Redistributive Taxation and Public Expenditures\textsuperscript{1}

Sanghamitra Bandyopadhyay\textsuperscript{\dag} and Joan Esteban\textsuperscript{\ddag}
\textsuperscript{\dag}Department of Economics, London School of Economics.
Houghton Street, London WC2A 2AE.
s.bandyopadhyay@lse.ac.uk
\textsuperscript{\ddag}Institut d’Analisi Economica, CSIC, 08193 Bellaterra-Barcelona, Spain
joan.esteban@iae.csic.es

April 3, 2008

\textsuperscript{1}We are grateful to Tony Atkinson, Jordi Caballe, Ramon Caminal, Frank Cowell, Oliver Linton and Laura Mayoral for helpful discussions. Joan Esteban gratefully acknowledges financial support from the Polarization and Conflict Project CIT-2-CT-2004-506084 funded by the European Commission-DG Research Sixth Framework Programme, Barcelona Economics (XREA), CICYT grant no. SEJ2006-00369, and the Instituto de Estudios Fiscales.
Abstract

We introduce a model of redistributive income taxation and public expenditure. Besides redistributing personal income by means of taxes and transfers, the government supplies goods and services. The government chooses the tax schedule that achieves consensus among the population. We show that there is a unique income tax schedule that is universally acceptable. The progressivity of the income tax is shown to depend on the composition of the public expenditure and on the substitutability between the goods and services supplied by the government and the consumption goods privately obtained through the market. We test the empirical implications of the model. Specifically, we use OECD data to observe the relationship between marginal tax rates and the distribution over the taxpayers of the benefits produced by the specific composition of the government expenditure in the provision of goods and services. We confirm that for lower elasticities of substitution between public and private goods, there is a negative relationship between marginal tax rates and pro-taxpayer-bias, and for higher elasticities, there is a positive relationship.

JEL-Classification: H23, H50, 050

Key-words: Government policy, Income Taxation, Public Expenditure
1 Introduction

In this paper we study the interdependence between the income redistribution policies and the composition of public expenditure. The literature on income taxation has mostly focused on the disincentive effect of taxation on labour supply. In contrast, we emphasise that besides redistributing income, income tax also finances the public provision of a set of goods and services that are valued by individuals. Higher taxes reduce private consumption but may increase the supply of the goods provided by the government. Therefore, the individual valuation of alternative tax and expenditure policies depends upon the balance between private consumption and public supply of goods. Instead of assuming that income taxes are selected by majority voting, as done in the standard literature, we focus on the tax schedules that achieve consensus. We show that there is a unique tax schedule which satisfies this acceptability criterion and examine its properties. The progressivity of the income tax is shown to depend on the composition of the public expenditure and on the substitutability between the goods and services supplied by the government and the consumption goods privately obtained through the market.

Thereafter, we test the empirical implications of the model. We use OECD data to observe the relationship between marginal tax rates and the distribution over the taxpayers of the benefits produced by the specific composition of the government expenditure in the provision of goods and services. We use threshold regressions to confirm that for lower elasticities of substitution between public and private goods, there is a negative relationship between marginal tax rates and pro-taxpayer-bias, and for higher elasticities, there is a positive relationship.

We divide the commodities into two sets as to whether they are obtained through the market or through the government. The government, besides redistributing income through taxes and cash transfers,\(^1\) supplies goods and services such as general administration, education, health, law-and-order, infrastructures, culture, or defence, which are in turn financed by the net revenue of the income tax (net of the social transfers). We investigate the relationship between the redistribution of money income through taxes and transfers and the composition of public expenditures in goods and services. These two aspects have mostly been studied in separate realms of public eco-

\(^1\)The redistributive task of the government has been the object of extensive studies by Moene and Wallerstein (2001a), (2001b) and (2003) and Alesina and Glaeser (2004). Their concern is the relationship between the pre-tax income inequality, the progressivity of the income tax and the size of the social transfers, and do not address the public provision of goods and services.
nomics, and not in conjunction.\textsuperscript{2} Our aim, however, is restricted to the choice of income taxes and hence consider expenditure policy to be exogenous.

The structure of public spending depends upon two types of decisions (both of which we consider to be exogenous). The first type of decision concerns which commodities are supplied by the government and those which are produced by the private sector. This includes regulations on whether the government reserves monopoly of supply, (also that of close substitutes), or permits various degrees of concurrence through the private market. These decisions determine both the composition and substitutability between public and private bundles. For instance, security or mail services were the monopoly of the government until a few decades ago. The stricter the monopoly of the state, and larger the number of commodities included, the smaller is the substitutability between the public and private bundles of commodities. Therefore, cross-country variation in the substitutability between the two bundles is largely imputable to government decisions rather than to variations in preferences. In our empirical tests we proxy this substitutability by the ratio of private to public expenditures in education and health.

The second type of decision - again, exogenous - refers to the quantities supplied of each commodity. The supply of goods and services by the government constitutes a large share of the public budget compared to social cash or near-cash transfers.\textsuperscript{3} Moreover, it is well-known that different types

\textsuperscript{2}The incidence and efficiency of income taxation has usually been analyzed in abstraction from public expenditures. Guesnerie and Roberts (1984) and Cremer and Gahvari (1997) show that welfare can be improved by some public supply of goods. Meltzer and Richard (1985), Bergstrom and Blomquist (1996), Pirttila and Tuomala (2002), and Blomquist and Christiansen (2007) focus on the publicly supplied good being complementary to labour (e.g. day care). An increase in the supply will increase the productivity and the tax collection. Besley and Coate (1991), Bergstrom and Blomquist (1996), and Blomquist and Christiansen (1995) study the case of publicly provided goods whose quantity/quality cannot be supplemented through the private market, for example public vs private education. Boadway and Marchand (1995) study a system that allows supplementation. Blomquist and Christiansen (1998) compare the relative merits of the two systems. In Epple and Romano (1996) individuals vote over the level of the public provision of a good and the budget balancing proportional income tax. Since the only redistribution possible is through the publicly supplied good this is (nearly) tantamount to choosing a budget balanced linear income tax. The macroeconomics literature has also given some attention to the impact of government spending on individual behaviour. The key issue here has been whether public and private expenditures are complements or substitutes. Barro (1981), Aschauer (1985), Christiano and Eichenbaum (1992), Baxter and King (1993), Karras (1994), Ahmed and Yoo (1995), Amano and Wirjanto (1998), and Cardia, Kozhaya, and Ruge-Murcia (2003) are all relevant contributions.

\textsuperscript{3}For the UK 2004/2005 the average yearly non-contributory social cash and near-cash benefits were 40 percent of the benefits in kind received from the public provision of education and health (Jones 2006).
of commodities and services have differential redistributive impact across the income distribution.\textsuperscript{4} Health tends to uniformly benefit the entire population and education is also strongly redistributive.\textsuperscript{5} Of course, there are other lines in the public budget, such as foreign service, culture or law-and-order, that accrue benefits increasing with income (or income taxes).\textsuperscript{6} Different compositions of government spending can be interpreted as different ways of distributing its benefits over the income distribution. A pro-rich expenditure policy may have a positive effect on those who bear the heavier part of the tax burden: the rich can enjoy a significant fraction of their taxes return to them in the form of goods and services they value most and this may make them more amenable towards income taxation. In our empirical test, we proxy the pro-taxpayer bias of public expenditure by the inverse of the share of public primary education.

