PRICE ADVERTISING AND COUPONS IN A MONOPOLY MODEL*

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The paper studies the pricing and advertising policies of a monopolist in a situation where consumers discover prices by costly search. The monopolist is shown to choose between no advertising and large scale advertising. This is shown to be optimal behavior even with decreasing returns to scale to advertising expenditures which would a priori argue for small advertising expenditures. The optimal advertising policy is undertaken in conjunction with a pricing policy that is characterized by downwardly rigid prices. Flexibility with regard to realizations of a cost parameter is achieved via couponing, which, even though it is untargeted, allows for discrimination between ex ante identical consumers.

I. INTRODUCTION

The failure of some prices to react to changes in demand and supply conditions has always attracted the attention of economists. Price rigidities are important for two reasons: first, they may cause and/or reflect inefficiencies in the allocation of resources; and, second, if prices are rigid the consequent quantity adjustments may have important macroeconomic implications.

A related issue is the potential discrepancy between posted and actual transaction prices, in particular given that the latter are sometimes significantly more flexible than the former. For example, retailers tend to post the prices recommended by the manufacturer, even though they might be ready to offer discounts in low demand states. In fact, in some durable goods markets the listed price is simply the starting price in a bargaining process between the seller and individual buyers.

In this paper I argue that, in some contexts, firms find it optimal to set relatively rigid posted prices due to the costs of price advertising, but can still achieve some degree of flexibility by sending out coupons to a fraction of consumers.

I start from the observation that in decentralized markets the transmission of price information from sellers to buyers is usually a costly activity. Consumers can engage in costly search to find out prices and firms can engage in costly advertising to disseminate price information. More specifically, I analyze a static monopoly model in which the seller, after privately observing his unit production

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cost, chooses both the price and the advertising intensity. A certain fraction of consumers remains uninformed, but they can still discover the actual price by incurring a (small) fixed cost to visit the firm. Other features of the model, including the characteristics and availability of the product and the ex-ante distribution of the unit production cost, are common knowledge. Given their information, consumers first decide whether to visit the firm or not and, once in the firm, whether or not to purchase. Finally, demand is satisfied and the game ends.

In the unique pure strategy equilibrium, the monopolist always chooses either not to advertise at all or to advertise its regular price to a large fraction of consumers, even through the advertising technology exhibits decreasing returns to scale. Associated with such a discontinuity the equilibrium price function is characterized by downward price rigidity, in the sense that the same price is set for different realizations of unit costs, and the price rarely falls below that level.

The existence of a constant price (and no advertising) for a certain interval of unit production costs is caused by a combination of several factors: (i) the costs of visiting the firm, which induce consumers with low reservation prices to visit the firm only if they receive a low price advertisement,\(^1\) (ii) the costs of advertising and (iii) the firm’s inability to price discriminate between informed and uninformed consumers. It is never optimal for the firm to advertise a low price to a small fraction of consumers since only a few additional consumers are attracted whilst the price is reduced for everyone. Hence, the firms finds it optimal to set a very low price and advertise it with high intensity, only if the realization of unit production costs is sufficiently low. Only in this case does the higher revenue obtained by setting a low price compensate for the high advertising costs incurred.

In the range of the unit production cost for which the regular price is constant, the seller has incentives to send out coupons. In other words, the firm finds it optimal to distribute messages which promise a reduced price only to those consumers receiving the message but keeping the regular price (the price charged to uninformed consumers) high.\(^2\) Even though couponing is untargeted, it allows the seller to price discriminate between ex ante identical consumers.

The paper is organized as follows. In the next section I discuss how the current paper is related to the literature. In Section III, the basic model of price advertising is presented and the unique equilibrium is characterized. In Section IV I extend the model by allowing the monopolist to issue coupons in combination with advertising. Some concluding remarks close the paper.

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\(^1\) Those consumers who do not receive an advertisement conjecture that the price is likely to be high. Thus, uninformed consumers with low reservation prices are not willing to sink the costs to find out the actual price, since they expect that with a high probability they will not purchase and the visiting cost will be wasted.

\(^2\) The type of coupon I have in mind is the one with the price printed on it. If the coupon is of the “cents off” variety then there is no price commitment and the monopolist may face a time inconsistency problem. In fact, in this model the monopolist is only willing to send out coupons with price commitment.
II. REVIEW OF THE RELATED LITERATURE

My model brings together two separate strands of the literature, one dealing with price advertising and the other with coupons and rebates. I discuss each in turn.

A monopolist's optimal pricing and advertising policies have been studied by Bester [1994]. In Bester's model there is no exogenous uncertainty, and price advertising arises to solve a time inconsistency problem. Given that consumers incur a cost $s$ to visit the store, their reservation prices at home and at the store are different and, as a result, a pure strategy equilibrium does not exist. If consumers expect a price $p_0$, only those with reservation prices greater than or equal to $p_0 + s$ will choose to visit the store. Given such a kinked demand function, the optimal firm's price is $p_0 + s$, which is inconsistent with consumers' expectations. Thus equilibrium must involve the use of mixed strategies. In Bester's model, advertising is essentially a commitment device although advertisements actually provide valuable information to consumers, because of the strategic uncertainty.4

LeBlanc [1993] introduces exogenous uncertainty in an oligopoly model in which firms' costs are private information. In the first stage, before uncertainty resolves, firms may costlessly announce an ex-ante pricing rule $p(c)$. In the second stage, after observing their actual costs, firms may still engage in costly price advertising.5 This model nicely illustrates how advertising enhances price competition. However, by allowing firms to precommit to a price rule, the time inconsistency problem is ruled out.

