On the time-consistency of optimal capital taxes

Begoña Domínguez*

Department of Economics, The University of Auckland, Commerce A Building, 3A Symonds Street, Private Bag 92019, Auckland, New Zealand

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Abstract

This paper studies the time-inconsistency problem of optimal capital taxes. In the absence of full-commitment, it is well known that debt restructuring cannot solve the time-inconsistency problem for economies with a private stock of capital. We re-examine this result by exploring the role of institutional delays in government policies. We show that, when the implementation of government policy requires time, debt restructuring can enforce commitment to the optimal capital taxes. We conclude that, since institutional delays characterize democratic decision making, the time-inconsistency problem of capital taxes is not so severe.

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1. Introduction

This paper investigates the time-inconsistency problem of capital taxation. From earlier literature we know that debt restructuring cannot make the optimal fiscal policy time-consistent
because of the severity of the capital levy problem. In this paper we investigate the nature
of the capital levy problem and explore a form of partial-commitment: an institutional
delay in the implementation of capital taxes. In this context, we show that debt
restructuring can make the optimal fiscal policy time-consistent.

In a seminal paper Lucas and Stokey (1983) show that the optimal management of the
maturity structure of government debt can make the optimal policy time-consistent in
economies with no private capital. This method has been called debt restructuring and has
been extended to many different scenarios. For example, Persson and Svensson (1986) and
Faig (1991) solve the time-inconsistency problem of optimal labor taxes for open
economies. Later Faig (1994) shows that debt restructuring can eliminate the time-
inconsistency problem for economies with endogenous government spending. More recently, Alvarez et al. (2004) address and solve, through debt restructuring, the
time-inconsistency problem of labor taxes and monetary policy. All in all, the
general principle of this literature is that, as long as the government has enough variety
of debt at its disposal, debt restructuring can solve the time-inconsistency problem
of fiscal policy in economies without private capital. However, the literature is silent about
how debt restructuring can alleviate the time-inconsistency problem of optimal capital
taxes.

The time-inconsistency problem of optimal capital taxes has been investigated since
benevolent government finds optimal to tax capital heavily in the short-run but promise
zero capital taxes in the long-run in order to encourage investment. However, if future
governments can revise the policy plan, then they would optimally deviate from the
announced zero capital taxes and tax capital income heavily in their current and
subsequent periods. The reason for this is that capital taxes have a different degree of
distortion depending on the planning date; taxing capital income is more distortionary in
the long-run than in the short-run and taxing current capital income is just not
distortionary. In view of these incentives to deviate from the previously announced policy,
governments face a time-inconsistency problem. This problem is so severe that debt
restructuring cannot make the optimal policy time-consistent in economies with a stock of
private capital.

Is indeed the capital levy problem so severe? In a paper on the theory and practice of the
capital levy, Eichengreen (1990) claims that it is not. He argues that delays between the
proposal and implementation of a capital levy generate an opportunity to capital flight and
make a capital levy non-desirable. He concludes that this explains the lack of historical
evidence of successful capital levies since delays in policy implementation are an intrinsic
feature of democratic policy making.

The advantages of institutional delays have been also pointed out by Chari (1988). He
indicates that “since delays in implementing policies allow the government to take account
of the effect of the policy on current decisions, they help in resolving the time-inconsistency
problem”. However, he also comments that, in a dynamic setting, the outcomes with
delays do not coincide in general with the Ramsey outcomes. In the present paper we show
that they do coincide provided a sufficiently large debt structure.

This paper studies debt restructuring and the time-inconsistency of optimal capital taxes.
We consider a closed economy populated by rational individuals, competitive firms and a
benevolent government. The government must choose taxes on labor and capital income
and the issues of debt to finance an exogenous stream of government spending. Following
Chari (1988) and Eichengreen (1990), we introduce a delay between the proposal and implementation of capital taxes. In particular, we assume that capital taxes take place with a one-period lag. Next, we characterize the full-commitment policy by maximal capital taxes in the short-run and zero capital taxes in the long-run. Then we demonstrate that the full-commitment policy can be made time-consistent through the optimal management of the maturity structure of debt indexed to consumption and to after-tax wage. The main reason behind is that the time delay allows individuals to react to all changes in government policy and, thus, all taxes are now distortionary. There an appropriate restructuring of government debt can give the right incentives to future governments to continue with the announced policy plan. Therefore, we can conclude that, since time delays in policy implementation form part of actual policy making in democratic societies, the time-inconsistency problem of optimal capital taxes is not so severe.

Several papers, among them Lucas (1990) and Atkenson et al. (1999), have advocated that governments should simply switch from their current tax systems to zero capital taxes. Following this suggestion, we extend our results by introducing a zero capital tax rule and showing that the full-commitment policy under such a rule can be made time-consistent through debt restructuring. Yet the debt structure required to sustain this equilibrium is more complicated than that without a capital tax rule.

This paper is related to that of Zhu (1995). He considers an economy with endogenous capital utilization, where capital is never in inelastic supply, and therefore, taxing the current and the future capital income are both distortionary. In this context, he shows that the careful management of the maturity of debt indexed to consumption, to the after-tax wage, and to the after-tax return on capital can solve the time-inconsistency problem. This and the present paper share that taxes on capital income are distortionary. However, in contrast to Zhu’s setup, we consider private capital as a stock, since once accumulated it is in inelastic supply as in the classical examples of Kydland and Prescott (1977) and Fischer (1980), and capital levies as distortionary due to delays in the implementation of government policy. By doing this, the present paper completes Zhu’s arguments on the time-inconsistency of capital taxes and debt restructuring. Moreover, this paper shows the crucial role of policy implementation delays in the severity of time-inconsistency problem.

The effects of a delay between proposal and implementation of taxes have been also studied by Domeij and Klein (2005). They show that the delay reduces the welfare provided by the policy under full-commitment. Moreover, the longer is the delay, the higher the welfare losses. In contrast to these results, we show that, without commitment, an implementation delay can improve welfare.

In an economy without commitment, Klein and Ríos-Rull (2003) study the properties of optimal time-consistent taxes under a one-period lag in the implementation of capital taxes. They rule out the possibility of issuing government debt and focus on the properties of Markov perfect equilibria. The present paper shows that, once governments can issue debt with a sufficiently large debt structure, their time-inconsistency problem vanishes.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 solves the optimal policy under full-commitment and a one-period lag in capital taxes. Section 4 shows that debt restructuring can make the full-commitment solution sustainable. Section 5 concludes. Proofs of the propositions can be found in the Appendix.
2. The model

The economy is populated by infinitely lived identical individuals and a benevolent
government. The government must finance a given stream of public spending through time-
variant tax rates on labor and capital income and through debt. We assume that government
debt can be issued with a sufficiently rich structure in terms of maturity calendar and debt-
type variety. More precisely, the government at date \( t \) can issue sequences \( \{t+1b_s^c, t+1b_s^w\}_{s=t+1}^{\infty} \), which enter in the economy at the end of date \( t \), of claims on debt indexed to consumption and to after-tax wage at date \( s \geq t+1 \), respectively.\(^1\) Through the issue of these types of
bonds, the government promises debt payments, interest and principal, which can be, respectively, viewed as additional units of consumption and net labor income that the
individual receives at a given future date.\(^2\)