Our work contains a second major novelty. This is how taxes are chosen. There are two major lines of research on how the current literature deals with the choice of taxation. One is that a benevolent government chooses the tax function which maximizes social welfare.\textsuperscript{7} Such a tax function, while with elegant efficiency properties, is not representative of the observed behaviour by tax authorities. The second line – known as positive, in contrast with the previous normative approach – conceives taxation as chosen by majority voting. Individuals have preferences over the available tax functions and, under some assumptions, the tax that obtains a majoritarian support is the one preferred by the median voter.\textsuperscript{8} This positive approach, while appealing because of its politico-economic flavour, has no efficiency properties and yields predictions with little or no empirical support. The median voter model appears to fail on three of its empirical implications. The first implication is that higher income inequality will lead to higher progressivity


\textsuperscript{5}On the distributional effects of taxes and benefits (education and health) in the UK, see Jones (2006) in the monthly publication Economic Trends.

\textsuperscript{6}In Adam Smith’s words: "The rich, in particular, are necessarily interested to support that order of things, which can alone secure them in the possession of their own advantages. (…) Civil government, so far as it is instituted for the security of property, is, in reality, instituted for the defence of the rich against the poor, or those who have some property against those who have none at all." (Book V, Chap. 1, Part II)

\textsuperscript{7}Modern literature on optimal, normative income taxation originates in Mirrlees (1971). A modern rigorous account of optimal income taxation can be found in Myles (1995), Chapter 5.

\textsuperscript{8}The key references are Romer (1975), Roberts (1977) and Meltzer and Richards (1981)
in taxation because of the increased distance between mean and median income. The second implication is that individuals with incomes higher than the mean would support zero income taxation, while people below the mean would support redistributive taxation, the more progressive the poorer they are. A third implication is that poor individuals refrain from voting for full expropriation because they are aware that labour supply will elastically respond to high taxation so that increases in progressiveness would reduce the tax collection and harm the transfer-receiving poor.

Our approach on how taxes are selected departs from the above. We assume that the government chooses a tax schedule that is acceptable to the largest share of the population possible. An individual considers a tax schedule acceptable if she does not wish to modify its steepness by means of an affine transformation that maintains the same net tax revenue. We will show that there is a unique tax schedule that is unanimously acceptable i.e. consensual. Furthermore, this tax schedule has the standard efficiency properties: the unanimously acceptable tax maximizes social welfare among the tax functions yielding the same net tax revenue. We thus provide a positive basis for a normative tax.

9Perotti (1996) finds no evidence of a significant relationship between inequality and redistribution in democracies. The implication that the median voter is decisive in the choice of the degree of redistribution has recently been empirically tested by Milanovic (2001) and Wong (2004). They find no evidence of such a decisive role of the median voter.

10Such “tax resistance” does not emerge from opinion polls. According to the US 1998 Gallup Poll – as cited in Fong (2001) – among American families with incomes of $10,000 or less, 35 percent report that the government should not redistribute wealth by heavy taxes on the rich and 21 percent believe that the poor should help themselves rather than this being the government’s responsibility. For the entire sample these percentages are 53.9 percent and 30.2 percent, respectively. More interestingly, Fong finds that income is a very poor predictor of redistributive attitudes. Wong (2004) also finds no evidence of pocketbook voting in income redistribution. In contrast, Corneo and Gruner (2002) obtain a strong and significant relationship between attitudes towards redistribution and expected benefit from more progressive income taxation. The reason for this divergent result might be the specificities of the set of countries they work with: six former socialist countries, four Anglo-Saxon countries, Norway and West Germany.

11There is no evidence of an elasticity of labour supply significantly different from zero. Blundell and MaCurdy (1999) summarize the different existing estimates of the uncompensated wage elasticity (all using different approaches and estimation techniques). The estimates reported there give nearly rigid labor supply functions: (i) US, zero (MacCurdy et al., 1990) and 0.05 (Triest, 1990); (ii) UK, 0.09 (Blundell et al,1988), (iii) France, 0.1 (Bourguignon and Magnac, 1990); (iv) Germany -0.004 (Kaiser et al,1992); and similar values for Sweden, Netherlands and Finland.

12While the voting models restrict to linear tax functions, in our case we place no restriction on neither the functional form of the tax schedules nor the net tax revenue.
In contrast with the existing literature, our model implies that the main determinants of redistribution are: (i) the composition of the public spending in the different commodities and services supplied by the government, and (ii) the substitutability between the commodities privately obtained through the market and the commodities publicly provided. Indeed, at the individual level income taxation is also seen as a mechanism by which disposable income is converted into publicly supplied goods. Therefore, the degree of substitutability between the two types of commodities plays a crucial role in determining the attitudes towards taxation. We examine how the changes in substitutability translate into the progressiveness of income taxation.

For the case of constant-elasticity preferences we obtain explicit, testable results. We focus on two implications: (i) the (constant) marginal redistributive factor (which we call marginal tax rate for simplicity) is negatively related to the degree of substitutability and (ii) a negative relationship between the marginal tax rate and the pro-taxpayer bias among countries with low substitutability and a positive relationship among countries with high substitutability. Both implications are empirically validated by our results using OECD country data. For the first result, we use standard ordinary least squares and kernel regressions (the latter, so as to not impose any functional form on the data), and for the second, we primarily use threshold regressions. This method allows the researcher to identify exact values of the elasticity of substitution at which the relationship under investigation (in our case marginal tax rate and pro-tax bias) switches from negative to positive. The relationship clearly holds for expenditures on primary education and health.

This paper goes only part of the way towards an integrated analysis of public taxation and expenditure. Our analysis, while bringing in the influence of government spending on the choice of income taxation, does not address the issue of the determinants of public spending policies themselves. This question is open to future research.

The structure of the paper is as follows. In section 2 we develop the model. Section 3 defines the notion of consensus taxation, proves the existence and uniqueness of a consensual income tax and shows that this tax is welfare efficient. Section 4 is devoted to the relation between income tax progressiveness, the pro tax-payer bias of public spending, and the degree of substitutability between the private and public bundles of commodities. Section 5 undertakes the empirical tests of the predictions of the model. Section 13 Note the similarity with the case of the voluntary provision of public goods. In that case too individuals see the tax paid as the cost necessary to obtain a useful commodity provided by the state. On this see Warr (1983), Bernheim (1986) and Bergstrom et al (1986).
2 The Model

2.1 Individuals

We assume that there is a continuum of individuals. Individual income is denoted by $y$, it is assumed to be exogenous, and distributed over the population accordingly with the cdf $F$ with support $[a, \infty)$.

We shall denote by $\mu$ the average per capita income.

The set of commodities is divided into two bundles, private (denoted $x$) and public (denoted $g$), accordingly with the mechanism by which they are allocated. Individual demand for the private commodities is satisfied through the markets: in view of market prices individuals choose how best to allocate their disposable income. The individual consumption of the publicly supplied commodities is fixed by the government through its expenditure policy.

We assume that all commodity prices are constant. This allows us to just focus on the aggregate expenditure on the two bundles of commodities.

We assume that individual preferences are defined on private and on public goods only and are represented by $u(x, g)$. Labour is thus assumed to be rigidly supplied.

On individual preferences we make the following standard assumptions:

**Assumption 1** $u_x > 0, u_g > 0, u_{xx} < 0, u_{gg} < 0$ and $u_{xg} > 0$. Further, we assume that for $g > 0$, $\lim_{x \to 0} u_x = \infty$ and $\lim_{x \to \infty} u_x = 0$, and for $x > 0$, $\lim_{g \to 0} u_g = \infty$ and $\lim_{g \to \infty} u_g = 0$.

