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This paper illustrates how convergence equations can be used to analyse the dynamics of the income distribution using a simple extension of standard techniques. Using data for a sample of OECD countries, I estimate an equation that relates growth in income per capita to the standard growth theory variables, government size and labour market performance indicators. The estimated model and the underlying data are then used in a convergence accounting exercise that yields quantitative estimates of the contribution of each of these variables to the relative growth performance of each country and to observed convergence in the sample.

INTRODUCTION

Following the work of Barro and Sala-i-Martin (1990, 1992) and Mankiw et al. (1992), a large number of studies have used convergence equations derived from different versions of an extended neoclassical model as a framework for the empirical analysis of growth. Despite its widespread use, this methodology has been subject to a number of criticisms. One of the most compelling ones, advanced mainly by Quah (e.g. 1993, 1996), is that this approach does not provide a suitable framework for the analysis of the dynamics of the income distribution. This author argues that the parameters of a growth regression, while providing a convenient way to summarize the behaviour of a ‘typical’ country or region, do not tell us anything about how or why the income distribution is changing over time or about how different countries or regions are moving within it.

In this paper I will argue that the convergence equation methodology can be easily extended to overcome this limitation. The idea is simply that there is no reason why the analysis should stop with the inspection of the estimated coefficients of the growth equation when this equation, together with the underlying data, can be used to generate a good deal of information about the immediate sources of changes in the distribution of income. To illustrate how this can be done, I will use a rather standard growth equation estimated with OECD data to construct quantitative measures of the sources of the differential growth performance of each country in a sample of industrial economies, and to perform an exercise in what may be called convergence accounting. For this purpose, I will develop some simple extensions of the standard techniques for the analysis of beta and sigma convergence that can be used to analyze the impact of each of the explanatory variables of the model on the evolution of the income distribution and to quantify their contribution to observed convergence in the sample. These techniques do not, of course, solve all the problems that beset the convergence equation approach, but they do extend it in a way that makes apparent its usefulness for the analysis of...
the determinants of distribution dynamics within a sound theoretical framework.

The paper is organized as follows. Section I sets the stage with a first look at the growth and convergence pattern in a sample of OECD countries. In Section II I set out and estimate a simple convergence equation relating the growth of income per capita to the usual ‘growth theory’ variables plus an indicator of government size and two measures of labour market performance. The estimated model and the underlying data are then used in Section III to provide a decomposition of the sources of growth and convergence. Section IV closes the paper with a brief summary and some conclusions.

I. GROWTH AND CONVERGENCE IN THE OECD: A FIRST LOOK

The beta convergence plot shown in Figure 1 provides a convenient point of departure for an analysis of the OECD’s growth experience during the last decades. This plot summarizes the relationship between the initial position of each country in a sample of 18 industrial economies in terms of relative income per capita (i.e. log GDP per capita in deviations from the contemporaneous sample average) and its differential growth rate during the period 1970–95. As expected in this sample, the slope of the fitted regression line is negative, indicating that poor countries have grown faster than rich ones on average. The estimated value of the convergence coefficient (the slope of the fitted regression line) tells us that a typical country eliminates each year 1.6% of its income differential with the sample average.

**FIGURE 1. β convergence in relative income per capita in the OECD, 1970–1995.**

Note: The fitted regression line is given by

\[ \text{GYPC.OBS} = 0.00 - 0.0159 \times \text{LYPCR70}, \quad t = 4.15 \quad R^2 = 0.5178 \]

where GYPC.OBS is the average annual change in relative income per capita and LYP7CR70 the initial value of relative income per capita.

