)(1 - \delta)^2 P + \lambda\alpha P$. Therefore, it is easy to see that we have two pure equilibria in this case: (R, C) and (C, R). Similar calculations as in Case 2 allow us to derive the mixed equilibrium. □

Proof of Corollary 1.

(i) From Figure 1 (which shows the equilibrium of the location game), it is clear that the maximal differentiation equilibrium (C, R) is obtained for intermediate levels of comparative advantage δ. In all other equilibria, there is always a positive probability that firms end up with the same design. According to Proposition 1 and its proof (C, R) is obtained when $\delta \in (Max\{\sqrt{1-\alpha}, 1 - \sqrt{1-\alpha}\}, \sqrt{\alpha})$ where $\frac{1}{2} < Max\{\sqrt{1-\alpha}, 1 - \sqrt{1-\alpha}\} < \sqrt{\alpha} < 1$ if $\alpha \in (\frac{1}{2}, 1)$.

(ii) Without loss of generality, we assume that $c_1 + c_2 = 1$ (this is a normalization). To show that the effort level is nonmonotonic in the comparative advantage of firm 1 we have to differentiate between two cases: Case 1 $\alpha > 0.75$ and Case 2 $\alpha < 0.75$.

Case 1. Let $\alpha > 0.75$. If $(1-\delta)^2 > 1-\alpha$ the equilibrium is (C, C) and the effort exerted by the firms are

$$e_1 = \frac{c_2 P}{(c_1+c_2)^2} = \delta P \text{ and } e_2 = \frac{c_1 P}{(c_1+c_2)^2} = (1-\delta)P.$$

If $(1-\delta)^2 < 1-\alpha$, and $\alpha > \delta^2 > 1-\alpha$, the equilibrium is (C, R) and the effort exerted by the firms are

$$e_1 = \frac{(1-\lambda)c_2 P}{(c_1+c_2)^2} = (1-\lambda)\delta P \text{ and } e_2 = \frac{(1-\lambda)c_1 P}{(c_1+c_2)^2} = (1-\lambda)(1-\delta)P.$$

Therefore, for a given α, when the cost advantage of firm 1 increases, the equilibrium changes from (C, C) to (C, R), and the firm's effort decreases. But if δ increases more such that $\delta^2 > \alpha$, we know from proposition 1 that the location stage has a mixed-strategy equilibrium and the expected effort exerted by the firms are:

$$E\{e_1\} = (1-\gamma)\delta P + \gamma(1-\lambda)\delta P \text{ and}$$
$$E\{e_2\} = (1-\gamma)(1-\delta)P + \gamma(1-\delta)(1-\lambda)(1-\delta)P,$$

where $\gamma = \beta^* + \sigma^* - 2\beta^*\sigma^*$. These effort levels are higher than those corresponding to the equilibrium (C, R).

Case 2. Let $\alpha < 0.75$. If $\delta^2 < 1-\alpha$ there is multiplicity of equilibria. But this implies, that with some probability the mixed-strategy equilibrium will be played. In the mixed-strategy equilibrium, there exists a positive probability $\epsilon > 0$ that firms locate at the same design. The *ex ante* equilibrium effort will be

$$E\{e_1\} = (1-\epsilon)\delta P + \epsilon(1-\lambda)\delta P \text{ and}$$
$$E\{e_2\} = (1-\epsilon)(1-\delta)P + \epsilon(1-\delta)(1-\lambda)(1-\delta)P.$$

For $\alpha > \delta^2 > 1-\alpha$, the equilibrium is (C, R) and the effort exerted by the firms are

$$e_1 = \frac{(1-\lambda)c_2 P}{(c_1+c_2)^2} = (1-\lambda)\delta P \text{ and}$$
$$e_2 = \frac{(1-\lambda)c_1 P}{(c_1+c_2)^2} = (1-\lambda)(1-\delta)P$$

Therefore, for a given α, when the cost advantage of firm 1 increases, the equilibrium changes from multiplicity of equilibria to (C, R) and firms' effort decreases. But if δ increases more such that $\delta^2 > \alpha$, we know from

Proposition 1 that the location stage has a mixed-strategy equilibrium and the expected effort exerted by the firms are

$$E\{e_1\} = (1-\gamma)\delta P + \gamma(1-\lambda)\delta P \text{ and}$$

$$E\{e_2\} = (1-\gamma)(1-\delta)P + \gamma(1-\delta)(1-\lambda)(1-\delta)P,$$

where $\gamma = \beta^* + \sigma^* - 2\beta^*\sigma^*$. These effort levels are higher than those corresponding to the equilibrium (C, R). □

Proof of Corollary 2. If both firms choose the same design (Case 1: $(1-\delta)^2 > 1-\alpha$) $\Pi_1 = \delta^2 P > \Pi_2 = (1-\delta)^2 P$, since $\delta = \frac{c_2}{c_2+c_1} > \frac{1}{2}$. If firm 1 locates at C, whereas firm 2 chooses a radical design (Case 3: $(1-\delta)^2 < 1-\alpha$ and $\alpha > \delta^2 > 1-\alpha$), $\Pi_1 = \delta^2 P > (1-\alpha)P > (1-\lambda)(1-\delta)^2 P + \lambda(1-\alpha)P = \Pi_2$. Finally, if both firms mix over design locations (Case 2: $(1-\delta)^2 < 1-\alpha$ and $\delta^2 > \alpha$.), observe that $\Pi_1 = \sigma^*\delta^2 P + (1-\sigma^*)((1-\lambda)\delta^2 P + \lambda\alpha P) > \alpha$ because $\delta^2 > \alpha$, and $\Pi_2 = (1-\beta^*)(1-\delta)^2 P + \beta^*((1-\lambda)(1-\delta)^2 P + \lambda(1-\alpha)P) < (1-\alpha)$ because $(1-\delta)^2 < 1-\alpha$. Therefore, $\Pi_1 > \alpha > 1-\alpha > \Pi_2$. This conclude the proof. □

Proof of Corollary 3. From Proposition 1, it can be seen that for some parameter values, the disadvantaged firm chooses the radical design, whereas the advantaged firm chooses the conservative design. We only have to show that when both firms choose a completely mixed design, location $\beta^* > \sigma^*$ always.

$$\beta^* - \sigma^* = \frac{(2\delta^2-1)(\alpha-(1-\delta)^2) - (1-2(1-\delta)^2)(\delta^2-\alpha)}{(1-2(1-\delta)^2)(2\delta^2-1)}.$$

Straightforward calculations show that

$$\beta^* - \sigma^* = \frac{(2\alpha-1)(\delta^2-(1-\delta)^2)}{(1-2(1-\delta)^2)(2\delta^2-1)} > 0.$$ □

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