Measuring Regional Fiscal Transfers Induced by National Budgets

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Abstract

National budgets typically induce a substantial redistribution of resources across regions. In this paper I propose an economic definition of such implicit fiscal transfers, which, in the absence of gains or losses out of centralizing fiscal policy, is particularly suitable for territorial equity discussions. In my view the fiscal transfer of a region is equivalent to the region’s willingness to pay for achieving fiscal independence. Such implicit transfers are also characterized in the context of a model where public debt is exclusively motivated by the tax-smoothing principle. It turns out that the fiscal transfer of a region can be computed by adding the region’s primary balance and an allocation of the national primary deficit according to a linear combination of the region’s share of receipts and expenditures. Thus, in general the computation of these implicit transfers requires detailed information about parameter values, which may not be available in practice. Some possible solutions are discussed.

Keywords: regional transfers, public debt

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1. Introduction

In most countries, the national budget induces a substantial redistribution of resources across regions. Regional redistribution is usually the result of both interpersonal redistribution at the national level, combined with regional heterogeneity, and discretionary regional policies. It is commonly assumed that the degree of regional redistribution can be summarized in a single variable: “the fiscal transfer of a region”; i.e., how much money a particular region transfers to the rest of the nation through the national budget. Despite the heated debates that are taking place in both academic and political spheres, it is not clear at all what exactly is behind the notion of regional fiscal transfers, and therefore how they should be measured in practice.

The empirical literature has provided estimates of the magnitude of inter-regional redistribution in different countries based on operational definitions of the regional fiscal transfer. For instance, the literature on the role of the national budget in smoothing region-specific shocks1 typically approximates inter-regional transfers by the difference between total tax revenue and total transfers of the national government in each region. Such an approximation is probably suitable to deal with risk sharing issues, given that taxes and transfers are the main automatic stabilizers. However, such a definition would clearly be inappropriate

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to discuss, for instance, equity considerations from a territorial point of view. In this case, we need a more comprehensive measure of inter-regional transfers which, besides taxes and transfers, also encompasses the level of consumption and investment of the national government in the region.

In this vein, many researchers have followed Mushkin (1956 and 1957) in constructing indicators of inter-regional transfers by allocating to the regions all the items of the national budget. Very recently, Wishlade et al (1997) have followed this road and confirmed that, in those European countries they studied, the degree of regional redistribution is quantitatively very important. Such an approach must provide answers to many delicate methodological questions; in particular, those related to the incidence of public goods.

This paper abstracts from the problems associated with measuring the regional incidence of taxes and expenditures and focuses on the incidence of national public deficits. First, I propose an economic definition of the regional transfer implicit in the national budget and, second, I characterize such a definition in terms of current fiscal variables, including a specific regional allocation of the national public deficits, in the context of a simple general equilibrium model.

In empirical work the regional transfer is typically defined as the primary balance of the region (the difference between receipts and expenditures of the national government in the region) plus an ad hoc imputation for debt-related items. For instance, in the case of Wishlade et al (1997) such an imputation is done using several different criteria, although no justification is provided. The view I take in this paper is that any operational definition of regional fiscal transfers must be derived from a notion that has a clear economic interpretation.

In a context in which there are neither efficiency gains nor losses out of centralizing fiscal policy, it is natural to define the region’s fiscal transfer as the opportunity cost of being part of the national fiscal policy. More precisely, consider an independent regional government which is free to select the fiscal policy of the region, subject to the constraint that in every period it has to transfer $R$ units to the rest of the nation. In this context, compute the level of $R$ that leaves the representative resident of the region indifferent between the national and the regional fiscal policies, which I shall denote by $\tilde{R}$. I consider $\tilde{R}$ to be the proper measure of the region’s transfer implicit in the national budget. We can also interpret $\tilde{R}$ as the region’s willingness to pay for the right to set its own tax and expenditure policies (for achieving fiscal independence).

Next, I characterize such a definition of the regional transfer in terms of current fiscal variables in the context of a general equilibrium model, in which the role of the national government is purely redistributive. It turns out that the fiscal transfer of a region can be computed by adding the region’s primary balance (the difference between receipts and expenditures of the national government in the region) and a certain allocation of the primary deficit of the national government (the difference between total expenditures and total receipts). The national primary deficit is allocated over the region according to a linear combination of the region’s share of receipts and expenditures. The weights assigned to these two variables closely mimic the precise determinants of national public debt. In fact, one of the main contributions of the paper is to demonstrate that we cannot compute the set of implicit inter-regional transfers unless we have...
a solid view on the reasons that induce national governments to issue debt. In other words, any characterization of regional transfers requires a theory of public debt determination.

I illustrate this point in a framework where public debt is issued because of Barro’s tax smoothing principle. The tax smoothing approach has successfully explained some of the historical evidence on public borrowing, although the recent theoretical and empirical literature has emphasized other complementary factors that may be particularly relevant in rationalizing the more recent experience. In any case, focusing exclusively on the tax smoothing motive seems a natural starting point.

One of the implications of the analysis is that, in practice, it will not be easy to compute from available information the exact values of inter-regional transfers implicit in national budgets. Nevertheless, I argue that, in the case of ignorance about the relevant parameter values we should compute two particular indicators that provide some kind of confidence interval. These two indicators allocate the national primary balance according to the regional distributions of expenditures and receipts, respectively. In our framework, each of these indicators provides an exact measure of the regional transfer for particular parameter values.