The elasticity of substitution between the two commodity bundles will play a key role in our analysis. A higher consumption of the commodities supplied by the government can be achieved only by accepting higher taxation. This is tantamount to substituting private for publicly provided consumption goods. How much individuals will be willing to give up on private consumption to increase the level of the public bundle depends upon their substitutability. Therefore, the individual attitudes towards taxation will be critically influenced by the elasticity of substitution between the two bundles of commodities.

2.2 Income Taxation

The government raises taxes/transfers in order to redistribute income across individuals. The net public revenue left over after performing the redistribu-
ution of incomes is spent for the provision of the public commodity bundle. Individuals expend their disposable income to purchase private commodities. To save on notation we denote disposable income by $x$.

We denote by $t(y)$ the tax (if positive) or the subsidy (if negative) allocated to each individual with income $y$. Hence,

$$x(y) = y - t(y).$$

(1)

Note that disposable income will exceed the pre-tax factor income when $t(y) < 0$.

We denote by $ar{t}$ the per capita aggregate net surplus/deficit left after income redistribution, i.e.

$$
\bar{t} = \int t(y) dF(y). 
$$

(2)

### 2.3 Public Expenditure

The role of taxing income is not only to achieve a given degree of income redistribution, but also to raise a net revenue to finance the provision of the publicly supplied commodities. This net revenue will be endogenously determined together with the income tax schedule. Therefore, besides analyzing how income redistribution depends upon the key factors, we shall also have something to say about their effect on the size of government: the share of the public supply of commodities over aggregate factor income.

The structure of the expenditure is essential because it establishes the distribution of its benefits. Transferring resources from support to arts to primary education will make the benefits of public spending more tilted in favor of poor taxpayers. Therefore, we consider the structure of the public expenditure as a commitment to a distribution of the benefits on the basis of the taxes paid. Specifically, we assume that the benefit from public spending obtained by an individual with income $y$, $g(y)$ is $g(y) = \gamma (t(y), \bar{g})$, where $\bar{g}$ is the per capita public expenditure.

The government’s budget is balanced and hence

$$
\bar{g} = \int g(y) dF(y) = \bar{t}. 
$$

(3)

In order to make the problem operational, we shall build on the above in which

$$
\gamma (t(y), \bar{g}) = \gamma t(y) + (1 - \gamma)\bar{g} = \gamma t(y) + (1 - \gamma)\bar{t}. 
$$

(4)

When $\gamma = 1$ individuals obtain publicly supplied goods of the same amount they have paid for in taxes. On the other extreme, when $\gamma = 0$
public spending is fully egalitarian. Throughout the analysis we shall assume that there are rigidities in the expenditure policy so that $\gamma$ cannot be modified by the government in the short run.

A budget balanced *fiscal policy* is fully characterized by $\gamma$ and the tax function $t(.)$.

The public expenditure policy will be considered exogenous. Hence, the elasticity of substitution and the value of $\gamma$ will be considered as given.

## 3 Consensus Income Taxation

### 3.1 Definition

We shall explore the existence of a consensus over the tax functions that is considered acceptable by the different individuals. A tax function $t(.)$ will be acceptable to an individual with income $y$ if she does not wish to vary its progressivity. Consequently, the tax function will be consensual if it is unanimously acceptable.

Now, consider a particular $t(.)$ with net tax revenue $\bar{t}$, as defined in (2). In order to operationalize the notion of "variation of the progressivity" of $t(.)$ we focus on affine transformations $\tilde{t}(.)$ such that

$$\tilde{t}(y) = \alpha + \beta t(y).$$

with

$$\int \tilde{t}(y)dF(y) = \int [\alpha + \beta t(y)]dF(y) = \int t(y)dF(y).$$

Because of (6) we obtain

$$\tilde{t}(y) = \bar{t} + \beta t(y) - \bar{t}.$$  

The parameter $\beta$ defines the degree of progressiveness of $\tilde{t}(.)$ relative to $t(.)$. Choosing $\beta > 1$ implies that all the individuals contributing below average will see their contribution diminished while the ones with incomes above will contribute more. The opposite holds for $\beta < 1$. Therefore, $\beta > 1$ increases [and $\beta < 1$ decreases] the progressiveness of $\tilde{t}(.)$ relative to $t(.)$. Notice that we are not restricting the sign of $\beta$. We are also allowing for individuals to prefer a negative value of $\beta$ so as to invert the direction of transfers between rich and poor.

We shall place very weak restrictions on the tax functions; we shall work with the set $\Theta$ of all functions from $\mathbb{R}$ to $\mathbb{R}$ that are strictly increasing. We wish to emphasize here that the set $\Theta$ is not conditioned to a particular
aggregate net tax revenue. The set $\Theta$ contains all the strictly increasing functions.

Consider any arbitrary $t(\cdot) \in \Theta$ and any given $\gamma$. The valuation of a change in progressiveness by $\beta$ will be

$$u(y - [\bar{t} + \beta(t(y) - \bar{t})], \gamma[\bar{t} + \beta(t(y) - \bar{t})] + (1 - \gamma)\bar{t}).$$

(8)

Given a tax function $t(\cdot)$ we denote by $\beta(t(\cdot), y)$ the change preferred by an individual with income $y$.

**Definition 2** A tax function $t(\cdot)$ is individually acceptable to a person with income $y$ if $\beta(t(\cdot), y) = 1$.

We denote by $\mathcal{X}(y)$ the set of all tax functions $t(\cdot) \in \Theta$ that are individually acceptable to earners of income $y$.

We assume that the government looks for the tax function that is acceptable to the largest share possible of the population. We explore here the most demanding acceptability requirement: consensus.

**Definition 3** A tax function $t(\cdot)$ is consensual, $t(\cdot) \in \mathcal{X}$, if it is unanimously acceptable; that is, if $t(\cdot) \in \bigcap_y \mathcal{X}(y)$.

As we shall now see such a stringent requirement on income taxation does not yield an empty set.

### 3.2 Existence of Consensus Income Taxation

The notion of "consensus taxation" as proposed is not meant to describe any explicit institutional process. On the contrary, we wish to capture the idea that the income tax schedule has been chosen so as to avoid any significant rejection on either side of the income ladder.

In short, we simply pose the following question: is there a tax schedule such that a vast majority of the population would not feel compelled to change its steepness? If the answer were in the affirmative one would expect to observe tax schedules close to the one attracting such widespread acceptance.

Surprisingly, the requirement of consensus is neither stringent nor too loose. For any given distribution of income there is always one and only one tax function in the set $\Theta$ that satisfies this property.

**Theorem 4** The set $\mathcal{X}$ is non-empty and contains one single element only.
Proof. Consider any arbitrary \( t(.) \), \( \bar{t} \) and \( \gamma \). The valuation of a \( \beta \) affine transformation, as in (8), will be

\[
u (y - (\bar{t} + \beta (t(y) - \bar{t})), \gamma (\bar{t} + \beta (t(y) - \bar{t})) + (1 - \gamma)\bar{t}). \tag{9}\]

It can be readily verified that the utility valuation is concave in \( \beta \). Hence, the first order condition fully characterizes the preferred \( \beta \).