Key: Po = Portugal, Jap = Japan, Sp = Spain, Ir = Ireland, It = Italy, Fin = Finland, Ost = Austria, Be = Belgium, Nor = Norway, Fr = France, Nl = Netherlands, Dk = Denmark, Swe = Sweden, Aus = Australia, Ge = West Germany, Can = Canada, UK = United Kingdom, US = United States.
The situation of each country in relation with the fitted regression line (which describes what may be considered the “typical” growth pattern in the sample) can be used as an indicator of its growth performance after eliminating a convergence effect that presumably reflects the relative advantages of initially backwards economies (such as a higher rate of return on investment if the technology exhibits decreasing returns to scale in reproducible factors, and the ability to profit from technological diffusion or to shift a large fraction of the labour force out of agriculture and into more productive activities). Hence, the “corrected” growth rate given by the distance to the fitted line shown in Figure 1 should be a better indicator of a country’s relative performance than its unadjusted growth rate.

Figure 2 shows that there are important differences between the raw and corrected growth rates of income per capita. The United States, for example, is among the slowest-growing countries in the sample, but, since it is also the richest, its corrected performance is actually among the best. At the other end of the scale, Spain and Portugal’s corrected growth rates are among the lowest, even though both countries grew at above-average rates.

Cross-country differences in growth performance are also quite sizeable. Unadjusted growth rates of relative income per capita range from Ireland’s 1.17% to Sweden’s −0.94%, and adjusted ones, from Norway’s 0.90% to Sweden’s −0.67%. In both cases, the growth differential is over 1.5 points per annum, a difference that, accumulated over 25 years, yields a change in relative income of almost 40 points.

II. AN EMPIRICAL GROWTH MODEL

In this section I set out to estimate the empirical model that will be used below to illustrate the techniques that are the main focus of the paper. The model tries to “explain” the cross-country growth differentials illustrated in the previous section in terms of the behaviour of three sets of variables. The first group includes the standard “growth theory” variables, that is, measures of factor
accumulation and variables that try to capture some of the convergence mechanisms identified in the literature (in particular, the operation of decreasing returns to scale and technological diffusion). In the second group I will include two indicators that summarize labour market performance (the changes in the unemployment and labour force participation rates). Although it is a common practice to estimate convergence equations using data on income per capita, I would argue that what the underlying theoretical model describes is the evolution of labour productivity (i.e. output per employed worker). Hence it should be expected that the growth of income per capita will also depend on the behaviour of participation and unemployment rates, and I have introduced these variables in the model in an attempt to capture their impact (without trying to explain them). Finally, I will also include among the explanatory variables an indicator of the size of the government sector that may serve as a proxy for the effects of public-sector activity on income levels, working through the efficiency of resource allocation and individual incentives for work and effort. The model is a simplified version of the one derived and estimated in de la Fuente (1997b) and I am using it here in part for convenience and in part because it yields what may be considered plausible estimates of the coefficients of the aggregate production function. The assumed form of this function is

\[
Y_{it} = \Theta^t K_{it}^\alpha H_{it}^\beta R_{it}^\gamma (A_{it}L_{it})^{1-\alpha-\beta-\gamma},
\]

where \(Y_{it}\) is aggregate output in country \(i\) at time \(t\), \(L\) is the level of employment and \(A\) is an indicator of the level of technical efficiency that grows at an exponential rate \(g\). The variables \(K, H\) and \(R\) denote, respectively, the stocks of physical, human and technological capital, and \(\Theta\) is a measure of the relative weight of the government sector in the economy. This formulation is completely standard except in that it allows national output to be a function of the relative size of government. The ‘government externality’ term \((\Theta^t)\) is meant to capture in the simplest possible way the fact that public activities may affect productivity in a variety of ways. Since some of the relevant effects are positive and others negative, the sign of the coefficient \(\gamma\) is unclear \textit{ex ante}, and may conceivably change with the expenditure level. My estimates, however, indicate that this is not the case within the range of values of \(\Theta\) observed in the sample.