My model embeds the time consistency problem into a model involving exogenous uncertainty. The combination of both issues produces the scope for rigid prices to emerge as optimal behavior. Coupons emerge as a better alternative than changing prices to accommodate needs of flexibility.

The literature has considered two types of coupons. The first type are coupons associated with repeat purchases. If a consumer buys one unit of the good from a particular seller, a discount is automatically received on future purchases from the same seller. Banerjee and Summers [1987] and Caminal and Matutes [1990] show that firms find it profitable to introduce such coupons in order to create artificial switching costs. In equilibrium, firms' profits are increased but social welfare is reduced.

The second type of coupons are the ones usually distributed in newspapers, magazines or by mail. These coupons have been rationalized as a price discriminating device. If coupons can be targeted to specific consumer groups,

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3 The early literature on informative advertising (for instance, Butters [1977], Grossman and Shapiro [1984] and Stegeman [1990] focused on the case in which advertisements convey information on both the existence of the product and the price quoted. Thus, consumers choose among the firms from whom they have received an advertisement. In this case, the advertising intensity is always strictly positive.

4 This model is extended to the oligopoly case by Bester and Petrakis [1995], who show that there may be multiple mixed strategy equilibria. See also Lal and Matutes [1994] for an analysis of price and advertising decisions in a multiproduct duopoly.

5 In his model the advertising decision is discrete: the firm either does not advertise or informs the entire market.
the optimal pricing policy consists of setting a high regular price and sending out
coupons to those consumers with relatively low reservation prices (see, for
instance, Bester and Petrakis [1994]. If couponing is untargeted, it still may be the
case that consumers with high reservation prices face higher costs of using
coupons (measured, for example, by the value of time spent collecting and
organizing the coupons). In this case, consumers self-select and only those with
low reservation prices end up using the coupons and purchasing at discount
prices (See Narasimhan [1984]; and Gertsner, Hess and Holthausen [1994].

In this paper, I consider the second type of coupons. Couponing is thus a price
discriminating device even though it is untargeted and all consumers who receive
the coupon use it if they purchase the product. Thus, price discrimination is only
a by-product of costly information transmission and of the endogenous
segmentation between informed and uninformed consumers.

III. A MONOPOLY MODEL WITH PRICE ADVERTISING

Consider a monopolist that produces a homogeneous good with unit production
cost c, which is a random variable distributed over the interval \([c, \bar{c}]\) according to
the probability density function \(h(c)\), \(h(c) > 0\) for all \(c\) in its support. There is a
continuum of risk-neutral consumers indexed by \(i\). Each consumer chooses to
purchase either one unit of the good or zero and has a willingness to pay denoted
by \(R_i\). Consumers’ reservation prices are uniformly distributed between 0 and \(a\)
with density 1. Also, each consumer has to pay a cost \(s\) to visit the firm.\(^6\) Thus, if
we denote by \(p\) the price charged by the monopolist, consumer \(i\)’s payoff from
shopping is equal to \(R_i - s - p\).

A crucial assumption is that consumers can only learn the price either by
receiving a message sent by the firm (advertisement) or by visiting the firm and
paying \(s\). Price advertising is costly for the firm. To inform a proportion \(\mu\) of
consumers the firm must pay \(\Psi(\mu)\). Advertising cannot be targeted to a specific
consumer group, i.e. the probability that consumer \(i\) receives a message is \(\mu\),
independently of \(R_i\). As is standard, I assume that price advertising involves full
commitment, i.e. a firm can not charge a price different from the one advertised.
Reputation arguments and legal sanctions are often invoked to justify this
assumption.

The timing of the game is the following. At the beginning of the period the
monopolist privately observes the realization of \(c\), sets the price \(p\) and chooses the
intensity of advertising \(\mu\). Thus, a strategy for the firm is a pair \([p(c), \mu(c)]\).
Informed consumers, those who received the message, choose to visit the firm
provided \(R_i - s - p \geq 0\). Uninformed consumers, those who did not receive any
message, update their beliefs about the distribution of prices and decide whether
to visit the firm or not. Once in the firm they learn the actual price and purchase
the good provided \(p \leq R_i\).

\(^6\) The parameter \(s\) must be thought of as a positive but very small number.
At the beginning of the game consumers hold conjectures about the firm's strategy, which are denoted by \( p^e(c), \mu^e(c) \). Based on the price conjecture, \( p^e(c) \), and on the probability distribution of \( c \), they decide whether to visit the firm or not. Unsophisticated consumers may simply use the ex-ante probability density function \( h(c) \) to compute the expected value of visiting the firm. However, rational consumers can potentially infer useful information about the distribution of prices from the fact that they did not receive a message. For instance, suppose consumers expect that the intensity of advertising is higher for relatively low realizations of \( c \). In this case, consumers interpret not having received a message as an indication that a high realization of \( c \) is in fact more likely than initially expected. More formally, the consumers' probability density function of \( c \), conditional on not having received any message, denoted by \( \lambda^e(c) \), can be computed by using Bayes' rule:

\[
\lambda^e(c) = \frac{[1 - \mu^e(c)]h(c)}{1 - \int \mu^e(c)h(c)dc}
\]

An uninformed consumer will visit the firm if and only if:

\[
\int_\xi \max[R_i - p^e(c), 0]\lambda^e(c) dc \geq s
\]

Define \( R_0 \) as the reservation price of the uninformed consumer indifferent between visiting the firm or not:

\[
\int_\xi \max[R_0 - p^e(c), 0]\lambda^e(c) dc = s
\]

Clearly, uninformed consumers choose to visit the firm if and only if \( R_i \geq R_0 \). Given such a demand structure, the monopolist's profits are given by:

\[
\pi(p, \mu) = (p - c)\{\mu(a - s - p) + (1 - \mu)[a - \max(R_0, p)]\} - \Psi(\mu)
\]

If the firm informs a proportion \( \mu \) of potential consumers that the price is \( p \), then it attracts \( \mu(a - s - p) \) of these informed consumers. Uninformed consumers decide to purchase the good if and only if \( R_i \geq p \). Hence, if \( p \geq R_0 \) the demand of uninformed consumers is \( (1 - \mu)(a - p) \) and if \( p \leq R_0 \), the demand is \( (1 - \mu)(a - R_0) \).