Differentiating with respect to \( \beta \) we obtain

\[
\frac{\partial u}{\partial \beta} = (t(y) - \bar{t}) \left[ -u_x \left( y - \bar{t}(y), \gamma(y, \hat{\gamma}) \right) + \gamma u_g \left( y - \bar{t}(y), \gamma(y, \hat{\gamma}) \right) \right].
\]

Note that for all \( t(.) \in \Theta \), \( (t(y) - t) \neq 0 \) except for at most one value of \( y \). Hence, \( \beta(y, t(y)) \) is implicitly characterized by the condition

\[
\frac{u_x (y - [\bar{t} + \beta (t(y) - \bar{t})), \gamma [\bar{t} + \beta (t(y) - \bar{t})] + (1 - \gamma)\bar{t})}{u_g (y - [\bar{t} + \beta (t(y) - \bar{t})], \gamma [\bar{t} + \beta (t(y) - \bar{t})] + (1 - \gamma)\bar{t})} = \gamma. \tag{10}\]

If \( t^*(y) \) is universally acceptable it has to be that \( \beta(y, t^*(y)) = 1 \) for all \( y \).

We start with an arbitrary parameter \( t \) and with the implicit definition of \( t(y) \) by

\[
\frac{u_x (y - t(y), \gamma t(y) + (1 - \gamma)t)}{u_g (y - t(y), \gamma t(y) + (1 - \gamma)t)} = \gamma. \tag{11}\]

Because of Assumption 1, the left-hand-side of (11) is strictly increasing in \( t(y) \), it goes to infinity as \( t(y) \to y \) and to zero as \( t(y) \to -\frac{1-\gamma}{\gamma} t \). Hence, for each \( t \) and \( y \) there exists a unique \( t(y) \) satisfying (11). We can thus write

\[
t(y) = \psi(y, t, \gamma). \tag{12}\]

It can be readily verified that \( \psi \) is continuous and strictly increasing in \( y \) and continuous and strictly decreasing in \( t \).

For an arbitrary \( t \), the average tax collection \( \bar{t} \) is

\[
\bar{t} = \int \psi(y, t, \gamma) dF(y) = \phi(t, \gamma).
\]

The socially acceptable tax-transfer policy \( t^*(.) \) is given by (12) evaluated at \( t^* \), where \( t^* \) satisfies \( t^* = \phi(t^*, \gamma) \).

We need now to show that \( \phi \) has a fixed point. Since \( \psi \) is continuous and strictly decreasing in \( t \), so is \( \phi \). From (10) we can easily obtain that for \( t = 0 \), \( \psi(y, 0, \gamma) > 0 \) for all \( y \). Therefore, we have that for \( t = 0 \), \( \phi(0, \gamma) > 0 \). Since \( \phi \) is continuous and strictly decreasing in \( t \), there exists a unique \( t^* \) such that \( t^* = \phi(t^*, \gamma) \). This completes the proof. \( \blacksquare \)
A clarifying remark is now in order. Individual acceptability as defined earlier only considers changes in the steepness of the tax function that did not modify the aggregate tax revenue. This restriction may raise concerns of a "hidden restriction" on the tax functions truly under consideration. Notice, however, that the set $\mathcal{S}(y)$ is obtained after having tested the acceptability of all possible strictly increasing functions with any arbitrary aggregate tax revenue. Therefore, the set $\mathcal{S}(y)$ contains tax functions yielding very different aggregate tax revenues. The restriction that a tax function is consensual, and hence acceptable to all, selects not only the consensual steepness of the tax function, but its net revenue as well. The aggregate tax revenue is determined together with the shape of the tax function.

### 3.3 Determinants of Tax Progressiveness

Before examining the progressiveness of the consensus income tax let us first show that the marginal tax rate is positive and does not exceed unity.

**Proposition 5** The marginal tax rate of the consensus tax function satisfies $0 \leq t^\prime(.) \leq 1$.

**Proof.** Totally differentiating (10) with respect to $t(.)$ and $y$ and rearranging we obtain

$$\frac{dt(.)}{dy} = \frac{u_{xy}u_x - u_{xx}u_g}{(u_{xy}u_x - u_{xx}u_g) + \gamma(u_{xy}u_g - u_{yy}u_x)}.$$ 

Observe now that the numerator and the first term in the denominator are the same and that they are positive because of Assumption 1. Because of the same Assumption 1 the second term in the denominator is positive as well. Therefore, the marginal tax rate is positive and below unity.

We have assumed that individual incomes are exogenous. What prevents poor individuals from accepting full income equalization only? The government policy allocates the publicly supplied commodity as an increasing function of one’s contribution in taxes. If a low-income earner demands more progressiveness — and hence a larger transfer — she will be trading-off more private consumption for less public consumption. Likewise, high-income earners might be willing to give up on private consumption in order to obtain higher levels of public consumption. At the consensus tax schedule, the marginal rate of substitution between the private and publicly supplied commodities will be equal across the population. It is worth noting that it is precisely this property that makes consensus tax functions welfare efficient.

We shall examine this issue for the class of CES preferences in Section 5.
4 Properties of the Consensus Income Taxation

4.1 Efficiency

We have uniquely characterized an income tax function based on the notion of individual acceptability, combined with a government seeking consensus. This places our approach within the positive, politico-economic approach to taxation. Yet, in contrast with this literature, the consensus income tax that we have characterized has interesting efficiency properties: the consensus income tax maximizes Social Welfare among all the tax functions in $\Theta$ that yield the same net tax revenue. Hence, our notion of consensus establishes a bridge between two quite independent branches of the literature on income taxation: positive and normative.

We define the (Utilitarian) Social Welfare as the sum of the individual utilities, that is,

$$W(t(.)) = \int u(y - t(y), \gamma t(y) + (1 - \gamma)\bar{t})dF(y). \quad (13)$$

We shall now show that $W(t^*(.)) \geq W(t(.))$ for all $t(.) \in \Theta$ with net tax revenue $t^*$.

**Proposition 6** Let $t^*(.)$ be a consensus income tax function with net tax revenue $t^*$. Then, $t^*(.)$ maximizes the Utilitarian Social Welfare over all tax functions $t(.) \in \Theta$ with net tax revenue $t^*$.

**Proof.** We can write

$$W(t^*(.)) - W(t(.)) =$$

$$= \int [u(y - t^*(y), \gamma t^*(y) + (1 - \gamma)t^*) - u(y - t(y), \gamma t(y) + (1 - \gamma)t^*)]dF(y).$$

Since $u(.,.)$ is concave in the tax function, and using (10), we can write

$$W(t^*(.)) - W(t(.)) \geq$$

$$\geq \int [t^*(y) - t(y)][-u_x(y - t^*(y), \gamma t^*(y) + (1 - \gamma)t^*) + u_y(y - t^*(y), \gamma t^*(y) + (1 - \gamma)t^*)]dF(y) = 0.$$

Let us emphasize that a consensus income tax maximizes welfare subject to a net revenue constraint. The implication is that had we fixed an arbitrary exogenous net tax revenue (different from $t^*$) we would have found no consensual tax function yielding this arbitrary revenue.
4.2 Nash Equilibrium Taxation

Consensus income taxation can also be interpreted as a Nash equilibrium of a tax-contribution game.

A tax function \( t(.) \) with net tax revenue \( t \) prescribes the tax to be paid by an individual with income \( y \). Let us denote by \( \delta(y) \) the deviation from the prescribed tax payment. Given the deviations by the others, individuals choose their best deviation.

We say that a tax function \( t(.) \) is a Nash equilibrium if no individual player deviates from the prescribed tax payment.

We can now state the following Proposition.