Starting from (1), the empirical specification is derived in the usual fashion by constructing a log-linear approximation around the steady state of an extended Solow model and then introducing in the resulting equation some ad hoc adjustments for labour market variables and technological diffusion. The estimated growth equation is of the form:

\[
GYPC_{it} = \Gamma_0 + \Gamma_1 t + \Gamma_2 t^2 + \Gamma_3 DLAGS + \Gamma_4 DLAGS^2 t + \Gamma_5 GTAC_{it}
+ \Gamma_6 DU_{it} - \beta^tLYPC_{it} + \gamma(GGOV_{it} + (\delta + g + n)\ln GOV_{it})
+ (\delta + g + n) \left( z_k \ln \frac{s_{kit}}{\delta + g + n_{it}} + z_h \ln \frac{s_{hit}}{\delta + g + n_{it}} \right)
+ z_r \ln \frac{s_{rit}}{\delta + g + n_{it}}
\]

\[
(2)
\]

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where $GYP_{it}$ is the growth rate of income per capita in country $i$ during the (five-year) sub-period that starts at $t$, and $LYPC$ the log of income per capita at the beginning of the sub-period. The coefficient $\beta$ measures the rate of convergence towards a pseudo steady state that would be attained asymptotically if the rest of the regressors remained constant over time.

The equation controls for the rates of accumulation of physical, human and technological capital, measured by the fraction of GDP invested in each of these assets ($s_{jit}$ with $j = k$, $h$, and $r$ for, respectively, physical, human and research capital). As in Mankiw et al. (1992), these variables enter the equation in logarithms and, normalized by the sum of the rates of depreciation ($\delta$), population growth ($n$) and technical progress ($g$), and their estimated coefficients can be used to recover the corresponding parameters of a Cobb–Douglas aggregate production function ($z_k$, $z_h$ and $z_r$). The labour market variables are the average annual increase in the unemployment rate during the sub-period ($DU$) and the growth rate of the labour force participation rate ($GTAC$). As for government size, it is measured by total government expenditure as a fraction of GDP ($GOV$), and both the level and the growth rate ($GGOV$) of this variable enter the equation in the way shown in (2). The model allows in a very simple way for a technological catch-up effect. As discussed in de la Fuente (1995), if technology diffuses across countries at a sufficiently rapid pace, those economies that are technically less advanced at the beginning of the period should grow faster than the rest, other things equal. This effect, however, will gradually exhaust itself as each country approaches an equilibrium level of relative technical efficiency that is determined by its own R&D effort and by the speed of diffusion. To try to capture this effect, I include a dummy ($DLAGS$) for initially backwards countries (Spain, Ireland, Greece, Portugal and Japan) and the product of this variable and a trend. I would expect the coefficient of the first variable to be positive, and that of the second to be negative. Finally, I allow for a trend in the rate of technical progress in order to capture the ‘productivity slowdown’ observed in this sample period. This effect is captured by the squared trend ($t^2$) term in the equation (and the coefficient of the trend is related to the initial rate of technological progress).

Following the standard practice in the literature, I impose a value of $\delta + g$ equal to 0.05. I also assume that the convergence parameter $\beta$ (which can be shown to be equal to $(1 - z_k - z_h - z_r) (\delta + g + n)$) is constant over time and across countries (even though the theoretical model suggests that it may vary with the rate of population growth) and will therefore interpret the variable $n$ which enters the term $(\delta + g + n)( = \beta/(1 - z_k - z_h - z_r))$ that multiplies the logs of the investment rates as the average rate of population growth in the sample as a whole. These simplifying assumptions have the advantage that they make the coefficients that multiply initial income and the logs of the investment rates constant, a feature that facilitates the growth accounting exercise undertaken in the next section.