The shape of the advertising cost function is important not so much in terms of the qualitative properties of the equilibrium but to make sure that a pure strategy equilibrium exists (and is unique). The assumptions below are sufficient, but clearly not necessary, conditions for existence. I assume throughout the paper that \( \Psi(\mu) \) is three times continuously differentiable, with \( \Psi(0) = \Psi''(0) = 0 \), \( \Psi'(\mu) > 0 \) for \( \mu > 0 \), \( \Psi''(\mu) \geq 0 \), \( \Psi'''(\mu) > -3\Psi''(\mu)/\mu \), and \( \lim_{\mu \to 1} \Psi(\mu) = \infty \).
These assumptions imply decreasing returns to scale in advertising. In particular, informing the first consumer is arbitrarily cheap ($\Psi'(0) = 0$), but informing the entire market is prohibitive ($\lim_{\mu \to 1} \Psi(\mu) = \infty$). The assumption of zero marginal advertising costs at $\mu = 0$ makes low scale advertising very attractive; nevertheless economic considerations will rule this possibility out. By assuming very high marginal advertising costs for $p$ close to 1 we make sure that, for some realizations of $c$, informed and uninformed consumers coexist in equilibrium, which is crucial to show existence of equilibria. It is also assumed that the third derivative is not too negative. This is a technical condition that implies well behaved pricing and advertising policies.

**Definition 1.** A price advertising equilibrium is a triple $[p(c), \mu(c), R_0]$ such that:

(i) Given $R_0$, $p(c)$ and $\mu(c)$ maximize the profit function (4), subject to $\mu \geq 0$. 
(ii) Given $\mu^e(c)$, beliefs are updated according to equation (1)
(iii) $R_0$ is implicitly given by equation (3)
(iv) Beliefs are correct, i.e. $p^e(c) = p(c)$, and $\mu^e(c) = \mu(c)$.

The following proposition provides sufficient conditions for existence and characterizes a price advertising equilibrium. These results are not completely straightforward because the firm’s profit function, as it is usually the case in this class of models, is not concave in $(p, \mu)$.

**Proposition 1** If $s$ is small enough then there exists a unique, pure strategy, price advertising equilibrium. In that equilibrium the price and advertising functions have the following structure:

$$p(c) = \begin{cases} \frac{a + c}{2} & \text{if } c \geq 2R_0 - a \\ R_0 & \text{if } c \in [c_A, \min(2R_0 - a, \bar{c})] \\ \frac{a = R_0 + \mu(R_0 + c - s)}{2\mu} & \text{if } c \in [\underline{c}, c_A] \end{cases}$$

$$\mu(c) = \begin{cases} 0 & \text{if } c \in [c_A, \bar{c}] \\ \tilde{\mu}(c) & \text{if } c \in [\underline{c}, c_A] \end{cases}$$

where $\tilde{\mu}(c)$ is implicitly given by

$$\mu^2(R_0 - c - s)^2 - (a - R_0)^2 = 4\mu^2\Psi'(\mu)$$

with $R_0 \in \left(\frac{a + \underline{c}}{2}, a\right)$ and $c_A \in (\underline{c}, \min(2R_0 - a, \bar{c}))$

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7 Decreasing returns to scale in advertising is widely assumed in this literature. The specific assumptions I make on the advertising cost function include as particular cases the advertising technology assumed in Butters [1977] and the constant-reach independent-readership technology analyzed in Grossman and Shapiro [1984].

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Figure 1
Remark 1. There is a discontinuity at \( c = c_A \) in both the price and advertising functions. More specifically:

\[
\begin{align*}
\lim_{c \to c_A^-} p(c) &< R_0 \\
\lim_{c \to c_A^+} \mu(c) &> 0
\end{align*}
\]

Remark 2 \( p(c) \) is increasing and \( \mu(c) \) decreasing.

The proofs are given in the Appendix and the graphical representation is provided in Figure 1. The intuition goes as follows. Consider the firm’s optimization problem for a given \( R_0 \). That is, given that all uninformed consumers with \( R_i \geq R_0 \) visit the firm and purchase the good as long as \( R_i \geq p \).

Thus, in the absence of advertising the monopolist faces a kinked demand function, with demand equal to \((a - p)\) for prices above \( R_0 \), and \((a - R_0)\) for prices below \( R_0 \). If \( c \) is high enough so that the full information monopoly price is above \( R_0 - s \), i.e. if \( a + c - s/2 \geq R_0 - s \), then the monopolist does not advertise and sets the optimal price given the demand function of the uninformed, \( a + c/2 \). The reason is twofold; first, advertising is costly; and second, it reduces total sales since informed consumers only visit the firm if \( R_i \geq p + s \).

If \( (a + c - s/2) < R_0 - s \), it could be optimal to advertise below \( R_0 \), provided the intensity of advertising is high. Advertising a price below \( R_0 - s \) to a small fraction of consumers (even if its cost is negligible) can never be optimal because few additional consumers are attracted while the price is also reduced to all uninformed consumers. Also, advertising with high intensity can only be profitable if the full information monopoly price is substantially below \( R_0 \), and the gains from setting a lower price make up for the advertising costs.