The representative individual is endowed with a given initial capital \( k_0 \), initial debt
claims maturing at date \( t \), and one unit of time per period that can be either devoted to
leisure \( 1-\ell_t \) or to output production \( \ell_t \). The individual derives utility from consumption \( c_t \) and leisure according to the function

\[
\sum_{t=0}^{\infty} \beta^t U(c_t, 1-\ell_t),
\]

with \( \beta \in (0, 1) \), and where \( U(\cdot, \cdot) \) is strictly increasing, concave and continuously
differentiable in both arguments. Taking prices and the government policy as given, the
individual chooses consumption, leisure and asset holdings (debt and capital \( k_t \)) to maximize his welfare (1) subject to the budget constraint

\[
 p_t \left[ c_t + k_{t+1} + \sum_{s=t+1}^{\infty} \frac{p_s}{p_t}(t+1b_s^c-i_t^c) + \sum_{s=t+1}^{\infty} \frac{p_s}{p_t}q_s(t+1b_s^w-i_t^w) \right] \\
\leq p_t\left[b_t^c + (1-\ell_t)w_t(l_t+1b_t^w) + R_tk_t\right],
\]

and the no-Ponzi-game conditions

\[
\lim_{t \to \infty} \sum_{s=t}^{\infty} p_{st}b_s^c = 0, \quad \lim_{t \to \infty} \sum_{s=t}^{\infty} p_{st}q_s b_s^w = 0, \quad \lim_{t \to \infty} p_t k_{t+1} = 0.
\]

Here \( p_t \) is the price of a final good, \( q_t \) the price of a bond indexed to after-tax wage in terms of
final goods, \( w_t \) the real wage, \( \ell_t \) the labor income tax rate, \( R_t \) the gross return on capital, after-tax \( \ell_t \) and depreciation \( \delta \) rates, and \( r_t \) the net return on capital, i.e. \( R_t = (1 + (1-\ell_t)r_t - \delta) \), at date \( t \). The first-order conditions for this optimization problem are the following:

\[
U_x(c_t, 1-\ell_t) = (1-\ell_t)w_t U_x(c_t, 1-\ell_t),
\]

\(^1\)If debt were indexed to before-tax wage, a government could default on debt payments by setting the labor tax
rate to one. Therefore, we consider debt indexed to after-tax wage. This variety of debt can be also found in Faig

\(^2\)Debt indexed to consumption can be identified with Treasury Inflation-Protected Securities that are issued
with a 5-, 10-, and 30-year maturity by the U.S. Treasury since 1997. These securities vary with the consumer price
index. We may identify debt indexed to after-tax wage with the promise of future social security pensions, which
are closely linked to the wage rate.
\[ U_c(c_t, 1 - l_t) = \beta (1 + (1 - \tau_{t+1}^k) r_{t+1} - \delta) U_c(c_{t+1}, 1 - l_{t+1}), \quad (5) \]

\[ \beta^t \frac{U_c(c_t, 1 - l_t)}{U_c(c_0, 1 - l_0)} = \frac{p_t}{p_0} \quad \text{and} \quad q_t = (1 - \tau_t^l) w_t, \quad (6) \]

where \( U_c(c_t, 1 - l_t) \) and \( U_x(c_t, 1 - l_t) \) denote the marginal utility with respect to consumption and leisure, respectively. Other derivatives of the utility and production functions follow a similar notation.

A representative competitive firm produces the final good using the technology \( y_t = \mathbf{f}(k_t, l_t) \), where \( \mathbf{f} \) is increasing, concave and continuous differentiable. Taking factor prices as given, the firm chooses capital and labor to maximize profits. This program yields the conditions

\[ r_t = f_k(k_t, l_t) \quad \text{and} \quad w_t = f_l(k_t, l_t). \]

The government must finance an exogenous stream of public spending \( \{g_t\}_{t=0}^{\infty} \) subject to the following budget constraint:

\[
p_t \left[ \sum_{s=t+1}^{\infty} p_s \left( (1 - \tau_s^w) b_s^w - b_s^c \right) + \sum_{s=t+1}^{\infty} q_s (1 - \tau_s^w) b_s^w + \tau_s^l b_s^l \right] \geq p_t [b_t^c + q_t b_t^w + g_t]. \quad (8)
\]

As commented earlier, we assume a one-period lag in the implementation of capital taxes. Moreover, as in Chamley (1986), capital tax rates must be lower or equal to a given upper bound, which for simplicity, equals one. Using the first-order condition for capital (5), this restriction can be written as

\[ U_c(c_{t-1}, 1 - l_{t-1}) \geq \beta U_c(c_t, 1 - l_t)(1 - \delta). \quad (9) \]

Next, we assume that the initial debt holdings \( \{b_t^c, b_t^w\}_{t=0}^{\infty} \) are such that distortionary taxes on labor and capital income are required in order to finance the public spending. Finally, in order to allow for long-run stationarity, we assume that the exogenous public spending converges to a constant value in the long-run and that the initial bonds \( \{b_t^c, b_t^w\}_{t=0}^{\infty} \) become constant for long maturities \( t \).

To finalize the model, we write down the resource constraint as

\[ c_t + k_{t+1} + g_t \leq f(k_t, l_t) + (1 - \delta) k_t, \quad (10) \]

and define a competitive equilibrium in what follows:

**Definition 1.** Given the policy \( \{\tau_t^k, \tau_t^l\}_{t=0}^{\infty} \), the exogenous government spending \( \{g_t\}_{t=0}^{\infty} \), the initial debt \( \{b_t^c, b_t^w\}_{t=0}^{\infty} \) and the initial private capital \( k_0 \), an allocation \( \{c_t, l_t, k_{t+1}\}_{t=0}^{\infty} \) is a competitive equilibrium allocation if and only if there exists a price sequence \( \{p_t, q_t, r_t, w_t\}_{t=0}^{\infty} \) such that: (i) the representative individual maximizes his welfare (1) subject to the budget constraint (2) and the no-Ponzi game conditions (3); (ii) factors are paid their marginal products (7); and (iii) all markets clear (the resource constraint (10) holds with equality).\(^4\)

\(^3\)For simplicity, we consider that labor taxes can be implemented without any delay. The results can be easily generalized to the introduction of a delay in the implementation of all tax rates.

\(^4\)Given that individual’s budget constraint (2) and the resource constraint (10) hold, the government budget constraint (8) is also satisfied in a competitive equilibrium.
3. The policy under full-commitment

In this section we assume that there is a full-commitment among the successive governments. In other words, future governments do not revise the policy plan, they just commit to follow the policy chosen by the government at date 0. In this context, the one-period lag in capital taxes only implies that the capital tax rate at date 0 is exogenously given.

To solve for the optimal fiscal policy, we follow the primal approach. We first determine the implementability constraint by adding the budget constraint (2) over time and plugging the transversality conditions

$$\lim_{t \to \infty} \sum_{s=1}^{\infty} \beta^t U_{c_t} b^C_s = 0, \quad \lim_{t \to \infty} \sum_{s=1}^{\infty} \beta^t U_{c_t}(1 - \tau^t_s)f_{l_t} b^W_s = 0, \quad \lim_{t \to \infty} \beta^t U_{c_t} k_{t+1} = 0,$$

and the first-order conditions (4)–(7), which yields

$$\sum_{t=0}^{\infty} \beta^t [(c_t - 0b^C_t)U_{c_t}(c_t, 1 - l_t) - (l_t + 0b^W_t)U_{x_t}(c_t, 1 - l_t)] = W_0 U_{c}(c_0, 1 - l_0),$$

where $W_0$ is the individual’s initial capital income, that is $W_0 = R_0k_0$. We now define the government’s optimization problem as follows. The government at date 0 chooses the sequences $\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize the welfare of the representative individual (1) subject to the resource constraint (10), the implementability condition (12), the upper bound on capital tax rates (9) and the transversality conditions (11), given the exogenous government spending $\{g_t\}_{t=0}^{\infty}$ and the initial conditions $\{0b^C_s, 0b^W_s\}_{s=0}^{\infty}$, $k_0$ and $\tau_0$.