The data on real income per capita, employment, investment and population growth are taken from Doménech and Bosca (1996), who essentially replicate Summers and Heston’s (1991) Penn World Table for the OECD using a set of purchasing power parities that are specific to this sample. My proxy for the level of investment in human capital ($s_h$) will be the total secondary and
university enrolment as a fraction of the labour force (from the UNESCO Yearbook). The series on R&D expenditure are constructed combining information from the UNESCO Yearbooks and the OECD's Basic Science and Technology Statistics, as discussed in de la Fuente (1997c). The last two variables are averaged over several sub-periods because it is expected that investment in education and R&D will affect output only with relatively long lags. In the case of $s_h$ I use the average value over the current five-year sub-period and the previous one, and for R&D ($s_r$), the cumulative average share of total R&D expenditure in GDP over the current and all preceding sub-periods. Finally, the data on government expenditures are taken from the OECD’s Statistical Compendium and from European Commission (1996). Owing to the lack of fiscal data, I have had to exclude Switzerland and New Zealand from the original sample. With this omission, the sample covers 19 countries and ends in 1990–95. In most cases the first observation corresponds to the period 1965–70 or 1970–75. The exception is Greece, whose data start in 1980.

The estimated coefficients of equation (2) are shown in Table 1. The parameters of the production function are significant, have the expected sign and display reasonable values. The coefficient of the stock of technological capital in the production function (0.0603) is similar to the one obtained by Lichtenberg (1992), and those of physical and human capital (0.306 and 0.204 respectively) and the convergence rate (0.034) are within the usual range in the literature. The coefficients of the terms that include the technological backwardness dummy are significant and have the expected sign. The values of these coefficients suggest that the contribution of technological diffusion to the growth of the poorer countries was quite important at the beginning of the sample period (around 1.8% per year) but has declined rapidly with the passage of time. As for the remaining regressors, the coefficients of the changes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coeff. $(t)$</th>
</tr>
</thead>
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<tr>
<td>Constant</td>
<td>$\Gamma_o$ 0.0835 (4.60)</td>
</tr>
<tr>
<td>Trend: $t$</td>
<td>$\Gamma_1 -0.00104 (2.15)$</td>
</tr>
<tr>
<td>Trend$^2$: $t^2$</td>
<td>$\Gamma_2 2.94*10^{-5} (2.41)$</td>
</tr>
<tr>
<td>Technology gap: $DLAG5$</td>
<td>$\Gamma_3 0.0188 (3.72)$</td>
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<tr>
<td>Tech. gap$^*$/trend: $DLAG5^*t$</td>
<td>$\Gamma_4 -0.00098 (4.71)$</td>
</tr>
<tr>
<td>Growth of participation rate: $GTAC$</td>
<td>$\Gamma_g 0.5267 (4.44)$</td>
</tr>
<tr>
<td>Change in unemployment: $DU$</td>
<td>$\Gamma_u -0.6496 (4.54)$</td>
</tr>
<tr>
<td>$-\log$ initial income per capita: $-LYPC$</td>
<td>$\beta 0.03394 (5.25)$</td>
</tr>
<tr>
<td>Invest. in physical cap.: $ln s_k/(\delta+g+n)$</td>
<td>$\alpha_k 0.3065 (5.07)$</td>
</tr>
<tr>
<td>Invest. in human capital: $ln s_h/(\delta+g+n)$</td>
<td>$\alpha_h 0.2041 (3.74)$</td>
</tr>
<tr>
<td>R&amp;D investment: $ln s_r/(\delta+g+n)$</td>
<td>$\alpha_r 0.0603 (2.22)$</td>
</tr>
<tr>
<td>Gov’t spend.: $(GGOV+/(\delta+g+n) ln GOV)$</td>
<td>$\gamma -0.1789 (4.51)$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.7817</td>
</tr>
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<td>103</td>
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</table>

Notes: $t$-statistics in parentheses next to each coefficient. $N$ is the number of observations. Estimation with pooled time-series cross-section data for quinquennial sub-periods.