**Proposition 7** The consensus income tax \( t^*(.) \) is a Nash equilibrium of the tax-contribution game.

**Proof.** Note first that no individual deviation can modify the net tax revenue \( t \). Thus, for an individual with income \( y \) the payoff of deviating by \( \delta(y) \) is

\[
u(y - (t(y) + \delta(y)), \gamma(t(y) + \delta(y)) + (1 - \gamma)\bar{t}).
\]

The first order condition for a maximum is

\[u_x(y - (t(y) + \delta(y)), \gamma(t(y) + \delta(y)) + (1 - \gamma)\bar{t}) = \]

\[= \gamma u_y(y - (t(y) + \delta(y)), \gamma(t(y) + \delta(y)) + (1 - \gamma)\bar{t}).
\]

In view of Theorem 1, it is straightforward that if \( t(.) = t^*(.) \) the optimal deviation satisfies \( \delta(y) = 0 \), and that this holds for all \( y \).

This property of consensus income taxation is the analog of the equilibrium in the case of voluntary contributions to public goods.

5 Income Tax Progressiveness and Public Expenditure: the CES case

We shall restrict individual preferences to be of the CES type. This will permit us to examine the effects of income inequality, expenditure bias and elasticity of substitution on the tax schedule and on the size of government.

The family of CES utility functions is given by:

\[
u(x, g) = \left[ \alpha x^{\frac{\sigma-1}{\sigma}} + (1 - \alpha) g^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (14)
\]
with the elasticity of substitution $\sigma > 0$.

The marginal utilities to the two types of consumption are

$$
\begin{align*}
    u_x &= \alpha x^{-\frac{1}{\sigma}} \left[ \alpha x^\frac{\sigma-1}{\sigma} + (1 - \alpha)g^\frac{\sigma-1}{\sigma} \right]^{\frac{1}{\sigma-1}}, \quad \text{and} \\
    u_g &= (1 - \alpha)g^{-\frac{1}{\sigma}} \left[ \alpha x^\frac{\sigma-1}{\sigma} + (1 - \alpha)g^\frac{\sigma-1}{\sigma} \right]^{\frac{1}{\sigma-1}}.
\end{align*}
$$

Therefore,

$$
\frac{u_x(x, g)}{u_g(x, g)} = \frac{\alpha}{1 - \alpha} \left[ \frac{y - t(y)}{\gamma t(y) + (1 - \gamma)t} \right]^{-\frac{1}{\sigma-1}} = \gamma. \quad (15)
$$

We can thus easily obtain that

$$
t(y) = \frac{y - (1 - \gamma)\lambda \tilde{l}}{1 + \gamma \lambda}, \quad (16)
$$

where

$$
\lambda = \left( \frac{\alpha}{(1 - \alpha)\gamma} \right)^\sigma. \quad (17)
$$

Integrating over the incomes $y$ we can obtain

$$
\tilde{t} = \frac{\mu - (1 - \gamma)\lambda \tilde{l}}{1 + \gamma \lambda}.
$$

Hence,

$$
\tilde{t} = \frac{\mu}{1 + \lambda} = g. \quad (18)
$$

Therefore, we obtain that the consensus income tax schedule is linear

$$
t(y) = \tau y - T, \quad (19)
$$

where

$$
\tau \equiv \frac{1}{1 + \gamma \lambda} \quad \text{and} \quad T \equiv \frac{1 - \gamma \lambda}{1 + \gamma \lambda} \frac{\lambda \mu}{1 + \lambda}. \quad (20)
$$

From (18) we obtain the size of the public sector to be

$$
\frac{\bar{g}}{\mu} = \frac{1}{1 + \lambda}. \quad (21)
$$

For all OECD countries $\frac{\bar{g}}{\mu} < \frac{1}{2}$. Therefore, in view of (21) and of (17) we can deduce that the empirically relevant parameter values satisfy:

$$
\lambda > 1, \text{and} \frac{\alpha}{(1 - \alpha)\gamma} > 1. \quad (22)
$$
We can now state our results on income taxation and the size of government. First, we discuss the effect of $\gamma$ and $\sigma$ on the marginal tax rate $t'(\cdot)$.

**Proposition 8** Let preferences be CES. Then: (i) the unique consensus income tax is linear; (ii) it is independent of the distribution of income; (iii) the (constant) marginal tax rate, $t'(\cdot) \equiv \tau$, increases (decreases) with the bias parameter $\gamma$ if the elasticity of substitution is high (low), $\sigma > 1$ ($\sigma < 1$); and (iv) when $\frac{\sigma}{\rho} < \frac{1}{2}$ (and hence (22) is satisfied) an increase in the elasticity of substitution reduces the marginal tax rate.

**Proof.** Statements (i) and (ii) follow immediately from (19).

Differentiating the marginal tax rate in (19) with respect to $\gamma$ and using (17) we obtain

$$\frac{d\tau}{d\gamma} = -\frac{1}{(1 + \gamma \lambda)^2} \frac{d\gamma \lambda}{d\gamma} = (\sigma - 1) \frac{\lambda}{(1 + \gamma \lambda)^2}.$$ \(\frac{d\tau}{d\gamma} = \frac{d\gamma}{d\tau} = \frac{\lambda}{(1 + \gamma \lambda)^2}.\)

This proves statement (iii). As for statement (iv) we similarly differentiate with respect to $\sigma$ and obtain

$$\frac{d\tau}{d\sigma} = -\frac{\gamma}{(1 + \gamma \lambda)^2} \frac{d\lambda}{d\sigma} = -\frac{\gamma}{(1 + \gamma \lambda)^2} \lambda \ln \left(\frac{\alpha}{(1 - \alpha)\gamma}\right).$$

Noting that because of (22) $\ln \left(\frac{\alpha}{(1 - \alpha)\gamma}\right) > 0$, statement (iv) obtains.

Without having made any assumption on the tax-transfer function, we obtain that for CES preferences, the unique $t(\cdot)$ that would be consensual would be a linear tax function. Therefore, any departure from linearity in taxes requires significant variations in the substitutability of the two bundles of commodities as income varies.

By the same argument, the inequality in the distribution of pre-tax income will not play a major role in determining the degree of income redistribution, $\tau$, unless individual preferences display a significant variation in the degree of substitution as real income changes.

The marginal tax rate depends upon the bias of government spending. The effect of a more egalitarian expenditure policy, $\gamma \to 0$ on the marginal tax rate $\tau$ critically depends upon the degree of substitutability between the two bundles of goods. For low substitutability, $\sigma < 1$, the marginal tax rate tends to unity and for high substitutability, $\sigma > 1$, it tends to zero.

Finally, in economies with a moderate share of government an increase in the substitutability between the two bundles of goods will decrease the marginal tax rate, $\tau$.
Let us now turn to the effects of $\gamma$ and $\sigma$ on the size of government $\frac{\bar{g}}{\bar{u}}$. From (21) we can easily obtain the following result.

**Proposition 9** Let preferences be CES. Then, the size of government $\frac{\bar{g}}{\bar{u}}$: (i) increases with the bias parameter $\gamma$; and (ii) decreases with the elasticity of substitution $\sigma$.

The results of Proposition 4 are not surprising; they are worth noting. The first result implies that the higher the pro-taxpayer bias in the public spending the larger will be the size of the government that the population will consider acceptable. The second result implies that increasing the substitutability between the market and the publicly supplied goods will induce a demand for a smaller size of the government.$^{14}$

It is worth discussing the case of the tax schedule being linear. Consider the effect of an increase by $\Delta$ of an income $y$. Due to the biased expenditure policy, a linear income tax implies that private and public consumption will also increase at the same rate. If preferences have a falling elasticity of substitution of private for public consumption, individuals with an income increased by $\Delta$ would prefer a more than proportional increase in the supply of the public good and hence would rather favor an increasing marginal tax rate. If the elasticity of substitution were to rise individuals would have a preference for declining marginal tax rates. Clearly, whether individuals unanimously support a tax function with increasing or decreasing marginal tax rates critically depends upon the change in the elasticity of substitution as the consumption levels rise. A similar argument explains why the tax rate is shown out to be independent of the distribution of income.