The solution of this problem satisfies constraints (9)–(12), and the following first-order conditions for consumption, labor and private capital, respectively:

$$\mu_{0t} = H^c_{0t}(c_t, l_t, 0b^C_t, 0b^W_t, \Theta_{0t}, \lambda_0),$$

$$f_{l_t} \mu_{0t} = H^x_{0t}(c_t, l_t, 0b^C_t, 0b^W_t, \Theta_{0t}, \lambda_0),$$

$$\mu_{0t} = \beta \mu_{0t+1}[1 + f_{k_{t+1}} - \delta],$$

with

$$H^c_{0t} = (1 + \lambda_0) U_{c_t} + \lambda_0 [U_{c_t}(c_t - 0b^C_t) - U_{c_t}(c_t - 0b^C_t)] + U_{c_t} \Theta_{0t},$$

$$H^x_{0t} = (1 + \lambda_0) U_{x_t} + \lambda_0 [U_{x_t}(c_t - 0b^C_t) - U_{x_t}(c_t - 0b^C_t)] + U_{x_t} \Theta_{0t},$$

and

$$\Theta_{0t} = \begin{cases} \phi_{01} - \lambda_0 W_0 & \text{for date } t = 0, \\
\phi_{0t+1} - (1 - \delta)\phi_{0t} & \text{for all dates } t > 0,
\end{cases}$$

where $\phi_{0t}$, $\mu_{0t}$, and $\lambda_0$ are the Lagrange multipliers associated with constraints (9), (10) and (12), respectively.\(^5\)

\(^5\)As pointed out by Lucas and Stokey (1983), second-order conditions are not clearly satisfied because they involve third and second-derivatives of the utility function. Therefore, we assume that an optimal solution interior exists.
The optimal taxes are obtained from the first-order conditions (4)–(5). The dynamics of these taxes depend on the number of periods the upper bound on capital taxes (9) is active; let $\Phi$ denote those periods. Then the optimal capital taxes are described as follows:

**Proposition 1.** The optimal tax policy under full-commitment satisfies:

(i) $\tau^k_s = 1$ for all periods $s \in \Phi$.

(ii) $\tau^k = 0$ at the steady state.

**Proof.** See Appendix.

Proposition 1 summarizes Chamley’s (1986) results. The optimal capital tax rate is zero at steady state. Moreover, prior to the steady state, there may be a number of periods in which capital is taxed at its maximal rate, unity. This is illustrated in the next lemma:

**Lemma 1.** Let $U(c_t, 1 - l_t) = \theta_c \ln c_t + \theta_x \ln(1 - l_t)$, $f(k_t, l_t) = Ak_t + Bl_t$ and $\delta = 0$. If $\Phi$ is non-empty, then the duration of the tax regime $\tau^k_s = 1$ is such that $\Phi = [1, T]$, where

$$T = \frac{1}{\ln(1 + A)} \left( \ln \left( 1 + \lambda_0 \frac{W_0}{c_0} - \frac{\phi_0}{c_0} \right) - \ln \left( 1 + \frac{\phi_0}{c_T} \right) \right).$$ (16)

**Proof.** See Appendix.

As Chamley (1986) also argues, we find that, for a given initial consumption $c_0$, the duration of the regime with high capital taxes (16) depends positively on the need of distortionary taxation (measured by $\lambda_0$) and the initial capital income $W_0$.

The constraints (9)–(12) and the first-order conditions (4)–(5) and (13)–(15) describe the optimal allocation and policy under full-commitment. However, in the absence of full-commitment, future governments have incentives to select a continuation policy different from the announced plan. These incentives to deviate from the announced policy come from the possibility of re-optimizing taking into account the new endowments. These incentives take two forms. First, once government debt has been purchased by the individuals, the government finds optimal to default on the current debt payments. Second, given that the degree of distortion of the taxes changes with the planning date, the optimal policy does no longer coincide with the announced plan. Suppose there is a commitment to honor debt, can debt-restructuring eliminate the remaining incentives?

4. Debt restructuring

From this section on, we assume that future governments can reconsider both taxation and spending plans, but they commit to honor debt payments. In this context, the one-period lag in the implementation of capital taxes implies that the government at date $t$ can choose the policy plan for all dates $s \geq t$ but the initial capital tax rate $\tau^k_t$ is inherited from the previous government plan.

Let us now consider the time-inconsistency problem. We have assumed that all governments commit to honor debt and that the initial capital tax rate is inherited from the past. By inheriting an initial capital tax, we eliminate the possibility of increasing the current capital tax as a non-distortionary source of additional funds. However, the government can still choose all future capital tax rates and all current and future labor tax
rates. Can the current government neutralize the future incentives to deviate from its announced policy through debt restructuring? Here is the answer:

**Proposition 2.** If the sequences \( \{c_t, l_t, k_{t+1}\}_{t=0}^{\infty} \) and \( \{e_t, e_t^*\}_{t=0}^{\infty} \) are the optimal allocation and policy with a one-period lag in the implementation of capital taxes, then it is always possible to choose a debt structure \( \{b_t^c, b_t^w\}_{t=1}^{\infty} \) at market prices (6) such that the continuation sequences \( \{c_t, l_t, k_{t+1}\}_{t=0}^{\infty} \) and \( \{e_t, e_t^*\}_{t=1}^{\infty} \) of the same allocation and policy are a solution for the government problem when it is reconsidered at date 1. This can be done through the following debt structure:

\[
1b_t^c - 0b_t^c = \left[ \frac{\lambda_0}{\lambda_1} - 1 \right] \left( 0b_t^c - c_t - e_t^c \right) - \left( \frac{1}{\lambda_1} \right) \Gamma_t^c,
\]

(17)

\[
1b_t^w - 0b_t^w = \left[ \frac{\lambda_0}{\lambda_1} - 1 \right] \left( 0b_t^w + l_t - e_t^w \right),
\]

(18)

where

\[
\Gamma_t^c = \begin{cases} 
(\phi_{t2} - \phi_{t1}) - (1 - \delta)(\phi_{t0} - \lambda_1 k_1) & \text{for maturity } t = 1, \\
(\phi_{t0+1} - \phi_{t+1}) - (1 - \delta)(\phi_{t0} - \phi_{t1}) & \text{for all maturities } t>1.
\end{cases}
\]

By induction, the same is true for all later periods.\(^6\)

**Proof.** See Appendix.

Proposition 2 shows that the debt structure (17)–(18) induces the government at date 1 to continue with the policy that was previously announced by the government at date 0. Since the same is also true for all later governments, the full-commitment policy with a delay in the implementation of capital taxes can be made time-consistent through debt restructuring.

How is the debt structure that makes the optimal policy time-consistent? For an economy without restrictions on taxes, Lucas and Stokey (1983) obtain that debt structure is uniquely determined. In the next proposition, we show that the upper bound on capital taxes gives rise to a multiplicity of optimal debt structures:

**Proposition 3.** If the upper bound on capital taxes (9) is not binding for all dates \( t \geq 2 \), then Eqs. (17)–(18) determine uniquely the optimal debt structure \( \{b_t^c, b_t^w\}_{t=1}^{\infty} \). Otherwise, there are multiple debt structures that solve (17)–(18) and make the optimal policy time-consistent.

**Proof.** See Appendix.

As can be seen in Proposition 2, the optimal debt structure (17)–(18) depends on the multipliers faced by the government at date 1, namely, that on the implementability condition \( \lambda_1 \) and those on the upper bound on capital taxes \( \{\phi_{t1}\}_{t=2}^{\infty} \). But the latter depend on the particular debt structure inherited at date 1. Proposition 3 shows that Eqs. (17)–(18) together with the implementability condition at date 1 determine a unique \( \lambda_1 \). Then, if the upper bound on capital taxes (9) is not active for all \( t \geq 2 \), this multiplier, \( \lambda_1 \), implies a unique optimal debt structure. Otherwise, if the bound is active from period 2 to \( T \), then the multiplier \( \lambda_1 \) determines uniquely the debt indexed to the after-tax wage \( \{b_t^w\}_{t=1}^{\infty} \) and

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\(^6\)The terms \( e_t^c \) and \( e_t^w \) are defined in the Appendix.
the debt indexed to consumption \( \{b_i^t\}_{t>T} \) for the maturities in which the upper bound (9) is not active. But there are infinite combinations of positive multipliers \( \{\phi_{1t}\}_{t \geq 2} \) and debt structures \( \{b_i^t\}_{t \geq 1} \) that solve the implementability condition at date 1 and also make the optimal policy time-consistent.