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in the unemployment and labour force participation rates have the correct sign and are close to their expected value (which would be equal in absolute value to labour’s coefficient in the production function, \(1 - \alpha_k - \alpha_h - \alpha_r\)). Finally, the coefficient of the government size variable is negative, significant and quite large. While the sign of this coefficient is not surprising, in view of the previous literature, its size is considerably larger than I expected—particularly because, since I am controlling for factor accumulation and the level of employment, the distortionary effects I am picking up exclude crowding out and part of the adverse labour supply responses. One possibility I have considered is an endogeneity bias. In de la Fuente (1997b), however, I investigate this possibility with some care and conclude that the results do not seem to be driven by reverse causation. Hence an increase in the size of the public sector seems to have a negative and quite sizeable effect on the level of productivity, even after controlling for employment and factor stocks. The analysis, however, sheds no light on the mechanisms behind this effect.

III. THE SOURCES OF CONVERGENCE IN THE OECD

In this section I will use the model estimated above to analyse the contribution of the different variables of interest to growth and convergence in the sample. As a starting point, I will use equation (2) to decompose each country’s growth differential with respect to the sample average into seven factors that reflect, respectively: (i) a convergence effect (\(CONV\)), which results from the operation of decreasing returns to scale and technological diffusion and tends to favour initially backwards countries; (ii) the impact of labour market performance on income per capita (\(LAB\)), which summarizes the contributions to the growth of income per capita of changes in the unemployment and labour force participation rates; (iii)–(v) the contribution of factor accumulation (investment in physical (\(K\)), human (\(H\)) and technological (R&D) capital), normalized by population growth in the manner suggested by the model; (vi) the impact of government size (measured by the share of total expenditures in GDP) on productivity (\(GOV\)); and (vii) an error term (\(ERROR\)), which is the difference between the observed growth differential and the model’s prediction for each country and period.

Table 2 summarizes the results of the exercise. For each country I show the average value (across sub-periods) of the relative growth rate of income per capita and its components. The first two columns of the table show the observed (\(OBS\)) growth rate of income per capita in differences with the contemporaneous sample average (excluding Greece), and the model’s prediction for the same variable (\(PRED\)). The remaining columns report the seven components of the differential growth rate described above. To ensure that the composition of the sample remains the same throughout the sample period, the figures shown in the table correspond to the period 1970–95 and exclude Greece (whose data start in 1980).

Using the information contained in Table 2, I will now undertake two simple exercises that will allow me to quantify the contribution of each of the relevant variables to observed (unconditional) convergence in income per capita. To facilitate the discussion, I will adopt a more compact notation than
<table>
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<th>OBS</th>
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<th>LAB</th>
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</table>

Note: Contribution of each of the explanatory variables in the model to the average growth differential (in annual rates) between each country and a hypothetical (unweighted) average economy.
the one I have used until now. Let \( y_{it} \) denote the relative income per capita of country \( i \) at time \( t \). I have decomposed the annual growth rate of this variable over the period 1970–95 (\( \Delta y_{it} \)) into a series of additive factors that I will denote by \( x_{k,t} \), so that

\[
\Delta y_{it} = \Delta_k x_{ik}.
\]

My strategy will be to reconstruct the (counterfactual) distribution of \( y_{it} \) in 1995 that would have been induced by each of the components of the relative growth rate when we set all the rest equal to zero. Comparing each of these hypothetical 1995 distributions with the one observed in 1970, I will construct in the usual way a series of partial sigma and beta convergence indicators that can be used to measure how much of observed convergence can be traced back to each of the variables included in the model. The extent to which these partial convergence measures add up to observed total convergence will be discussed below and, more formally, in the Appendix.

To obtain the desired indicators of partial beta convergence, I will regress each of the components of the relative growth rate on initial relative income per capita; that is, for each \( k \) I estimate a cross-section equation of the form

\[
(3) \quad x_{ik} = a_k - \beta_k y_{io}.
\]

The slope coefficient of each of these component regressions, \( \beta_k \), will give us the rate of (unconditional) beta convergence that would have been observed in a hypothetical world in which the relative income of each country changed as a result of only one of the factors under consideration, with all economies displaying average behaviour in terms all other variables. It is easy to show\(^9\) that, if we include the error term as one of the components of the relative growth rate, the partial convergence rates \( \beta_k \) will add up exactly to the observed rate of ‘total’ (unconditional) convergence, \( \beta_u \), which can be obtained by regressing the relative growth rate (\( \Delta y_{i} \)) on initial income (\( y_{io} \)). Since the contribution of the error term to observed convergence is likely to be negligible in practice (around 2% of the total in the current sample), the sum of the partial convergence rates associated with the explanatory variables in the model will be approximately equal to the total rate of convergence.