In other words, the firm faces a kinked demand function, with the absolute value of the slope of the demand curve at prices below \( R_0 \) being inversely related to the intensity of advertising. Thus, the discontinuity in the price function can be explained by the standard arguments. What is special in this model is that both the kink and the slope of the demand function are endogenous and closely related to the costs of advertising.

According to Proposition 1, existence of pure strategy equilibria is guaranteed if \( s \) is small enough.\(^8\) In Bester [1994] a pure strategy equilibrium does not exist because the cost of visiting the firm is relatively large with respect to the uncertainty on the firm’s profit function (which is zero). Conversely, in our model existence requires the visiting cost to be relatively small with respect to the

\(^8\)If the advertising cost function is linear up to a certain intensity \( \mu^* \), and infinite from thereafter, the sufficient condition for existence of pure strategy equilibria involves \( s \leq s^* \), where \( s^* \) can be explicitly computed and it is a decreasing function of the marginal advertising cost.
uncertainty on the firm's payoff function. The role of some of the assumptions on the advertising cost function can be now better understood. I have assumed that the limit of $\Psi(\mu)$, as $\mu$ goes to 1, is infinite (informing all the consumers is impossible). In the absence of such a condition, it could be the case that the firm chooses either $\mu = 0$ or $\mu = 1$ (as it would happen if the advertising cost function were not too convex in all the range). In this case, for $c < c_A$, $\lambda(c) = 0$ and the left hand side of equation (3) is always 0, and hence a pure strategy equilibrium does not exist.

Allowing for an arbitrarily low marginal advertising cost at low advertising levels was important to show that the discontinuities in the equilibrium price and advertising functions are essentially due to the firm's inability to price discriminate between informed and uninformed consumers and are not implied by the advertising technology. Finally, the condition on the third derivative implies that the firm's optimization problem has a unique local maximum.

III(i) Comparative statics and welfare

Let us first consider the limiting equilibrium as $s$ goes to zero. In this case, the "downward rigidity" is more transparent since the price function becomes an inverse $L$ (see Figure 2), i.e. the pricing policy is characterized by the existence of a regular price, $R_0$, which prevails for a certain range of realizations of the random variable; the price can be higher if the realization of the shock is high enough, but it is only set below $R_0$ with probability zero. In order to see this point we can rewrite equation (3) using the fact that in equilibrium the price function is increasing:

$$\int_{c_A}^{c} [R_0 - p(c)]\lambda(c) = s$$

Clearly, as $s$ goes to zero, $c_A$ goes to $c$ and hence it is optimal to set a price below $R_0$ only if $c = c$.

If $s$ is small enough as we increase the dispersion of shocks, but keeping the support of the distribution unchanged, the average price also increases. The reason is that as $s$ goes to zero the price function becomes convex and independent of the distribution of shocks. In other words, in the limiting equilibrium, $R_0$ only depends on the advertising cost function, since it is simply the price that makes the firm with $c = c$ indifferent between ($\mu = 0$, $p = R_0$) and ($p(c)$, $\bar{\mu}(c)$), given in Proposition 1. In this case as we put more weight on the tails

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9 More generally, the present analysis can be understood as a purification argument along the lines of Harsanyi [1973]. According to Harsanyi any mixed strategy equilibrium can almost always be obtained as the limit of a pure strategy equilibrium of a sequence of slightly perturbed games. In other words, mixed strategy equilibria of a complete information game can be purified by introducing some uncertainty. In our case, this implies that the equilibrium distribution of $[p(c), \bar{\mu}(c)]$ that is generated by the random variable $c$ should converge to the mixed strategy equilibrium in Bester [1991] as cost uncertainty vanishes.
the average price clearly increases. If $s$ is strictly positive the price function is not independent of the distribution of shocks and moreover is not convex everywhere. However, if $s$ is small enough the previous effect dominates.\footnote{The fact that the average price increases with the dispersion of shocks does not imply that average welfare decreases, since welfare is not a concave function of the price.}

Changes in advertising costs have complicated effects on equilibrium payoffs. Clearly, as advertising costs go to zero, the equilibrium approaches the full information case. If $s$ is arbitrary small, a reduction in advertising costs decreases $R_0$.\footnote{This is not necessarily the case if $s$ is relatively large. The reason is that if $R_0$ falls the intensity of advertising for $c$ below $c_A$ may actually decrease, and as a result prices in this range may increase. In fact, if this is not the case, the increase in advertising intensities implies an increase in $c_A$, which again make the impact of advertising costs on the average price level potentially non-monotonic.} As a result, both firm's profits and consumer surplus increase.

Since advertising costs are fully paid by the firm it is immediate that in equilibrium the level of advertising is inefficiently low. In other words, the monopolist does not internalize the increase in consumer surplus brought about by any additional advertisement. The implication of such a remark is that in this model a government subsidy on advertising expenses would increase social welfare. However, this result relies heavily on the absence of competition. In a competitive framework advertising also aims at stealing other firms' customers, which is (at least partially) a social loss. In general there are two countervailing
effects: (i) a market size effect (advertising makes some mutually beneficial trade possible), which has a positive impact on social welfare, and (ii) a business stealing effect, which tends to have a negative impact on social welfare. Thus, equilibrium could be characterized by excessive advertising and in this case the optimal policy would be to tax advertising.\(^{12}\)

IV. COUPONS AS A PRICE DISCRIMINATING DEVICE

The discontinuity in the equilibrium price function found in the previous section was due to the fact that for some realizations of the marginal cost the firm did not find it profitable to advertise a price reduction to a small fraction of consumers because this would have implied losses from the uninformed consumers. However, the monopolist could increase its profits if it were able to price discriminate between informed and uninformed consumers.