Other studies have found that the optimal debt structure depends on the intertemporal elasticity of substitution, the relative cost of distortionary taxation \( \lambda_0/\lambda_1 \), the allocation, and the initial debt structure.\(^7\) For our economy with capital, we find those determinants and, in addition, we find the effects of both the initial stock of capital and the different efforts to satisfy \( \tau^k_i \leq 1 \) on the debt indexed to consumption with one-period maturity and with longer maturities, respectively. Consider, for example, the utility function

\[
u(c_t, 1 - l_t) = \frac{[c_t^\beta(1 - l_t)^{(1-\beta)}]^{-\sigma}}{1 - \sigma}.
\]

For this function, we have \( \varepsilon_t^c = -(1/\sigma)c_t \) and \( \varepsilon_t^x = -(1/\sigma)x_t \), where \( 1/\sigma \) is the intertemporal elasticity of substitution. Let us consider \( \sigma > 1 \). If the cost of distortionary taxation is relatively lower at date 1, i.e. \( \lambda_0/\lambda_1 > 1 \), then Eqs. (17)–(18) imply that the government should sell bonds indexed to the after-tax wage at all maturities and buy (sell) bonds indexed to consumption at maturities \( t > T \). The debt indexed to consumption with maturities \( 1, \ldots, T \) is undetermined. Suppose we choose the one that implies multipliers \( \phi_{1t} = \phi_{0t} \) for all \( t \), then the government should buy (sell) debt indexed to consumption with two-period and longer maturities. The one-period debt indexed to consumption depends on the sign of \( \phi_{01} - \lambda_1k_1 \), which is negative in our numerical simulations.\(^8\) If \( \lambda_0/\lambda_1 \leq 1 \), the direction of the above results reverse.\(^9\) Other combinations of multipliers and debt indexed to consumption at those maturities can be also obtained.

We provide a numerical example in Table 1. This table reports an optimal allocation and policy and some examples of optimal debt structure that make that allocation and policy time-consistent. To construct this table, we assume the utility function (19) and the production function \( f(k_t, l_t) = Ak_t^{\beta}l_t^{1-\beta} \).\(^10\) For the initial conditions \( k_0 = 8.1164, \tau^k_0 = 1, b_i^c = 0 \) and \( b_i^w = 0 \) for all \( t \geq 0 \), the upper bound on capital taxes is active for three periods, then there is one period with lower but positive capital taxes followed by the steady state with zero capital taxes. Labor taxes are negative in the first two periods and increase towards the steady state. This generates a budget surplus at date 0. In an environment with full-commitment, these taxes implement the optimal allocation \( \{c_t, l_t, k_t\}_{t=0}^\infty \) shown in the table. Moreover, without commitment, the government at date 0 can influence the government at date 1 through the debt structure. In particular, given this budget surplus, the government can choose any of the debt structures \( \{b_i^c, b_i^w\}_{t=1}^\infty \) shown in Table 1 (or any other that satisfies (17)–(18)) and for that debt and initial conditions \( k_1 = 7.8437, \tau^k_1 = 1 \), the government at date 1 optimally chooses to

\(^7\)See, for example, Faig (1994).

\(^8\)For an economy without tax constraints, the optimal debt structure would be uniquely determined and would resemble the one specified for the multipliers \( \phi_{1t} = \phi_{0t} \) for all \( t \).

\(^9\)Faig (1994) shows that the sign of \( \lambda_0/\lambda_1 - 1 \) is opposite to the sign of the government cash flow at date 0 (the difference between tax revenue and government expenditure). For our economy with capital, we can show that the sign of \( \lambda_0/\lambda_1 - 1 \) is linked to the sign of the government cash flow but it is not always the opposite.

\(^10\)We consider the parameter values \( \beta = 0.97, \sigma = 2, \theta = 0.36, A = 2, \alpha = 0.33, \delta = 0.05, \) and \( g_t = g = 0.25 \) for all \( t \geq 0 \).
continue with the policy chosen at date 0. Thus, the optimal policy and allocation are made time-consistent.

Table 1 shows some examples of optimal debt structure. For this budget surplus at period 0, the cost of distortionary taxation in period 1 is relatively lower and, thus, the bonds indexed to the after-tax wage at all maturities and to consumption at maturities $t > T$ satisfy the above description. To characterize the bonds indexed to consumption at maturities $1, \ldots, T$, we consider different streams of positive multipliers $\phi_{1t}$. The first example corresponds to $\phi_{1t} = \phi_{0t}$ for all $t$. There we can see the effect of the initial capital stock in the one-period debt indexed to consumption. The one-period bond indexed to consumption is negative and large in order to neutralize the effect of the initial wealth on the government at date 1. Longer maturities are proportional to consumption and negative. The next three examples correspond to positive multipliers $\phi_{1t}$ that do not coincide with $\phi_{0t}$ for all $t$. There we can see that positive amounts of debt indexed to consumption can be sold at some maturities as long as they are correctly compensated with negative debt at other maturities. The debt structures shown in Table 1 are just mere examples, other debt structures that satisfy (17)–(18) would also make the government at date 1 optimally selects the same taxes as those that were announced at date 0.

We can use Lemma 1 to understand our result better. This lemma says that the government at date 0 sets capital tax rates equal to 100% in all periods $[1, T]$. Since the debt structure (17)–(18) guarantees that the allocation and policy chosen at date 1 coincide with those announced at date 0, the upper bound on capital taxes (9) is active from period 2 to $T$ and the duration of the regime with high capital taxes is now $[2, T]$. When period $T + 1$ arrives, the government does not want to tax any future capital at the maximal rate but, if it could, it would tax very heavily the current capital income. However, the institutional delay between the proposal and the implementation of the capital tax rules out that possibility and enables debt restructuring to solve the time-inconsistency problem.

In this section we have shown that debt restructuring can make the optimal fiscal policy time-consistent. This government policy is characterized by an initial regime of maximal

### Table 1
Optimal allocation, taxes and examples of optimal debt structure with one-period lag in capital taxes

<table>
<thead>
<tr>
<th>Period</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>$t \geq 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocation</td>
<td>$c_t$</td>
<td>1.5307</td>
<td>1.4387</td>
<td>1.3525</td>
<td>1.2718</td>
<td>1.2064</td>
</tr>
<tr>
<td></td>
<td>$l_t$</td>
<td>0.3339</td>
<td>0.3312</td>
<td>0.3289</td>
<td>0.3268</td>
<td>0.3253</td>
</tr>
<tr>
<td></td>
<td>$k_t$</td>
<td>8.1164</td>
<td>7.8437</td>
<td>7.6451</td>
<td>7.5177</td>
<td>7.4595</td>
</tr>
<tr>
<td></td>
<td>$y_t$</td>
<td>1.9138</td>
<td>1.8823</td>
<td>1.8574</td>
<td>1.8394</td>
<td>1.8293</td>
</tr>
<tr>
<td>Taxes</td>
<td>$\tau^c_t$</td>
<td>0.0636</td>
<td>-0.0044</td>
<td>0.0535</td>
<td>0.1095</td>
<td>0.1562</td>
</tr>
<tr>
<td></td>
<td>$\tau^l_t$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.8668</td>
</tr>
<tr>
<td>Debt structure</td>
<td>$(b_{1c}^t,b_{1w}^t)$ &amp; $(-3.1085,0.0294)$ &amp; $(-0.2999,0.0294)$ &amp; $(-0.0281,0.0293)$ &amp; $(-0.0267,0.0293)$ &amp; $(-0.0267,0.0293)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(b_{2c}^t,b_{2w}^t)$ &amp; $(-0.0000,0.0294)$ &amp; $(-2.9829,0.0294)$ &amp; $(-0.0281,0.0293)$ &amp; $(-0.0267,0.0293)$ &amp; $(-0.0267,0.0293)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(b_{3c}^t,b_{3w}^t)$ &amp; $(-0.0000,0.0294)$ &amp; $(-0.0000,0.0294)$ &amp; $(-2.8619,0.0293)$ &amp; $(-0.0267,0.0293)$ &amp; $(-0.0267,0.0293)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(b_{4c}^t,b_{4w}^t)$ &amp; $(0.1734,0.0294)$ &amp; $(0.2184,0.0294)$ &amp; $(-3.2259,0.0293)$ &amp; $(-0.0267,0.0293)$ &amp; $(-0.0267,0.0293)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
capital taxes and by zero capital taxes at the steady state. What about if governments would reform their current tax systems and switch directly to zero capital taxes? In the next section, we explore whether there is an appropriate debt structure that would allow governments to sustain a zero capital tax rule.