To construct a similar decomposition of sigma convergence, I calculate the reduction in inequality across countries that would have been induced by each of the components of relative income growth, setting the rest equal to zero. The counterfactual end-of-period relative income level of country \( i \) induced by the \( k \)th component of productivity growth, \( x_{ik} \), would be given by

\[
(4) \quad y_{i95}^k = y_{i70} + x_{ik} T,
\]

where \( T = 95–70 \) is the length of the sample period. After computing \( y_{i95}^k \) for each country \( i \), I calculate the standard deviation of the resulting counterfactual distribution, \( \sigma_{i95}^k \). Comparing this figure with the observed value of the same inequality indicator at the beginning of the period, \( \sigma_{70} \), I obtain an estimate of the degree of sigma convergence induced by the behaviour of the \( k \)th component of relative growth. Because the covariances across the different components of \( \Delta y_{i} \) will in general be different from zero (the squares of) the partial sigma convergence measures will not in general add up exactly to the observed total convergence (measured by the reduction in the variance of
### Table 3

**Immediate Sources of Convergence in Income Per Capita**

<table>
<thead>
<tr>
<th></th>
<th>Std. dev. of relative income</th>
<th>Unconditional convergence reg.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1970</td>
<td>1995</td>
</tr>
<tr>
<td><strong>Observed values:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>induced by:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Labour market</td>
<td>0.239</td>
<td>0.170</td>
</tr>
<tr>
<td>2. Converg. effects</td>
<td>0.239</td>
<td>0.083</td>
</tr>
<tr>
<td>3. Physical capital</td>
<td>0.239</td>
<td>0.230</td>
</tr>
<tr>
<td>4. Human capital</td>
<td>0.239</td>
<td>0.262</td>
</tr>
<tr>
<td>5. R + D</td>
<td>0.239</td>
<td>0.283</td>
</tr>
<tr>
<td>6. Govt. expend.</td>
<td>0.239</td>
<td>0.241</td>
</tr>
<tr>
<td>Sum 1–6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: A negative value of the beta coefficient indicates divergence, i.e. that the relevant variable contributes to increase the level of inequality by inducing faster growth in richer countries.*
relative income per capita). They will, however, give us a rough idea of ‘how much’ of observed sigma convergence can be traced back to the behaviour of each of the explanatory variables of the model.

The results of the application of these techniques to the sample at hand are presented in Table 3. The first three columns of the table show the initial and (observed or counterfactual) final values of the standard deviation of relative income per capita and the percentage reduction of this inequality indicator during the sample period. In the fourth column, this last variable is shown as a fraction of the observed reduction in the dispersion of income, which is normalized to 100. The remaining columns show the total and partial beta convergence rates, the normalized values of these coefficients (with the observed rate of ‘total’ beta convergence set to 100) and some summary statistics of the corresponding regressions. The first row shows the observed values of these variables. In the remaining rows I present counterfactual values which isolate the (beta and sigma) convergence effects of each of the determinants of the growth rate. Figures 3–8 show the corresponding scatters and the fitted regression lines.