In our model, the supply of a subset of commodities is the monopoly of the government. This monopoly provides the government with the coercive power to make individuals accept taxation on incomes. How effective this power is critically depends upon the substitutability between this bundle of commodities and the commodities individuals can purchase in the market. Hence, our approach suggests that the rich will lobby more strongly for increasing the substitutability between public and private goods by privatizing as many as possible rather than about the shape of the income tax schedule.

$^{14}$This result seem in contradiction with Karras’s (1994) argument that the larger is the public sector the more the goods and services supplied will be substitutes for the goods provided through the market. Two points are in order. First, Karras does not take into account that the substitutability between public and private goods critically depends on the political decision of allowing or not the private supply of substitutes (e.g. security, mail service, prisons,...). Second, Karras’s argument does not consider whether such an increase in the size of government would be considered acceptable. This precisely is our point: if the government allows for higher substitutability the policy that will be found consensual will consist of a smaller size of the government sector.
6 Empirical results

6.1 Testable Implications

For the case of constant-elasticity preferences, our model has well-defined, testable implications. Specifically, we shall focus on two implications:

1. the (constant) marginal redistributive factor (which we call marginal tax rate for simplicity) $\tau$ is negatively related to the degree of substitutability and

2. we should observe a negative relationship between the marginal tax rate and the pro tax-payer bias among countries with low substitutability and a positive relationship among countries with high substitutability.

Both implications are empirically validated by our results.

6.2 Empirical Strategy and Data

We shall now test the empirical validity of the relationship between $\tau$, $\gamma$ and $\sigma$ implied by our results. We do not have direct data on any of the three variables and hence we have to work with reasonable proxies. Furthermore, our choice of proxies has been severely conditioned by the need of a consistent set of basic information available for a sufficiently large number of countries. We have therefore tested our empirical implications using the OECD database\textsuperscript{15} that includes fifteen countries, listed in the Appendix.

Let us discuss our proxy for $\tau$. This is the slope of the affine tax function that turns out to be acceptable with CES preferences. In our paper the tax function merges together the income tax schedule and the different money transfers. In other words, $t(.)$ is the difference between factor income (plus retirement payments and minus retirement contributions) and disposable income.

In purity, we would have had to test whether the difference between the two individual incomes can be represented by an affine function. The only data base available with individual information of this kind is the Luxembourg Income Study database. We performed this exercise but discarded this option for two reasons. First, the estimated parameters were unreasonably unstable from year to year, hinting towards some possible deficiencies in the raw data. Secondly, $\gamma$ and $\sigma$ would have to be estimated from a completely different source.

\textsuperscript{15}We use the OECD Statistical database obtainable at www.oecd.org/statistics.
We have opted instead to use the maximum marginal tax rate in each country reported in the OECD database as a proxy for $\tau$. Hence, we have implicitly assumed that there is a stable relationship between the maximum marginal tax rate and the slope of the affine function that would approximate the difference between factor and disposable individual incomes.

Let us now turn to our estimates for $\gamma$. As defined in Section 2, $\gamma$ captures the pro-taxpayer bias in the public provision of goods and services, which we denote by $G$. This bias depends upon the share in the government budget of the expenditures that mostly benefit the low incomes versus those that mostly benefit the rich taxpayers. For some countries, discussed earlier, there are estimates of the distribution of the benefits of specific lines in the government expenditure (essentially, education and health).\footnote{There is a large literature which discusses and identifies the redistributiveness of public expenditures. (See Le Grand, 1982 for the redistributive effects of health and education in the UK - he concludes that of the two education is more redistributive). A large debate in the 1980s (Hansen and Weisman 1969, Pechman 1970) contest the redistributive effects of higher education in particular, concluding that the redistributive effects of higher education were debatable, and that existing measurement methodologies were not successful in effectively measuring their effects. We do not perform any statistical analysis to test for the relative redistributiveness of the different types of public expenditures in the OECD countries studied; this would entail a separate econometric exercise beyond the purview of this paper.} We are however interested in the distribution of the benefits of the entire government supply of goods and services (including general administration and law-and-order, among others). Therefore, we have had to estimate our own proxies for $\gamma$.

We first estimate $G$, the total government expenditure in the provision of goods and services, from OECD data sources. $G$ is estimated by subtracting the amounts that are spent on money transfers from the total amount of government expenditure, detailed in the Appendix. All estimates are done at constant 1995 US dollars. Of the total amount of government expenditure, $G$, we focus on two redistributive, pro-poor public expenditures - health and education. For education, we focus on all three types of expenditures - primary, secondary and tertiary.

We estimate $\gamma$ with the following expression:

$$\gamma = 1 - \frac{\text{government expenditure on redistributive goods}}{G}$$

Finally, we need an estimate for the elasticity of substitution between the two bundles of commodities, private and publicly provided, $\sigma$. The substitutability between public and private expenditure has been a recurrent topic in Macroeconomics. Since the work of Barro (1981) there have been
numerous attempts at estimating the elasticity of substitution. Aschauer (1985) finds a significant degree of substitutability between the two variables for the United States. Karras (1994) finds that they are complementary or unrelated, using data for 30 countries. Evans and Karras (1996) provide additional evidence supporting the complementarity using data for 54 countries. More recently, Amano and Wirjanto (1998) for the US show that the two variables are unrelated or have very weak complementarity. For Japan, Hamori and Asako (1999) find a significant degree of substitutability, while for Okubo (2003) the two bundles are complementary or unrelated. Finally, Bouakez and Rebei (2006) with the same specification of preferences as ours - but with habit formation - estimate $\sigma = 0.332$.

Unfortunately, we will not be able to make use of these [quite contradictory] estimates for the following reasons. First, most of the Macro literature has defined substitutability by the sign of the cross derivative and not by the value of the elasticity, $\sigma$. Second, the models are all inter-temporal and this aspect proves to be critical for the estimates. Ni (1995) empirically finds that when the two expenditures add linearly in the preferences, the estimates indicate substitutability, while if the two expenditures enter the utility function non-separably one obtains complementarity. Third, the estimates are perplexedly contradictory. Finally, most of the literature tries to estimate a world elasticity using panel data, but we are interested in country estimates which can be used to make a ranking across countries. Kwan (2006), for example, using co-integration methods, has found that for nine East Asia countries while the two bundles are substitutes on the average, in some countries they are complements. In sum, we cannot base our empirical work on these estimates.

Our approach to the estimation of $\sigma$ is therefore as follows. The substitutability between the two bundles of commodities depends upon the nature of individual preferences and on the degree of monopoly that the government keeps for itself for some subset of commodities, as discussed earlier. For many OECD countries the postal system or security has been a public monopoly until fairly recently. Today, however, rich people can supplement the public supply of police force, for instance, by purchasing additional private security. Similarly, in many countries education and health have high degrees of publicness while in others, a good share of the demand is satisfied through the private market. The larger the share of the expenditure channelled through the market, the higher is the substitutability between the public and the private provision of these goods. For our purposes, therefore, we estimate a proxy of the elasticity of substitution using a metric $\lambda$, which equals the ratio of private over the total of public and private expenditures. Using this ratio, we estimate a proxy of the elasticity of substitution, $\sigma = \lambda/(1 - \lambda)$. If all
is private, and $\lambda = 1$, then elasticity is infinity. Comparative data of public and private expenses have been used for both health and education for the OECD countries in this study\textsuperscript{17}. We estimate $\lambda$ for all three types of education - primary, secondary and tertiary - and for health, obtained from OECD sources. Estimates of $\lambda$ are available at http://darp.lse.ac.uk/expenditures/.