4.1. A zero capital tax rule

In a seminal paper Kydland and Prescott (1977) advocated rules rather than discretion and claimed that “reliance on policies such as … constant tax rates constitute a safer course of action”. In this section, we consider a zero tax rule on capital income. Under this rule, the current and future governments choose their policy subject to a constant capital tax rate equal to zero for all dates. This restriction can be written as

\[ U_{c_{t-1}} = \beta U_{c_t}(1 + f_{k_t} - \delta). \]  

(20)

The government’s problem is to choose \( f_{c_t}, l_t, k_{t+1} \) to maximize the welfare of the representative individual (1) subject to the resource constraint (10), the implementability condition (12), the zero tax rate constraint on capital income (20) and the transversality conditions (11), given the exogenous government spending \( g_t \) and initial conditions \( f_{0t}, l_0, k_0, \delta \).

From this problem, we obtain the following first-order conditions for consumption, labor and private capital, respectively:

\[ \mu_{0t} = J^c_{0t}(c_t, l_t, k_{t+1}, \Theta^c_{0t}) \]

(21)

\[ f_{lt} = J^x_{0t}(c_t, l_t, k_{t+1}, \Theta^x_{0t}), \]

(22)

\[ \mu_{0t} = \beta \mu_{0t+1}[1 + f_{k_{t+1}} - \delta] - \beta \xi_{0t+1} f_{k_{t+1}} U_{c_{t+1}} \]

(23)

where

\[ J^c_{0t} = (1 + \lambda_0) U_{c_t} + \lambda_0 [U_{c_t, c_t}(c_t - 0b_t) - U_{c_t, l_t}(l_t + 0b_t)] + U_{c_t, \Theta^c_{0t}}, \]

\[ J^x_{0t} = (1 + \lambda_0) U_{x_t} + \lambda_0 [U_{x_t, c_t}(c_t - 0b_t) - U_{x_t, l_t}(l_t + 0b_t)] + U_{x_t, \Theta^x_{0t}}, \]

with

\[ \Theta^c_{0t} = \begin{cases} \xi_{0t} - \lambda_0 W_0 & \text{for date } t = 0, \\ \xi_{0t+1} - (1 + f_{k_t} - \delta) \xi_{0t} & \text{for all dates } t > 0, \end{cases} \]

\[ \Theta^x_{0t} = \begin{cases} \xi_{0t} - \lambda_0 W_0 - f_{k_{0t}} U_{c_t, \Theta^c_{0t}} \lambda_0 k_0 & \text{for date } t = 0, \\ \xi_{0t+1} - (1 + f_{k_t} - \delta) \xi_{0t} - f_{k_{0t}} U_{c_t, \Theta^c_{0t}} \xi_{0t} & \text{for all dates } t > 0. \end{cases} \]

Here \( \mu_{0t}, \lambda_0 \) and \( \xi_{0t} \) are the Lagrange multipliers associated with constraints (10), (12) and (20), respectively. The optimal labor tax rates can be obtained from (4).

Let us assume that governments commit to honor debt. Then, we can show that the following debt structure eliminates the time-inconsistency problem:

\[ \text{ARTICLE IN PRESS} \]

\[ \text{B. Domínguez / Journal of Monetary Economics 54 (2007) 686–705} \]

11As in the previous section, we assume that an optimal interior solution exists.
Proposition 4. If the sequences \{c_t, l_t, k_{t+1}\}_{t=0}^{\infty} and \{\tau_t^c, \tau_t^x\}_{t=0}^{\infty} are the optimal allocation and policy under a zero capital tax rule, then it is always possible to choose a debt structure \{b_t^c, b_t^w\}_{t=1}^{\infty} at market prices (6) such that the continuation sequences \{c_t, l_t, k_{t+1}\}_{t=1}^{\infty} and \{\tau_t^c, \tau_t^x\}_{t=1}^{\infty} of the same allocation and policy are a solution for the government problem when it is reconsidered at date 1. This could be done through the following debt structure:

\[
b_t^c - b_t^c = \left[\frac{\lambda_0}{\lambda_1} - 1\right] (g_t^c - c_t - \epsilon_t^c) - \frac{1}{\lambda_1} (\Gamma_t^c - \Psi_t^c), \tag{24}
\]

\[
b_t^w - b_t^w = \left[\frac{\lambda_0}{\lambda_1} - 1\right] (g_t^w + l_t - \epsilon_t^x) - \frac{1}{\lambda_1} (\Gamma_t^x - \Psi_t^x), \tag{25}
\]

where

\[
\Psi_t^m = \beta(\xi_{0t+1} - \xi_{1t+1})v_t^m U_{c_t,k_{t+1}} f_{k_{t+1}}
\]

\[
+ \sum_{j=t+1}^{\infty} \beta R_{j} \beta(\xi_{0j+1} - \xi_{1j+1}) U_{c_{j+1},k_{j+1}} f_{k_{j+1}},
\]

for \(m = \{c, x\}\) and

\[
\Gamma_t^c = \begin{cases} 
(\xi_{01} - \xi_{12}) - (\xi_{01} - \lambda_1 k_1)(R_1 - \lambda_1 f_{k_{11}}) & \text{for maturity } t = 1, \\
(\xi_{0t+1} - \xi_{1t+1}) - (\xi_{0t} - \xi_{1t})(R_t - \lambda_1 f_{k_{t1}}) & \text{for all maturities } t > 1,
\end{cases}
\]

\[
\Gamma_t^x = \begin{cases} 
-(\xi_{01} - \lambda_1 k_1)\lambda_1 f_{k_{11}} & \text{for maturity } t = 1, \\
-(\xi_{0t} - \xi_{1t})\lambda_1 f_{k_{t1}} & \text{for all maturities } t > 1.
\end{cases}
\]

By induction, the same is true for all later periods.\(^{12}\)

Proof. See Appendix.

Proposition 4 guarantees that under that debt structure the full-commitment policy under a zero capital tax rule can be made time-consistent through debt restructuring.

As in the previous section, it can be easily proved that the restriction on capital taxes generates a multiplicity of optimal debt structures. Moreover, the multiplicity is greater because (i) the zero capital tax constraint (20) is active for much longer, (ii) debt indexed to the after-tax wage depends on the multipliers on the tax constraint and (iii) the multiplier \(\lambda_1\) depends on the particular debt structure.

We find the same determinants of the optimal debt structure but the effect of the different efforts to satisfy \(\tau_t^k = 0\) instead of \(\tau_t^k \leq 1\). For the utility function (19), \(x > 1\) (\(x \leq 1\)) and the multipliers \(\xi_{1t} = \xi_{0t}\) for all \(t\), equations (24) - (25) imply that, if \(\frac{\lambda_0}{\lambda_1} > 1\), then the government should sell bonds indexed to the after-tax wage and buy (sell) bonds indexed to consumption at all maturities \(t > 1\). Conversely, if \(\frac{\lambda_0}{\lambda_1} \leq 1\). The one-period debt depends on the sign of \(\xi_{01} - \lambda_1 k_1\). We can also find other combinations of multipliers and debt structures.

A numerical example is provided in Table 2.\(^{13}\) For the initial conditions \(k_0 = 8.1164, \tau_0^k = 0, \ 0b_t^c = 0\) and \(0b_t^w = 0\) for all \(t \geq 0\), the zero capital tax rate condition is always active

\(^{12}\) The terms \(v_t^m\) and \(\chi_t^m\) for \(m = \{c, x\}\) are defined in the Appendix.