By sending out coupons the firm can let some consumers know that it is willing to sell the good at reduced prices but at the same time does not have to cut the price charged to uninformed consumers. In this section we analyze this issue by adding to the model of the previous section the possibility of sending out coupons.

The firm, after learning the realization of \(c\), can inform a fraction \(\mu\) of the population of its regular price \(p\) by paying a cost \(\Psi(\mu)\). Also, it can send out coupons to a fraction \(\delta\) of the population by paying a cost \(\Omega(\delta)\). Each coupon allows the receiver to purchase the good at a price \(q\). This is equivalent to saying that the coupon could announce the regular price \(p\) and allow for a discount \(d\), in which case the actual price would be \(q = p - d\).

I assume that regular price advertising and sending out coupons are independent activities; i.e. given the firm’s choice \((\delta, \mu)\), the probability that a particular individual receives a message about the regular price is \(\mu\) (independent of \(\delta\)) and the probability that receives a coupon is \(\delta\) (independent of \(\mu\)).

In this case, a strategy for the firm is a vector \([p(c), q(c), \mu(c), \delta(c)]\). We can distinguish several types of consumers:

a) Uninformed consumers (those who did not receive a coupon or a message about the regular price). Their ex-post beliefs are given by:

\[
\lambda^*(c) = \frac{[1 - \mu^*(c)][1 - \delta^*(c)]h(c)}{\int_c^\infty[1 - \mu^*(c)][1 - \delta^*(c)]h(c)dc}
\]

If the conjectured price function is monotone increasing, then an uninformed consumer \(i\) chooses to visit the firm if and only if \(R_i \geq R_0\), where \(R_0\) is given by equation (3).

\(^{12}\)This is the case, for instance, in Bester and Petrakis [1995]. In the models where ads convey information about both prices and the existence of the firm, the efficiency results are also ambiguous. For instance, in Butters [1977] the level of advertising is socially efficient, in Grossman and Shapiro [1984] there is excessive advertising, but in Stegeman [1990] advertising is below the level that maximizes social welfare.
b) Consumers who only receive a coupon. They choose to visit the firm, and make the purchase, if and only if \( R_i \geq q + s. \)

c) Consumers who receive both the advertisement about the regular price and the coupon. Obviously, for this group the message about the regular price is redundant and thus they behave like the group of consumers who receive only the coupon.

d) Consumers who only receive the advertisement about the regular price. They choose to visit the firm, and make the purchase, if and only if \( R_i \geq p + s. \)

Without loss of generality suppose \( p \geq q, R_0 \geq q, \) then firm’s profits are given by:

\[
\pi(p, q, \mu, \delta) = \delta(q - c)(a - s - q) \\
+ (1 - \delta)(p - c)[\mu(a - s - p) + (1 - \mu)[a - \max(p, R_0)]] \\
- \Psi(\mu) - \Omega(\delta)
\]

The assumptions on \( \Psi(\mu) \) made in the previous section are maintained. In a similar spirit I assume that \( \Omega(\delta) \) is twice continuously differentiable, with \( \Omega(0) = \Omega'(0) = 0, \Omega'(\delta) > 0 \) for \( \delta > 0, \Omega''(\delta) \geq 0, \lim_{\delta \to 1} \Omega(\delta) = \infty. \)

It would make sense to assume that \( \Omega(z) \geq \Psi(z) \) for all \( z > 0. \) The reason is that coupons are simply a more specialized type of advertising, since each message reaches exclusively a single individual. Thus, the firm can announce the regular price simply by using the coupon sending technology but also can use alternative technologies (radio, TV, etc. which have a public good characteristic). Thus, sending out coupons to a certain fraction of the population must be at least as costly as informing them of the regular price. However, it turns out that, given the assumptions made above, the relative inefficiency of the coupons sending technology does not matter for existence.

**Definition 2.** A price advertising and coupon equilibrium is a vector \( [p(c), q(c), \mu(c), \delta(c), R_0] \) such that:

(i) Given \( R_0, p(c), q(c), \mu(c), \delta(c) \) maximize the profit function (6), subject to the non-negativity constraints \( \mu, \delta \geq 0. \)

(ii) Given \( \mu^*(c), \delta^*(c) \), beliefs are updated according to equation (5).

(iii) \( R_0 \) is implicitly given by equation (3).

(iv) Beliefs are correct, i.e. \( p^*(c) = p(c), \delta^*(c) = \delta(c), \) and \( \mu^*(c) = \mu(c). \)

**Proposition 2.** If \( s \) is small enough, then there exists a unique, pure strategy, price advertising and coupon equilibrium. In that equilibrium the regular and discount prices and the intensity of regular price advertising and

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\(^{13}\) No condition on the third derivative of \( \Omega(\delta) \) is needed.