Inspection of the partial convergence indicators contained in Table 3 shows that, as expected, the combination of the two convergence effects clearly favours the poorer countries and would have induced, other things equal, a rate of convergence about twice as high as the one we actually observe in the sample (with partial convergence measures of $\beta = 173.58$ and $\Delta \sigma = 228.69$, after normalizing to 100 the observed ‘total’ values of these indicators). Hence, the combined effect of the remaining variables has been clearly divergent. This is particularly so in the cases of R&D investment and labour market variables, with partial convergence rates of $-0.71\%$ and $-0.57\%$, respectively.
Although the dispersion of R&D expenditure levels has fallen significantly during the period, rich countries continue to devote a considerably larger share of their GDP to technological investment than poor ones (see Figure 7). The
resulting growth differentials are sizeable, and have helped to weaken the tendency towards income equalization. The same is true of labour force participation and unemployment rates. Although the fit of the corresponding partial convergence regression is not very good (see Figure 3), labour market
performance has been best in the United States, and worst in the three poorer European countries. The pattern, finally, is less clear when we consider investment in physical and human capital and the impact of government size. Investment in physical capital has contributed somewhat to income convergence, but this effect is mitigated by the low investment rates of the three poorer countries of the sample (Figure 5). In the case of human capital, the correlation between investment and income levels is weak, and the positive slope coefficient is due almost exclusively to Portugal, which is an extreme outlier (Figure 6). Finally, the government spending component of the growth rate bears no relationship to income levels, but it does tend to favour the Anglo-Saxon countries, and especially the United States, over continental Europe (Figure 8).

IV. SUMMARY AND CONCLUSIONS

The standard techniques for the analysis of sigma and beta convergence provide some useful summary measures of how a distribution is changing over time. In this paper I have shown how these indicators can be approximately decomposed into the sum of a series of partial sigma and beta convergence measures that describe the contribution to observed convergence of each of the explanatory variables included in a growth equation.

To illustrate the proposed methodology, I have explored the sources of convergence in a sample of OECD countries. I have estimated an equation that relates growth in income per capita to factor accumulation rates, the size of the public sector and two labour market variables, and have analysed the impact of

\[ GYP\_GOV = 0.00 - 0.0003*LYPCR70, \quad t = 0.15 \quad R^2 = 0.0014 \]

**Figure 8.** \( \beta \)-convergence in relative income per capita induced by public expenditure, 1970–1995.
each of these factors on the growth performance of the countries in the sample and on the evolution of cross-country inequality. I find that convergence in income per capita in this sample has been driven almost exclusively by diminishing returns and technological diffusion, with most other factors working against it or playing no significant role. R&D investment and labour market performance have worked clearly against convergence, while investment in human and physical capital and government expenditures have been roughly neutral on the whole.

These results also provide some insight into the immediate factors behind each country’s relative growth performance. The United States, for example, has been able to maintain its lead, in spite of a strong convergence effect and low investment rates, through a flexible labour market, high R&D investment and a fair amount of fiscal discipline. The poorer European countries, on the other hand, have been hampered by rigid labour markets and low investment rates.

APPENDIX: ADDITIVITY PROPERTIES OF THE PARTIAL CONVERGENCE MEASURES

Let \( y_{it} \) denote the relative income per capita of country \( i \) at time \( t \), where \( t \) ranges from 0 to \( T \). Letting \( \Delta y^k_{it} = x_{ki}T \) (where \( x_{ki} \) is the \( k \)th component of the growth rate of relative income, as described in the text), the counterfactual end of period income induced by the \( k \)th component of the growth rate is given by

\[
(A.1) \quad y^k_{iT} = y_{io} + \Delta y^k_{i}
\]

and actual end-of-period income can be written

\[
(A.2) \quad y_{iT} = y_{io} + \sum_k \Delta y^k_{i},
\]

provided we include the error term among the components of the growth rate. Using (A.1) and (A.2), the total reduction in the variance of \( y_{it} \) during the sample period is given by

\[
\Delta \sigma^2 = \text{var} \ y_{io} - \text{var} \ y_{iT}
= -\sum_k \text{var} \ \Delta y^k_{i} - 2 \sum_k \text{cov} \ (y_{io}, \Delta y^k_{i}) - 2 \sum_k \sum_{p \neq k} \text{cov} \ (\Delta y^k_{i}, \Delta y^p_{i})
\]

and the reduction in variance induced by the \( k \)th component of the growth rate is

\[
\Delta \sigma^2_k = \text{var} \ y_{io} - \text{var} \ y^k_{iT} = -\text{var} \ \Delta y^k_{i} - 2 \text{cov} \ (y_{io}, \Delta y^k_{i}).
\]