\(^{13}\) We consider the same utility function, production function, parameter values and initial conditions (except for \(\tau_0^k = 0\)) as in Table 1.
and generates a very long transition towards the steady state. Labor taxes are positive and increasing. If, for the given cash flow at date 0, the government restructures the debt appropriately (some examples of appropriate debt structure are given in Table 2) then it guarantees that its policy announcement will be indeed carried out.

Table 2 provides four examples of optimal debt structure. The first corresponds to $x_{1t} = x_{0t}$ for all $t$, which highlights the effect of the initial capital stock on the one-period debt. The initial capital stock affects now both debt indexed to consumption and to the after-tax wage with one-period maturity. The amounts of traded debt at all maturities are smaller (compared to those in Table 1) because the cost of distortionary taxation at date 1, measured by $\lambda_1$, does not fall as much as in the case with initial maximal capital taxes. The other three examples are such that $x_{1t} \neq x_{0t}$ for some $t$. These examples illustrate the multiplicity of optimal debt structures.

5. Conclusions

This paper has investigated the role of debt restructuring in the time-inconsistency problem of optimal fiscal policy for an economy with private capital. The time-inconsistency of optimal taxation arises when the time passes, the endowments change and the government finds the possibility of setting a less distortionary taxation than that of the policy plan announced by the previous government. If the passing of time brings along the possibility of exploiting a pure rent, the time-inconsistency problem cannot be solved through the debt restructuring. This paper shows that an institutional delay between the proposal and implementation of government policies eliminates that possibility and creates an appropriate environment to employ the debt restructuring method. In this environment, we demonstrate that debt restructuring can make the optimal fiscal policy time-consistent.

An important contribution of this paper is that we show that the existence of institutional delays in policy implementation reduces dramatically the severity of time-inconsistency problems. In particular, we demonstrate that the policy and allocation with
delays coincide with the Ramsey policy and allocation provided a sufficiently rich debt structure.

A common result in the literature of debt restructuring is that the number of debt instruments required to solve the time-inconsistency must be equal to the number of policy choices. To solve the problem of labor taxes, Lucas and Stokey (1983) require bonds indexed to consumption for all possible maturities. In this paper, the government can choose both labor and capital tax rates and, therefore, we require an extra debt instrument: bonds indexed to the after-tax wage for all maturities. An interesting extension would be to assess whether all these types and maturities of debt are required when governments are trying to build a reputation. In an economy without capital, Domínguez (2005) shows that a rich debt structure is not necessary, and that, moreover, we would rather observe a concentration of government debt in few maturities under a reputation. Would that still hold in an economy with private capital? Moreover, how beneficial would institutional delays be under a reputation?

One important question that remains unanswered is how the government can commit to credibly honor its debt payments. In addition, it should be evaluated whether delays in the implementation of a government default can help enforce such a commitment.

Appendix A

Proof of Proposition 1. The proof of (i) is obvious. If the upper bound on capital taxes (9) is binding at date $s$, then the capital tax rate is equal to 1 at date $s$. Next, we prove (ii), that the capital tax rate is zero at steady state. First, note that the upper bound on capital taxes (9) cannot be active at the steady state. If the constraint (9) was binding, the marginal utility of consumption would be increasing over time, which contradicts the definition of a steady state. Second, given the assumptions on the initial debt structure, the first-order condition for consumption (13) satisfies that $H_{0t}$ becomes constant at the steady state. Finally, combining the first-order conditions for capital for the government (15) and for the individual (5), we obtain that the steady state capital tax rate is zero.

Proof of Lemma 1. As proved above, the upper bound on capital taxes (9) cannot be active at the steady state. Moreover, by the same reasoning, there should be a finite date $T$, which is the last date the upper bound (9) is binding. Next, we show that, if the constraint (9) is binding at date $T$, then it is binding in all periods $[1, T]$. For the given specified functions, the first-order conditions (5) and (15) can be, respectively, written as

$$c_T = \beta c_{T-1}, \quad (26)$$

$$\left(1 - \frac{\Theta_{0T-1}}{c_{T-1}}\right) c_T = \beta (1 + A) \left(1 - \frac{\Theta_{0T}}{c_T}\right) c_{T-1} \quad (27)$$

By assumption, we have $\phi_{0T}/c_T > 0$ and $\phi_{0T+1}/c_{T+1} = 0$. Combining Eqs. (26)–(27), we obtain

$$\left(1 - \frac{\phi_{0T}}{c_T} + \frac{\phi_{0T-1}}{c_T}\right) = (1 + A) \left(1 + \frac{\phi_{0T}}{c_T}\right),$$

which implies that $\phi_{0T-1}/c_{T-1} > 0$. Then, considering Eqs. (26)–(27) at date $T-1$ and earlier dates, it is very easy to see that $\phi_{0s}/c_s > 0$ for all $s \in [1, T]$. Next, we solve
Eqs. (26)–(27) recursively to obtain

\[ c_T = \beta^T c_0, \]

\[ \left(1 - \frac{\Theta_{00}}{c_0}\right) c_T = (\beta(1 + A))^T \left(1 - \frac{\Theta_{0T}}{c_T}\right) c_0. \]

Combining these last two equations, we get

\[ (1 + A)^T = \left(1 + \frac{\lambda_0 W_0}{c_0 - \phi_{01}/c_0} \right), \]

which proves Eq. (16). It can also be shown that \( \phi_{01} \) depends positively on \( \lambda_0 W_0 \) and that the overall effect of \( \lambda_0 W_0 \) on \( T \) is also positive. \( \square \)

**Proof of Proposition 2.** We consider the policy plans for the governments at date 0 and at date 1. We show that the government at date 0 can select a particular debt structure \( \{b_{t+1}^c, b_t^w\}_{t=1}^\infty \) such that the same allocation \( \{c_t, l_t, k_{t+1}\}_{t=1}^\infty \) and policy \( \{\tau_t^l, \tau_{t+1}^k\}_{t=1}^\infty \) solve both optimization problems at date 0 and at date 1. Let us present the policy plans for the two governments.

The government plan at date 0 satisfies the constraints (9)–(10), the first-order conditions (4)–(5) and (13)–(15) for all dates \( t \geq 0 \), the transversality conditions (11) and the implementability constraint (12), for a given initial debt \( \{b_{s}\}_{s=0}^\infty, b_s^w\), capital \( k_0 \) and capital tax rate \( t_0^k \). The government plan at date 1 satisfies the constraints (9)–(10) and the first-order conditions for the individual (4)–(5) and for the government

\[ \mu_{1t} = H_1^c(c_t, l_t, b_{t+1}^c, b_t^w, \Theta_{1t}, \lambda_1), \]

\[ f_{1t, \mu_{1t}} = H_1^k(c_t, l_t, b_{t+1}^c, b_t^w, \Theta_{1t}, \lambda_1), \]

\[ \mu_{1t} = \beta \mu_{1t+1}[1 + f_{k_{t+1}} - \delta], \]

for all dates \( t \geq 1 \), the transversality conditions (11) and the implementability constraint

\[ \sum_{i=1}^\infty \beta^{i-1}[(c_{t-1} - b_t^c)U_c(c_t, 1 - l_t) - (l_{t+1} + b_t^w)U_c(c_t, 1 - l_t)] = W_1 U_c(c_1, 1 - l_1), \]

given some initial conditions \( \{c_t, l_t, k_{t+1}, \tau_{t}^l, \tau_{t+1}^k\}_{t=0}^\infty \) that solves the policy plan at date 0 can solve the policy plan at date 1. First, since this sequence solves the constraints (9)–(10) and the conditions (4)–(5) at all dates \( t \geq 0 \), the sequence also solves those equations at all dates \( t \geq 1 \). By the same argument, the transversality conditions (11) also hold. Moreover, given the one-period implementation lag, the initial capital tax rate of the government plan at date 1 equals the capital tax rate for date 1 chosen by the government at date 0. Second, we require two debt instruments at each maturity to guarantee that the same allocation that solves (13)–(15) also solves the first-order conditions (28)–(30). Finally, by imposing the path for the debt instruments \( \{b_{t+1}^c, b_t^w\}_{t=1}^\infty \) into the implementability condition (31), we find the multiplier \( \lambda_1 \) that makes this condition hold. Therefore, debt restructuring can make the plan at date 0 optimal when re-considered at date 1.