\(^{14}\) There is no loss of generality in restricting attention to the case that \( p(c) \) is monotone increasing.
coupon sending have the following structure:

\[
p(c) = \begin{cases} 
\frac{a + c}{2} & \text{if } c \geq 2R_0 - a \\
R_0 & \text{if } c \in [c_A, \min\{2R_0 - a, \bar{c}\}] \\
\frac{a - R_0 + \mu(R_0 + c - s)}{2\mu} & \text{if } c \in [\underline{c}, c_A] 
\end{cases}
\]

\[
q(c) = \frac{a + c - s}{2} \quad \text{for all } c \in [\underline{c}, \bar{c}]
\]

\[
\mu(c) = \begin{cases} 
0 & \text{if } c \in [c_A, \bar{c}] \\
\tilde{\mu}(c) & \text{if } c \in [\underline{c}, c_A]
\end{cases}
\]

\[
\delta(c) = \begin{cases} 
0 & \text{if } c \in [c_B, \bar{c}] \\
\tilde{\delta}(c) & \text{if } c \in [c_A, c_B] \\
\bar{\delta}(c) & \text{if } c \in [\underline{c}, c_A]
\end{cases}
\]

where \(\tilde{\mu}, \tilde{\delta}\) and \(\bar{\delta}\) are implicitly given by:

\[
(a - 2R_0 + c)^2 = 4\Omega'(\bar{\delta})
\]

\[
\frac{(R_0 - c)^2}{4} - \frac{(a - R_0)^2}{4\tilde{\mu}} = \frac{\Psi'(\tilde{\mu})}{1 - \delta}
\]

\[
\frac{(R_0 - c)^2}{4} - \frac{(a - R_0)^2}{4\tilde{\mu}} = \frac{\Omega'(\bar{\delta})}{1 - \bar{\mu}}
\]

and \((a + c/2) < R_0 < a, \underline{c} < c_A < c_B < \min\{2R_0 - a, \bar{c}\}\)

**Remark 3.** There is a discontinuity at \(c = c_A\) in the price, advertising and coupons sending costs functions:

\[
\lim_{c \to c_A^-} p(c) < R_0 \\
\lim_{c \to c_A^-} \mu(c) > 0 \\
\lim_{c \to c_A^+} \delta(c) < \lim_{c \to c_A^+} \delta(c)
\]

**Remark 4.** \(p(c)\) is increasing and \(\mu(c)\) decreasing.

The firm's equilibrium strategy is depicted in Figure 3. Likewise in the previous section the discontinuities survive as we take the limit of \(s\) going to zero. Also in the limit \(c_A\) goes to \(\underline{c}\), and \(c_B\) goes to \(2R_0 - a\) (provided \(2R_0 - a \leq \bar{c}\)).

Since coupons allow the firm to price discriminate between informed and uninformed consumers, if \(c\) is below but close to \((2R_0 - a)\) the firm does not find it optimal to set a regular price below \(R_0\), but it can increase its profits by sending
Figure 3
out coupons and charging to the group of receivers the full information monopoly price. If $c$ is very small and since the marginal costs of sending out coupons is increasing, advertising a lower regular price becomes optimal.\footnote{It is also quite intuitive that if the advertising technology becomes relatively more efficient than the coupons sending technology then the equilibrium value of $R_0$ falls.}

Given consumers' beliefs, that is given $R_0$, the possibility of sending out coupons improves the degree of price flexibility, and increases both the firm's profits and consumer surplus. However, $R_0$ changes as firms are allowed to use coupons and hence the comparison between the two equilibria is not straightforward.\footnote{It is not clear that either the firm or society are made better off by the possibility of using coupons. In principle, it could be the case that the firm would like to precommit not to send out coupons (or, perhaps, the optimal government policy could be to forbid the use of coupons). The reason is that $R_0$ increases when coupons are allowed, and hence the comparison between the two types of equilibria is not trivial. This issue is left for future research.}

V. CONCLUDING REMARKS

In this paper I analyze the interaction between a firm and a large number of consumers in a world in which the transmission of price information from sellers to buyers is a costly activity. The firm may advertise its price but alternatively consumers may find out the price by visiting the firm and hence paying a small cost. In equilibrium the price exhibits downward rigidity, in the sense that it rarely falls below the "normal" level and when it does so it falls discretely, and the firm advertises such an unusual low price with high intensity.

The model clearly indicates that such a price rigidity arises from the fact that the firm is required to charge the same price to those consumers who received the advertisement and those who did not. In this context coupons can be an efficient mechanism to price discriminate between informed and uninformed consumers. If we keep consumers' conjectures unchanged, allowing for coupons reduces the degree of price rigidity and increases profits as well as social welfare. However, the comparison across equilibrium types (with and without coupons) is much more problematic.

Two extensions of this model seem to be particularly promising. First, by considering strategic firm interaction we would be able to understand how firms' incentives to advertise their pricing policies (and thus the degree of downward price rigidity) change with the degree of competition. Second, by making the model dynamic we may be able to provide an interesting theory of endogenous price adjustment costs.

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Proof of Proposition 1

If all the potential consumers visit the firm then the firm’s optimal strategy is obviously not to advertise ($u = 0$) and set the price that maximizes $(a - p)(p - c)$, i.e.

$$p = \frac{a + c}{2}$$

If $c \geq 2R_0 - a$, then clearly the firm will do so. Let us now consider the case $c < 2R_0 - a$, and define $V(c, \mu)$ as follows:

$$V(c, \mu) = \max((p - c)[\mu(a - s - p) + (1 - \mu)(a - R_0)]) \quad \text{subject to } p \leq R_0$$

That is $V(c, \mu)$ is the maximum amount of profits, before paying the advertising costs, that the firm can make for a given $\mu$. The solution to this optimization problem is:

$$p(c, \mu) = \begin{cases} \frac{a - R_0 + \mu(R_0 + c - s)}{2\mu} & \text{if } \mu \geq \mu_0 \\ \frac{R_0}{\mu} & \text{otherwise} \end{cases}$$

where $\mu_0$ is given by:

$$\mu_0 = \frac{a - R_0}{R_0 - c + s} \in (0, 1)$$

Thus

$$V(c, \mu) = \begin{cases} (R_0 - c)(a - R_0 - \mu s) & \text{if } \mu \leq \mu_0 \\ \frac{[a - \mu(c + s) - (1 - \mu)R_0]^2}{4\mu} & \text{if } \mu > \mu_0 \end{cases}$$