Indeed, these estimates are very rough proxies for the "true" elasticities of substitution. However, our empirical exercise essentially rests on the "ranking" of the countries by their degree of substitutability more than on its absolute value.

### 6.3 Relationship between $\tau$ and $\sigma$

In Section 4.2 theory predicts that the relationship between $\tau$ and $\sigma$ should be negative. Here, we investigate this relationship using the data and estimates that have been discussed in the previous section, for all four types of elasticities - primary and pre-primary, secondary and tertiary education, and health. We estimate both OLS regressions and kernel regressions to ascertain the functional nature of the relationship (positive or negative) between the two variables. The latter method is used to allow the data to determine the true nature of the relationship, rather than imposing a known functional structure.

The model we estimate is given as

$$\tau_{it} = f(\sigma_{it}) + \varepsilon_{it},$$

where $\tau_{it}$ is the marginal tax rate for country $i$ in time $t$, $\sigma_{it}$ is the elasticity of substitution between the publicly and privately provided good for country $i$ in time $t$, $f(.)$ is a generic function and $\varepsilon_{it}$ is an error term. We do not impose any structure on $f(.)$, except for the case when we estimate the above relationship using OLS.

The kernel regressions support a negative relationship. This is also supported by OLS regressions, except for that of $\sigma_{tertiary}$, given in Table 1. All other three $\sigma$s are significant at 1% level of significance.

In the figures below, we present the kernel regressions for each of the four relationships. The Epanechnikov estimator (Silverman 1986) is used for the kernel regression estimates. We present results with a smaller bandwidth\textsuperscript{17}

\textsuperscript{17}The data source for these estimates is the OECD Social Expenditure database. Data has been available only for specific years - 1995, 2000 and 2004.
(rather than a large one) to reveal the true relationship as closely as possible. For each of the four $\sigma$s it is clear that the dominant relationship between $\tau$ and $\sigma$ is a negative one.

### Table 1: OLS Regressions for Relationship between estimates of Marginal Tax Rate, $\tau$ and Elasticity of Substitutions, $\sigma$

<table>
<thead>
<tr>
<th>co-efficient</th>
<th>$\sigma_{primary}$</th>
<th>$\sigma_{secondary}$</th>
<th>$\sigma_{tertiary}$</th>
<th>$\sigma_{health}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>374</td>
<td>374</td>
<td>374</td>
<td>374</td>
</tr>
</tbody>
</table>

Table 1: OLS Regressions for Relationship between estimates of Marginal Tax Rate, $\tau$ and Elasticity of Substitutions, $\sigma$

**6.4 Relationship between $\tau$ and $\gamma$**

In this section, we investigate the relationship between $\tau$ and $\gamma$. Theory in Section 5 predicts that for low values of $\sigma$, there exists a negative relationship between the marginal tax rate and the pro-tax bias; for higher values of $\sigma$, there exists a positive relationship. We use threshold regressions to determine the value of $\sigma$ which splits the sample into two parts, $\sigma^*$, such that for values of $\sigma$ less than the estimated threshold value, $\sigma^*$, there is a negative relationship between $\tau$ and $\gamma$, and for values of $\sigma$ above the estimated threshold value $\sigma^*$, there is a positive relationship between $\tau$ and $\gamma$.  

Figure 1: Kernel Regression of $\tau$ on $\sigma_{primary}$
Figure 2: Kernel Regression of $\tau$ on $\sigma_{\text{secondary}}$

Figure 3: Kernel Regression of $\tau$ on $\sigma_{\text{tertiary}}$
Threshold regressions thus determine the different "regimes" of the relationship between τ and γ. Alongside the threshold regressions we estimate kernel regressions, to ascertain the exact nature of the relationship between τ and γ. Figures 7 to 14 present the kernel regressions between the marginal tax rate for each observation, τ, and the pro-tax bias, γ, where the data is sorted for increasing values of σ. First we will determine the threshold values of σ.

To estimate the threshold values and the regimes, we estimate the following model:

$$
\tau_{it} = \alpha_1 \gamma_{it} 1(\sigma_{it} \leq \sigma^*) + \alpha_2 \gamma_{it} 1(\sigma_{it} > \sigma^*) + u_{it},
$$

where for each \( i \), \( \tau_{it} \) is the dependent variable (the marginal tax rate corresponding to country \( i \) in time \( t \)), \( \gamma_{it} \) is the explanatory variable (the pro-tax bias corresponding to country \( i \) in time \( t \)), \( \sigma_{it} \) is the threshold variable (corresponding to the country \( i \) in time \( t \)), assumed to be strictly exogenous, \( \sigma^* \) is the threshold parameter, \( \alpha_1 \) and \( \alpha_2 \) are the slope parameters that will differ according to the value of \( \sigma_{it} \), and \( u_{it} \) is a random disturbance term. \( 1(\sigma_{it} \leq \sigma^*) \) is an indicator variable that takes the value 1 if \( \sigma_{it} \leq \sigma^* \) and 0 otherwise. The threshold value of σ is the estimate at which likelihood function achieves a local minimum.\(^{18}\)

\(^{18}\)There is no asymptotic theory to obtain p-values corresponding to the threshold value obtained, thus our reported threshold values of σ depends on the value obtained by minimising the likelihood function. Estima-
Table 2 presents the regimes that we obtain, using $\sigma_{\text{primary}}$ and $\sigma_{\text{health}}$, with the OLS estimates of the slope parameters. For $\sigma_{\text{primary}}$, we observe that for values of $\sigma_{\text{primary}} \leq 0.21$, there is a negative relationship, and for values greater there is a positive relationship between $\tau$ and $\gamma$. Likewise, for $\sigma_{\text{health}} \leq 0.59$, we observe a negative relationship and for greater values, a positive relationship.

<table>
<thead>
<tr>
<th>Regimes</th>
<th>$\sigma_{\text{primary}}$</th>
<th>$\alpha$</th>
<th>$\sigma_{\text{health}}$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>$\sigma_{\text{primary}} \leq 0.21304$</td>
<td>-1.1283$^*$</td>
<td>$\sigma_{\text{health}} \leq 0.59$</td>
<td>-0.00164$^*$</td>
</tr>
<tr>
<td>Regime 2</td>
<td>$\sigma_{\text{primary}} &gt; 0.21304$</td>
<td>1.4481$^*$</td>
<td>$\sigma_{\text{health}} &gt; 0.59$</td>
<td>0.03612$^*$</td>
</tr>
</tbody>
</table>

Notes: $^*$: Significant at the 1% level  
†: Significant at the 5% level  
‡: Significant at the 10% level

Table 2: Regimes obtained by Threshold Regression Estimations for Elasticity of Substitution for Primary Education and Health

The likelihood function plots revealing the threshold value at which it is minimised are presented in Figures 5 and 6.

We now present the kernel regressions corresponding to the above relationships explored. Kernel regressions are estimated to observe the nature of the relationship between $\tau$, the actual marginal tax rate, and $\gamma$, the pro-tax bias, without imposing a functional specification. The model estimated is given by

$$\tau_{it} = k(\gamma_{it}) + \nu_{it},$$  \hspace{1cm} (26)$$

where $\tau_{it}$ is the marginal tax rate for country $i$ in time $t$, $\gamma_{it}$ is the elasticity of substitution between publicly and privately provided good for country $i$ in time $t$, $f(.)$ is a generic function and $\nu_{it}$ is an error term. We again use the Epanechnikov kernel estimator (Silverman 1986). We also provide further kernel estimates using the Nadaraya-Watson estimator, which are available for the interested reader at the extended Appendix at http://darp.lse.ac.uk/expenditures/.