Let us now find the debt structure \( \{b_{t+1}^c, b_t^w\}_{t=1}^\infty \) that provides time-consistency. Given that the first-order conditions at the initial date are different from those at later dates, we need to determine the following: (i) debt indexed to consumption maturing at the first date \( b_1^c \);
(ii) debt indexed to consumption maturing at the second date and later \(\{b_{t}^{c}\}_{t=2}^{\infty}\); (iii) debt indexed to after-tax wage with one-period maturity \(b_{1}^{w}\); and (iv) the issues of debt indexed to after-tax wage maturing at the second and later dates \(\{b_{t}^{w}\}_{t=2}^{\infty}\).

We next compute the issues of debt indexed to after-tax wage with two-period and higher maturities, that is, \(\{b_{t}^{w}\}_{t=2}^{\infty}\). To do that, we first derive which condition must be met so that the same allocation satisfies the first-order conditions for consumption and leisure under the plan at date 0, (13)–(14), and under the plan at date 1, (28)–(29). These conditions can be summarized into:

\[
H_{0t}^{c}(c_{t}, l_{t}, 0b_{t}^{c}, b_{t}^{c}, \Theta_{0t}, \lambda_{0}) - f_{l}H_{0t}^{c}(c_{t}, l_{t}, 0b_{t}^{c}, b_{t}^{c}, \Theta_{0t}, \lambda_{0}) = 0, \tag{32}
\]

\[
H_{1t}^{c}(c_{t}, l_{t}, 0b_{t}^{c}, b_{t}^{c}, \Theta_{1t}, \lambda_{1}) - f_{l}H_{1t}^{c}(c_{t}, l_{t}, 0b_{t}^{c}, b_{t}^{c}, \Theta_{1t}, \lambda_{1}) = 0. \tag{33}
\]

Equating the LHS of Eqs. (32) and (33), we find

\[
[\lambda_{0} - \lambda_{1}]((U_{x_{t}} - f_{l}U_{c_{t}}) + (U_{c_{x_{t}}} - f_{l}U_{c_{c_{t}}})c_{t} - (U_{x_{c_{x_{t}}}} - f_{l}U_{c_{c_{c_{t}}}})l_{t}) = - (U_{c_{x_{x_{t}}}} - f_{l}U_{c_{c_{x_{t}}}})(\lambda_{1}b_{t}^{c} - \lambda_{0}b_{t}^{c} + ((\phi_{0t+1} - \phi_{1t}) - (1 - \delta)(\phi_{0t} - \phi_{1t})))), \tag{34}
\]

We divide Eq. (34) by \(- (U_{x_{x_{t}}} - f_{l}U_{c_{x_{t}}})\lambda_{1}\), add and subtract \([\lambda_{0}/\lambda_{1} - 1]b_{t}^{w}\) and rearrange to obtain

\[
\left[\frac{U_{x_{x_{t}}} - f_{l}U_{c_{x_{t}}}}{U_{c_{x_{x_{t}}} - f_{l}U_{c_{c_{x_{t}}}}}}\right] \left[\frac{\lambda_{0}}{\lambda_{1}} - 1\right] = \frac{1}{\lambda_{1}}((\phi_{0t+1} - \phi_{1t+1}) - (1 - \delta)(\phi_{0t} - \phi_{1t})). \tag{35}
\]

Second, we find the condition under which \(\mu_{1t} = \mu_{0t}\) for all \(t \geq 2\), which allows the same allocation to solve conditions (15) and (30). We first substitute \(\mu_{0t}\) and \(\mu_{1t}\) by their respective values. Eqs. (14) and (29), and divide the resulting equation by \(- U_{x_{x_{t}}}\lambda_{1}\). Next, we add and subtract \([\lambda_{0}/\lambda_{1} - 1]b_{t}^{w}\) to get

\[
\left[\frac{U_{x_{x_{t}}} - f_{l}U_{c_{x_{t}}}}{U_{c_{x_{x_{t}}} - f_{l}U_{c_{c_{x_{t}}}}}}\right] \left[\frac{\lambda_{0}}{\lambda_{1}} - 1\right] = \frac{1}{\lambda_{1}}((\phi_{0t+1} - \phi_{1t+1}) - (1 - \delta)(\phi_{0t} - \phi_{1t})). \tag{36}
\]

Finally, equating the LHS of (35) and (36), it results that

\[
1b_{t}^{w} - 0b_{t}^{w} = \left[\frac{\lambda_{0}}{\lambda_{1}} - 1\right] \left[0b_{t}^{w} + l_{t} - \frac{U_{x_{x_{t}}} - f_{l}U_{c_{x_{t}}}}{U_{c_{x_{x_{t}}} - f_{l}U_{c_{c_{x_{t}}}}}}\right] - \lambda_{1}(\phi_{0t+1} - \phi_{1t+1}) - (1 - \delta)(\phi_{0t} - \phi_{1t})).
\]

which are the optimal issues of debt indexed to after-tax wage maturing at date \(t \geq 2\).

The remaining types of debt and maturities can be found by following similar steps. We have used the notation

\[
e_{t}^{c} = \left(\frac{U_{c_{x_{t}}} - U_{x_{t}}U_{c_{x_{t}}}}{U_{c_{x_{x_{t}}} - U_{x_{t}}U_{c_{c_{x_{t}}}}}}\right) \text{ and } e_{t}^{w} = \left(\frac{U_{x_{x_{t}}} - U_{x_{t}}U_{c_{x_{t}}}}{U_{c_{x_{x_{t}}} - U_{x_{t}}U_{c_{x_{x_{t}}}}}}\right).
\]

To sum up, we have shown that the government at date 0 can choose a particular debt structure \(\{b_{t}^{c}, b_{t}^{w}\}_{t=1}^{\infty}\) that makes the allocation and policy that solve the plan at date 1.
identical to those of the government plan at date 0. We can replicate this procedure for all later dates. Therefore, the policy plan under full-commitment at date 0 can be made time-consistent through debt-restructuring. □

**Proof of Proposition 3.** From the previous proof, we know that the debt structure (17)–(18) makes the first-order conditions for the government at date 0, (13)–(15), and, in turn, those equations determine the same first-order conditions for the government at date 1, (28)–(30), identical to those of the government plan at date 0. We can replicate this procedure for all dates.

By introducing the debt structure (17)–(18) into the implementability condition (31), this condition becomes a function of the multipliers λ1 and {φ1t}t≥2. Using the upper bound on capital taxes (9), all the multipliers {φ1t}t≥2 cancel out. Therefore, the implementability condition (31) determines uniquely λ1. Moreover, if the upper bound on capital taxes (9) is not active for all dates t≥2, then the multiplier λ1 determines a unique debt structure {1t bT1, bTw} t≥1 through Eqs. (17)–(18). Otherwise, the multipliers {φ1t}T ≥2 are not pinned down and there are infinite combinations of positive multipliers {φ1t}T ≥2 and debt structure (17)–(18) that solve the implementability condition (31) and make the optimal policy time-consistent. □

**Proof of Proposition 4.** We basically follow the same steps as for the proof of Proposition 2. We first consider the policy plans for the governments at date 0 and at date 1. The government plan at date 0 satisfies the resource constraint (10), the capital tax rule (20) and the first-order conditions (4)–(5) and (21)–(23) for all dates t≥0, the transversality conditions (11), and the implementability constraint (12), for some initial conditions. The government plan at date 0 satisfies the resource constraint (10), the capital tax rule (20) and the first-order conditions (4)–(5) and (21)–(23) for all dates t≥0, the transversality conditions (11), and the implementability constraint (12), for some initial conditions.