Let us now study some of the properties of this function:

$$\frac{\partial V(c, \mu)}{\partial \mu} = \begin{cases} -s(R_0 - c) & \text{if } \mu \leq \mu_0 \\ \frac{\mu^2(R_0 - c - s)^2 - (a - R_0)^2}{4\mu^2} & \text{if } \mu > \mu_0 \end{cases}$$

$$\frac{\partial^2 V(c, \mu)}{\partial \mu^2} = \begin{cases} 0 & \text{if } \mu \leq \mu_0 \\ > 0 & \text{if } \mu > \mu_0 \end{cases}$$

Thus, $V(c, \mu)$ is linear and decreasing for $\mu < \mu_0$, and increasing and concave for $\mu > \mu_0$. The optimal intensity of advertising is the solution to

$$\max\{V(c, \mu) - \Psi(\mu)\}$$

subject to $\mu \geq 0$

If $c$ is close to $(2R_0 - a)$ then $\mu_0$ is close to 1. So clearly the firm chooses $\mu = 0$. Also notice that $\mu_0$ monotonically increases with $c$. Next, we study how $V(c, \mu) - V(c, 0)$ changes with $c$:

$$\frac{\partial [V(c, \mu) - V(c, 0)]}{\partial c} = \frac{(a - R_0) - \mu(R_0 - c - s)}{2} < 0$$

$$\frac{\partial^2 [V(c, \mu) - V(c, 0)]}{\partial c^2} = \frac{\mu}{2} > 0$$

for $\mu > \mu_0$

Hence, if $c$ is sufficiently low there exists a $\mu > 0$ that maximizes $V(c, \mu) - \Psi(\mu)$. Denote by $c_A$ the value of $c$ that makes the firm indifferent between $\mu = 0$ and a positive $\mu$. Thus,
for $c$ between $c_A$ and $(2R_0 - a)$, the firm’s optimal strategy is to set $p = R_0$ and $\mu = 0$. For $c$ lower than $c_A$, the optimal $\mu$ is given by the first order condition:

$$\frac{\partial V(c, \mu)}{\partial \mu} = \frac{-(a - R_0)^2 + \mu^2(R_0 - c - s)^2}{4} = \Psi''(\mu)$$

The second order condition is:

$$\frac{(a - R_0)^2}{2\mu^3} - \Psi''(\mu) < 0$$

To guarantee that there is only one local maximum it is enough to check that $\mu^3\Psi''(\mu)$ monotonically increases, i.e.:

$$\Psi''(\mu) > -\frac{3\Psi''(\mu)}{\mu}$$

Finally, let us check that $c_A$ is below $(2R_0 - a)$. If we compute $V(2R_0 - a, \mu)$:

$$V(2R_0 - a, 0) - V(2R_0 - a, 1) = s(a - R_0) - \frac{s^2}{4} > 0$$

Even if advertising were free, the firm does not have incentives to advertise when $c = 2R_0 - a$. With costly advertising a value of $c$ much below $2R_0 - a$ is required to find advertising profitable.

So far we have characterized the optimal pricing and advertising policies, for a given $(R_0, c_A)$. Let us now check existence and uniqueness of an advertising equilibrium.

At $c = c_A$ the firm is indifferent between charging $R_0$ without advertising, or setting a price below $R_0$ and advertise it with a strictly positive intensity. From our previous discussion of the optimal policy, it follows that:

$$[a - R_0 + \mu(R_0 - c_A - s)]^2 - (R_0 - c_A)(a - R_0) - \Psi(\mu) = 0$$

where $\mu$ is implicitly given by:

$$(R_0 - c_A - s)^2 - \frac{(a - R_0)^2}{\mu^2} = 4\Psi'(\mu)$$

Using equation (2), the rational expectations assumptions and the optimal pricing and advertising policies derived above, we can rewrite equation (4) as follows:

$$G(c_A, R_0) \equiv \int_{c_A}^{c_A} \left[ R_0 - p(c) \right] \frac{[1 - \mu(c)]h(c)}{1 - \int_{c_A}^{c_A} \mu(c)h(c) \ dc} \ dc - s = 0$$

Thus, we only need to check that the locus defined by $H(c_A, R_0) = 0$ and $G(c_A, R_0) = 0$ cross only once in the set

$$[c, \bar{c}] \times \left[ \frac{a + \varepsilon}{2}, a \right].$$

Using the implicit function theorem we can check that:

$$\frac{d c_A}{d R_0} \bigg|_{H=0} > 0$$
and $\frac{d c_A}{d R_0} \bigg|_{G=0}$ goes to zero as $s$ goes to zero.
In fact, \( c_A \) goes to \( c \) as \( s \) goes to zero. And since
\[
H(c_A, a) = \Psi(\mu) \left[ \frac{\Psi'(\mu)\mu}{\Psi(\mu)} - 1 \right] > 0
\]
\[
H(c_A, \frac{a + c_A}{2}) < 0
\]
we can conclude that if \( s \) is small enough there exists a unique pair
\[
(c_A, R_0) \in [\varepsilon, \bar{c}] \times \left[ \frac{a + c}{2}, a \right]
\]
that satisfies both equations. The only final requirement is to check that at \( c = c_A \), advertising intensity is strictly positive. For an arbitrary small \( s \), \( \mu(c_A) \) is given by:
\[
(R_0 - c)^2 - \frac{(a - R_0)^2}{\mu^2} = 4\Psi'(\mu)
\]
Since both sides of the equation increase in \( \mu \), a solution to this equation exists since
\[
(R_0 - c)^2 - (a - R_0)^2 > 4\Psi'(0) = 0
\]

Proof of Remark 1
For \( c \leq c_A \),
\[
R_0 - p(c) = \frac{\mu(R_0 - c + s) - (a - R_0)}{2\mu} > 0
\]
since \( \mu > \mu_0 = \frac{a - R_0}{R_0 - c + s} \)
The last inequality also shows the discontinuity in \( \mu(c) \).