Hence

\[
(A.3) \quad \Delta \sigma^2 = \sum_k \Delta \sigma^2_k - 2 \sum_k \sum_{p \neq k} \text{cov} \ (\Delta y^k_{i}, \Delta y^p_{i})
\]

so the partial convergence indicators will add up exactly to total convergence only if the different components of the growth rate are uncorrelated in the sample (and there is no prediction error).

Consider now a cross-section convergence regression of the form

\[
(A.4) \quad \Delta y_i = \alpha - \beta y_{io} + e_i
\]

where \( \Delta y_i \) is the observed average growth rate of relative income over the sample period and \( e_i \) is a random disturbance. Since all our variables are measured in deviations from sample averages, \( \alpha \) will be equal to 0 and the OLS estimator of \( \beta \) can be obtained by
minimizing the sum of squares,
\[ \sum_i (\Delta y_i + \beta y_{io})^2 \]
to obtain
\[ \beta = -\frac{\sum_i \Delta y_i y_{io}}{\sum_i y_{io}^2}. \]

In a similar way, the partial convergence coefficient \( \beta_k \) obtained by regressing \( x_{ik} \) on \( y_{io} \) is given by
\[ \beta_k = -\frac{\sum_i x_{ik} y_{io}}{\sum_i y_{io}^2}. \]

Using the fact that \( \sum_k x_{ik} = \Delta y_i \), we then have
\[ \sum_k \beta_k = -\frac{\sum_i \sum_k x_{ik} y_{io}}{\sum_i y_{io}^2} \]
\[ = -\frac{\sum_i (\sum_k x_{ik}) y_{io}}{\sum_i y_{io}^2} \]
\[ = -\frac{\sum_i \Delta y_i y_{io}}{\sum_i y_{io}^2} \]
\[ = \beta \]
as was to be shown.

**ACKNOWLEDGMENTS**

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**NOTES**

1. See de la Fuente (1997a) for a survey of this literature.
2. Among these, I would emphasize two: the statistical fragility of some of its results (Levine and Renelt, 1992) and the tendency for panel data specifications to yield implausible estimates of the parameters of the aggregate production function (e.g. Islam, 1995; Caselli et al., 1996). I would not, however, consider the standard Galton’s fallacy criticism (Friedman, 1992; Quah, 1993) a serious one, as it essentially reduces to the observation that sigma and beta convergence are not equivalent—a well-known fact that has never been questioned in the literature. There is a fair amount of work on these issues (see e.g. Sala-i-Martin, 1997; de la Fuente, 1998) but this is not the place to discuss it.
3. Much of the recent empirical growth literature has focused on the convergence implications of decreasing returns to scale in reproducible factors (see e.g. Barro and Sala-i-Martin, 1992; Mankiw et al., 1992). Dowrick and Nguyen (1989) and de la Fuente (1995) also allow for technological diffusion.
4. See this paper for a derivation of the growth equation and a discussion of various econometric issues that arise in its estimation.
5. As in de la Fuente (1997b), equation (2) is estimated jointly with an investment equation using a SUR procedure. This second equation is not reported, since it will not be used below.
7. I use equation (2) to compute the period-by-period ‘raw’ components of the growth rate in the obvious way. These components are then measured in deviations from the unweighted sample average and averaged over time for each country to obtain the figures reported in Table 2.
8. Even though some of the residuals are very large, the explanatory power of the model is quite satisfactory, particularly when we work with averages over the entire sample period. The predicted relative growth rate shown in Table 2 explains 80% of the variation in the observed relative growth rate.
9. See the Appendix.
REFERENCES

——— (various years). Basic Science and Technology Statistics. Paris: OECD.