Following the same arguments as in the proof of Proposition 2, it is clear that we need two debt instruments maturing at each period so as to make the same allocation satisfy the government’s first-order conditions from the plan at date 0, (21)–(23), and from the plan at date 1, (37)–(39). Once we pin down the debt structure, the same allocation also solves the remaining constraints and conditions.

As before, we have four types of bonds. We next compute the issues of debt indexed to after-tax wage with two-period and higher maturities, that is, ∞ t bTw t. The pair of first-order conditions for consumption and leisure under the plan at date 0, (14)–(22), and the plan at date 1, (37)–(38), can be written, respectively, as follows:

\[ J^X_{0t}(c_t, l_t, 0b_t, 0b_w, \Theta^X_{0t}, \lambda_0) - f_{lt} J^y_{0t}(c_t, l_t, 0b_t, 0b_w, \Theta^y_{0t}, \lambda_0) = 0, \]  

\[ J^X_{1t}(c_t, l_t, 1b_t, 1b_w, \Theta^X_{1t}, \lambda_1) - f_{lt} J^y_{1t}(c_t, l_t, 1b_t, 1b_w, \Theta^y_{1t}, \lambda_1) = 0, \]
Equating the LHS of Eqs. (40) and (41), we find
\[
[\lambda_0 - \lambda_1]((U_{x,t} - f_{l,t} U_{c,t}) + (U_{c,x,t} - f_{l,t} U_{c,c}) c_t - (U_{x,x,t} - f_{l,t} U_{c,c}) l_t) = - (U_{c,x,t} - f_{l,t} U_{c,c})(l_{11} b_t^w - \lambda_0 b_t^c + ((\xi_{0t+1} - \xi_{1t+1}) - R_t(\zeta_{0t} - \zeta_{1t}))) + U_{c,c} f_{k,l,t} (\zeta_{0t} - \zeta_{1t}) - (U_{x,x,t} - f_{l,t} U_{c,c})(\lambda_{11} b_t^w - \lambda_0 b_t^c).
\]
(42)

We divide Eq. (42) by \(-(U_{x,x,t} - f_{l,t} U_{c,c})\lambda_1\), and add and subtract \(0 b_t^w [\lambda_0 / \lambda_1 - 1]\) to obtain
\[
\left(\frac{U_{x,x,t} - f_{l,t} U_{c,c}}{U_{c,c} - f_{l,t} U_{c,c}}\right) \left( \left(\frac{\lambda_0}{\lambda_1} - 1\right) \left( l_t + 0 b_t^w - \left( \frac{U_{x,t} - f_{l,t} U_{c,t}}{U_{c,c} - f_{l,t} U_{c,c}} \right) \right) - (1 b_t^w - 0 b_t^w) \right) + \frac{1}{\lambda_1} \left( \frac{U_{c,c} f_{k,l,t}}{U_{c,c} - f_{l,t} U_{c,c}} \right) (\xi_{0t} - \xi_{1t}) = b_t^c - \frac{\lambda_0}{\lambda_1} b_t^c + \left( \frac{\lambda_0}{\lambda_1} - 1 \right) c_t + \frac{1}{\lambda_1} (\xi_{0t+1} - \xi_{1t+1}) - R_t(\zeta_{0t} - \zeta_{1t}).
\]
(43)

We find now the condition to allow the same allocation solve both
\[
\mu_{0t} - \beta \mu_{0t+1} [1 + f_{k_{t+1}} - \delta] + \beta \zeta_{0t+1} f_{k_{t+1} k_{t+1}} U_{c,t+1} = 0,
\]
\[
\mu_{1t} - \beta \mu_{1t+1} [1 + f_{k_{t+1}} - \delta] + \beta \zeta_{1t+1} f_{k_{t+1} k_{t+1}} U_{c,t+1} = 0.
\]
Equating the LHS of these two equations, substituting \(\mu_{0t}\) and \(\mu_{1t}\) by their values, dividing by \(-\lambda_1 U_{c,c}\), adding and subtracting \((U_{c,c} / U_{c,c})(\lambda_0 / \lambda_1 - 1)\) and rearranging terms, we obtain
\[
- \left( b_t^w - \frac{\lambda_0}{\lambda_1} b_t^c + \left( \frac{\lambda_0}{\lambda_1} - 1 \right) c_t \right) - \frac{1}{\lambda_1} ((\xi_{0t+1} - \xi_{1t+1}) - R_t(\zeta_{0t} - \zeta_{1t})) + \left( \frac{U_{c,c}}{U_{c,c} - f_{l,t} U_{c,c}} \right) \left( \left( \frac{\lambda_0}{\lambda_1} - 1 \right) \left( b_t^c + l_t - \left( \frac{U_{c,c}}{U_{c,c} - f_{l,t} U_{c,c}} \right) \right) \right) - (1 b_t^w - 0 b_t^w) = \beta R_{t+1} \left( \frac{U_{c,c} f_{k_{t+1} c_{t+1}}}{U_{c,c} - f_{l,t} U_{c,c}} \right) + \beta \frac{1}{\lambda_1} \left( \frac{U_{c,c}}{U_{c,c} - f_{l,t} U_{c,c}} \right) (\xi_{0t+1} - \xi_{1t+1}) f_{k_{t+1} k_{t+1}}.
\]
(44)
We introduce the LHS of Eq. (43) into Eq. (44) and it results that
\[
b_t^w - 0 b_t^w = \left( \frac{\lambda_0}{\lambda_1} - 1 \right) (0 b_t^w + l_t - c_t^\chi) + \frac{1}{\lambda_1} ((\xi_{0t} - \xi_{1t}) \chi_{t} f_{k,l,t}) + \beta \frac{1}{\lambda_1} (\xi_{0t+1} - \xi_{1t+1}) \left( \frac{U_{c,c} - f_{l,t} U_{c,c}}{U_{c,c} - f_{l,t} U_{c,c}} U_{c,c} - U_{c,c}^2 \right) U_{c_{t+1}} f_{k_{t+1} k_{t+1}}.
\]
Finally, substituting iteratively \( (1 b_{t+j}^w - 0 b_{t+j}^w) \) for all \( j \geq 1 \), we obtain

\[
1 b_t^w - 0 b_t^w = \left( \frac{\lambda_0}{\lambda_1} - 1 \right) \left( b_t^w + l_t - \xi_t \right) + \frac{1}{\lambda_1} \left( \xi_0 - \xi_t \right) f_{k_t} \]

\[
+ \frac{1}{\lambda_1} \left( \beta (\xi_0 - \xi_t) \right) \left( \frac{U_{c_t} - f_{l_t} U_{c_t}}{U_{c_t} U_{x_t} - (U_{c_t})^2} \right) U_{c_t} f_{k_{t+1}} \]

\[
+ \sum_{j=1}^{\infty} \left( \frac{U_{c_t} - f_{l_t} U_{c_t}}{U_{c_t} U_{x_t} - (U_{c_t})^2} \right) \left( \prod_{s=1}^{t} R_s \right) \beta (\xi_0 - \xi_t) U_{c_t} f_{k_j k_{j+1}} \right),
\]

which are the issues of debt indexed to after-tax wage maturing at date \( t \geq 2 \).

This procedure needs to be replicated for the two types of debt and the different maturities. We have used the notation

\[
v_t^c = \left( \frac{f_{l_t} U_{x_t} - U_{x_t}}{U_{c_t} U_{x_t} - (U_{c_t})^2} \right), \quad v_t^x = \left( \frac{U_{c_t} - f_{l_t} U_{c_t}}{U_{c_t} U_{x_t} - (U_{c_t})^2} \right),
\]

\[
\chi_t^c = \left( \frac{U_{c_t} U_{c_t}}{U_{c_t} U_{x_t} - (U_{c_t})^2} \right) \quad \text{and} \quad \chi_t^x = \left( \frac{U_{c_t} U_{c_t}}{U_{c_t} U_{x_t} - (U_{c_t})^2} \right).
\]

### References