Proof of Remark 2
For \( c \leq c_A \), using the implicit function theorem in the first order conditions of the firm’s optimization problem we can check that \( \mu'(c) < 0 \). Moreover, for \( c \geq c_A \), \( \mu(c) = 0 \). \( p(c) \) is clearly increasing for \( c \geq c_A \). For \( c \leq c_A \):
\[
\frac{dp}{dc} = \frac{\partial p}{\partial c} + \frac{\partial p}{\partial \mu} \frac{d\mu}{dc} > 0
\]
since the first term is positive and the two last derivatives are both negative.

Proof of Proposition 2
If \( c \geq 2R_0 - a \), it is clear that the optimal strategy consists of setting \( \mu = \delta = 0 \), and
\[
p = \frac{a + c}{2}
\]
If \( c < 2R_0 - a \), then the firm’s optimization problem is to choose \( p, q, \mu, \delta \) in order to maximize:
\[
\pi(c, p, q, \mu, \delta) = \delta(a - s - q)(q - c) + (1 - \delta)[\mu(a - s - p) + (1 - \mu)(a - \max(R_0, p))][p - c] - \Psi(\mu) - \Omega(\delta)
\]
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The first order conditions w.r.t. \( p \) and \( q \) are

\[
q = \frac{a + c - s}{2}
\]

\[
p = \begin{cases} 
R_0 & \text{if } \mu \leq \mu_0 \\
\frac{a + \mu(c - s) - (1 - \mu)R_0}{2\mu} & \text{if } \mu \geq \mu_0 
\end{cases}
\]

where \( \mu_0 = \frac{a - R_0}{R_0 - c + s} \)

Define

\[
\tilde{V}(c, \mu, \delta) \equiv \max_{p, q} \pi(c, p, q, \mu, \delta)
\]

Then,

\[
V(c, \mu, \delta) = \delta \frac{(a - s - c)^2}{4} + (1 - \delta)V(c, \mu)
\]

where \( V(c, \mu) \) has been defined in the proof of Proposition 1. Thus, the optimal levels of \((\mu, \delta)\) are the solution to

\[
\max_{\mu, \delta} \{V(c, \mu, \delta) - \Psi(\mu) - \Omega(\delta)\}
\]

The first order conditions for an interior solution are

\[
(1 - \delta)\frac{\delta V(c, \mu)}{\delta \mu} = \Psi'(\mu)
\]

\[
\frac{(a - s - c)^2}{4} - V(c, \mu) = \Omega'(\delta)
\]

As in Proposition 1, if \( c \) is very close to \((2R_0 - a)\) the optimal strategy is \( p = R_0 \) and \( \mu = 0 \). Hence, the optimal value of \( \delta \) will be given by:

\[
\frac{(a - s - c)^2}{4} - (a - R_0)(R_0 - c) = \Omega'(\delta)
\]

The left hand side decreases with \( c \) and it is negative for \( c = 2R_0 - a \). Let us denote by \( c_B \) the value of \( c \) that equals the left hand side to zero. Thus, if \( c \in [c_B, 2R_0 - a] \), the firm sets \( \delta = 0 \), and if \( c < c_B \) \( \delta(c) \) monotonically decreases.

Like in Proposition 1 it is immediate to show that if \( c \) is low enough then the firm will choose to advertise its regular price, i.e. there exists a \( c_A \) such that for that realization of the random variable the firm is indifferent between advertising its regular price (and simultaneously sending out some coupons) and not advertising but sending out a certain amount of coupons. The value of \( c_A \) is implicitly given by

\[
H(c_A, R_0) \equiv \frac{\delta(a - c_A)^2}{4} + (1 - \delta)(a - R_0)(R_0 - c_A) - \Omega(\delta)
\]

\[
- \frac{\delta(a - c_A)^2}{4} - (1 - \delta)\frac{[(a - R_0) + \bar{\mu}(R_0 - c_A)]^2}{4\bar{\mu}} + \Psi(\bar{\mu}) + \Omega(\bar{\delta}) = 0
\]
where \( \tilde{\delta}, \tilde{\delta}, \tilde{\mu} \) are given by:

\[
(a - 2R_0 + c)^2 = 4\Omega(\tilde{\delta})
\]

\[
\frac{(R_0 - c)^2}{4} - \frac{(a - R_0)^2}{4\tilde{\mu}^2} = \frac{\Psi(\tilde{\mu})}{1 - \tilde{\delta}}
\]

\[
\frac{(R_0 - c)^2}{4} - \frac{(a - R_0)^2}{4\tilde{\mu}} = \frac{\Omega(\tilde{\delta})}{1 - \tilde{\mu}}
\]

The system is closed by the consistency condition:

\[
G(c_A, R_0) \equiv \int_{c_e}^{c_A} [R_0 - p(c)]\lambda^e(c) dc - s = 0
\]

where \( \lambda^e(c) \) is given by equation (6) in the text, with \( \mu^e(c) = \mu(c) \) and \( \delta^e(c) = \delta(c) \).

In order to show existence and uniqueness of equilibrium, we follow the same procedure as in Proposition 1: checking that the two locuses given by \( H = G = 0 \) cross only once provided \( s \) is arbitrarily small.

Remarks 3 and 4 are either immediate or analogous to Remarks 1 and 2 and, therefore, need no proof.

REFERENCES


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