The results suggest similar, if not identical, results to those obtained with the Epanechnikov estimator.

Figures 7 to 14 suggest that for the values of $\sigma \leq \sigma^*$, the relationship between $\tau$ and $\gamma$ is negative, while for values of $\sigma_{it} > \sigma^*$, where $\sigma^*$ is the...
Figure 5: Likelihood function with sigma\_primary as threshold variable.

Figure 6: Likelihood function with sigma\_health as threshold variable.
threshold values of the elasticity of substitution, the relationship is positive. The sample size for the full data set is 375 - the data set consists of observations from 15 OECD countries (listed in the Appendix), over the years 1975 to 1999.

The sample is split into two sets on the basis of the threshold value of $\sigma$ (corresponding to primary education and health). The relationship between $\tau$ and $\gamma$ is then observed for the two samples, one corresponding to values of $\sigma \leq \sigma^*$ and another corresponding to values of $\sigma_{it} > \sigma^*$. The threshold values of $\sigma$ that has been used to split the samples are given below. We have two sets of results - one set estimating the relationship between $\tau$ and $\gamma_{\text{primary}}$ ($\gamma$ corresponding to expenditures on primary education) and another estimating the relationship between $\tau$ and $\gamma_{\text{total}}$ ($\gamma$ corresponding to expenditures on education and health).

- For $\sigma$ corresponding to that of primary and pre-primary education expenditures, the threshold values are $\sigma_{\text{primary}} < 0.21$ and $\sigma_{\text{primary}} > 0.21$
- For $\sigma$ corresponding to that of health expenditures, the threshold values are $\sigma_{\text{health}} < 0.59$ and $\sigma_{\text{health}} > 0.59$.

Figures 7 to 10 present the kernel regressions for the relationship between $\tau$ and $\gamma_{\text{primary}}$. It is clear that for values of $\sigma$ below the threshold value, we have a negative relationship between $\tau$ and $\gamma_{\text{primary}}$, and for values higher, there is a positive relationship.

We now repeat kernel regression estimates of the relationship between $\tau$, and $\gamma_{\text{total}} - \gamma$ estimated for total expenditures on education and health. A cursory glance at Figures 11 to 14 reveals an identical relationship as before - for lower values of $\sigma$, we have a negative relationship between $\tau$ and $\gamma_{\text{total}}$ and for higher values of $\sigma$, a positive one. The threshold values for $\sigma_{\text{primary}}$ and $\sigma_{\text{health}}$ obtained are the same as before, as follows:

- For $\sigma$ corresponding to that of primary and pre-primary education expenditures, the threshold values are $\sigma_{\text{primary}} < 0.21$ and $\sigma_{\text{primary}} > 0.21$
- For $\sigma$ corresponding to that of health expenditures, the threshold values are $\sigma_{\text{health}} < 0.59$ and $\sigma_{\text{health}} > 0.59^{19}$

\[19\text{There may be concerns for endogeneity between the regressor, }\gamma, \text{ and the threshold}\]
Figure 7: Kernel Regression of $\tau$ on $\gamma_{primary}$, for $\sigma_{primary} < 0.21$

Figure 8: Kernel Regression of $\tau$ on $\gamma_{primary}$, for $\sigma_{primary} > 0.21$
Figure 9: Kernel Regression of $\tau$ on $\gamma_{\text{primary}}$, for $\sigma_{\text{health}} < 0.59$

Figure 10: Kernel Regression of $\tau$ on $\gamma_{\text{primary}}$, for $\sigma_{\text{health}} > 0.59$
The relationships observed support a negative relationship between the marginal tax rate $\tau$ and the pro-taxpayer bias (for both $\gamma$s defined for primary education and that for total expenditure on education and health), for lower values of the expenditures’ elasticities between private and public provisions, and a positive relationship for higher values of the elasticities. These results are further confirmed by kernel regressions that are estimated using the Nadaraya-Watson estimator, a local polynomial averaging estimator, which are available in the extended Appendix at http://darp.lse.ac.uk/expenditures/ for the interested reader.

### 7 Conclusion

In this paper we have jointly treated public taxation and spending. We have proposed a new notion of tax selection, which is more in line with available evidence of attitudes towards redistribution. This model allows us to address novel issues such as the interdependence between income taxation, the variable, $\sigma$, due to both having been derived from expenditure data on public and private health and education sectors. We have searched for estimates of the elasticities of substitution (between public and private expenditures) of these individual sectors, but currently these do not exist. There also exists no theory under threshold regression that would account for the possible endogeneity due to the threshold variable. However, for the concerned reader, GMM methods may be applied to check for the presence of endogeneity, using the lagged values of $\sigma$ as an instrument. GMM would produce consistent and $\sqrt{n}$ asymptotically normal estimators which can be shown using the central limit theorem derived in Pakes and Pollard (1989).
Figure 12: Kernel Regression of $\tau$ on $\gamma_{total}$, for $\sigma_{primary} > 0.21$

Figure 13: Kernel Regression of $\tau$ on $\gamma_{total}$, for $\sigma_{health} < 0.59$
composition of public spending and the substitutability between public and private goods. The main results are that higher substitutability produces a lower progressivity of the income tax and a smaller size of the public spending over the GDP and that stronger pro-taxpayer bias in public spending is compatible with a lower/higher progressivity in the tax function as the substitutability is low or high. Both implications are validated by our empirical exercise using OECD country data, using kernel regressions and threshold regressions.

The paper has substantial room for improvement on both counts: theoretical and empirical. While redistributive activity of the government through taxes and transfers has attracted the interest of researchers, the role of public spending has been comparatively neglected.

We know too little about the redistributive impact of the different components of the government budget $G$. In the countries where redistributive effects are regularly estimated, such as the UK, they focus on five budget lines only: education, health, housing subsidies, travel subsidies, and school meals. For most countries these estimates simply do not exist. This lack of information is paralleled by a similar lack of modelling on how the change in the structure of government spending affects the consumer behavior and well-being.

The analysis of the substitutability between the private and publicly provided goods and services is in still a much weaker position. We are aware of

\[20\text{Except for the recent contribution by Schwabish et al (2006).}\]
no empirical work estimating this degree of substitutability nor of any formal modeling of the effect of the regulation of the private substitutive supply of goods and services that are being furnished by the state.

There is much to be gained by the joint analysis of public taxation and expenditure. Our work is but a first step in this direction.

References


A Countries used in the study - OECD database

The countries which are used for our analysis are as follows. Data has been obtained from the OECD database, at www.oecd.org/statistics

Australia
Belgium
Denmark
Finland
France
Germany
Ireland
Italy
Netherlands
Norway
Poland
Sweden
Switzerland
United Kingdom
United States

The data is available from 1975 to 1999, with no missing years.

B Definition of social transfers, OECD

The social transfers data, and expenditures on education and health, which have been used to estimate $G$, have been obtained from the OECD Social Expenditures database. The variables which are used to estimate the social transfers, obtained from the OECD data base, are given below. These social transfers are subtracted from total government expenditures to obtain the total amount of government expenditures on public services. This will constitute our estimate of $G$.

- GD1P: Compensation of employees; payable
- GD3P: Subsidies; payable
- GD62_631XXP: Social benefits and Social transfers in kind (via market producers); payable
- GD7P: Other current transfers; payable
- GD9P: Capital transfers; payable.

All estimates are converted to constant 1995 US dollars.