The next section presents the model. In Section 3 I analyze how national fiscal policy is determined, which includes the regional distribution of receipts and expenditures and the aggregate level of public debt. The definition and characterization of implicit transfers is provided in Section 4. The final section discusses some possible extensions.

2. The Basic Model

Let us consider a nation with two regions, indexed by \( i, i = A, B \), which are identical in all respects, except for a single parameter which measures labor productivity. Time is a discrete variable, indexed by \( t \), and we consider only a two-period horizon, \( t = 1, 2 \). Each region is populated by a representative consumer, whose utility depends on private consumption, public spending and labor. Preferences are the same in both regions and given by the following time-separable utility function:

\[
U_i^t = \sum_{t=1}^{2} \beta^{t-1} \left[ c_i^t - \frac{1}{2\sigma}(l_i^t)^2 + 2\delta(y_i^t)^{0.5} \right]
\]

(1)

where \( c, l, g \) denote private consumption, labor and public spending, respectively. Greek letters \( \sigma, \beta, \gamma, \delta \) denote exogenous parameters. Since \( \sigma \) and \( \delta \) do not play any role in the analysis they have been normalized to one: \( \sigma = \delta = 1 \). Parameter \( \beta \) is the discount factor. In order to simplify the presentation, I set \( \beta = \frac{1}{1+r} \); i.e., the time discount rate equals the rate of return on savings, \( r \). Thus, the rate of return on savings is also exogenously given. Such an assumption can be rationalized using standard arguments: either by assuming that our economy is small and has perfect access to an international capital market; or, alternatively, by postulating a constant returns to scale storage technology. Parameter \( \gamma \)
reflects consumers’ preferences over the time profile of government spending. The higher the value of $\gamma$, the steeper the optimal time profile of government spending.

Private consumption goods can be produced out of labor, by using a constant returns to scale technology:

$$y^i_t = \theta^i_t l^i_t, \quad t = 1, 2, \quad i = A, B$$

where $y$ denotes the level of production, and $\theta$ is an exogenous parameter that reflects the productivity level, which varies across regions and time periods. Given the absence of other factors of production, and assuming that both goods and labor markets are perfectly competitive, then wages absorb all (after-tax) income.

I shall consider two rather extreme scenarios of fiscal policy determination. In the first case, all fiscal variables will be determined exclusively by the national government. In the second case, regional governments will be able to choose their own tax code and spending levels, subject to an exogenous transfer to the other region. In any case, I shall assume that governments obtain revenue exclusively through a proportional labor income tax. Let us denote the (constant) tax rate by $\tau$, and total tax revenue by $T_i$. Thus, $T_i^t = \tau^i_t y^i_t, t = 1, 2, i = A, B$.

In each region, the representative agent chooses the levels of consumption and labor supply that maximize the objective function (1) subject to the following intertemporal budget constraint:

$$c^i_1 = \beta c^i_2 \leq (1 - \tau^i_1) \theta^i_1 l^i_1 = \beta (1 - \tau^i_2) \theta^i_2 l^i_2 + s^i_0$$

where $s^i_0$ denotes the gross return on past savings. The solution to such an optimization problem is characterized by the following conditions:

$$l^i_t = (1 - \tau^i_1) \theta^i_t \quad t = 1, 2 \quad (2)$$

$$c^i_1 + \beta c^i_2 = (1 - \tau^i_1) \theta^i_1 l^i_1 + \beta (1 - \tau^i_2) \theta^i_2 l^i_2 + s^i_0 \quad (3)$$

Equation (2) indicates that labor supply in each period is determined exclusively by the current tax rate and productivity levels. This is one of the advantages of the specific functional forms that I am using: Labor supply exhibits zero intertemporal substitutability, which highly simplifies the analysis. Also notice that the optimal consumption path is undetermined because of the constant marginal utility of consumption. Equation (3) only determines the present value of consumption at the optimum, but not its time distribution.

As a result of the equilibrium behavior of agents, output and tax revenue are given respectively by:

$$y^i_t = \theta^i_t l^i_t = (1 - \tau^i_t) \theta^i_t$$

$$T^i_t = \tau^i_t y^i_t = \tau^i_t (1 - \tau^i_t) \theta^i_t$$
Plugging equations (2) and (3) into the utility function (1), I obtain the indirect utility function (omitting a constant):

$$V_i^t = \sum_{t=1}^{2} \beta^{t-1} \left[ \frac{1}{2} \left( 1 - \tau_i^t \right)^2 \left( \theta_i^t \right)^2 + 2 \left( \gamma^{t-1} g_i^t \right)^{0.5} \right]$$  \hspace{1cm} (4)

It is important to emphasize that utility not only decreases with the tax rate, but it decreases at an increasing rate. This reflects the fact that the labor income tax distorts labor supply and that such a dead-weight loss increases with the tax rate. It is precisely such a feature of the (second best) optimal tax plan, plus an assumption about the optimal time profile for public spending, that will induce the government to issue public debt to smooth tax rates out.

3. The Optimal Policy of the National Government

In this section I consider the fiscal policy chosen by the national government, who cares about the welfare of agents in both regions. I assume that regions differ only in their labor productivity$^7$, $\theta_A^t > \theta_B^t$. Thus, region $A$ is relatively “richer” than region $B$. Also, for simplicity, productivity growth rates are equal across regions, $\theta_i^t = \lambda \theta_i^1$, $i = A, B$. Finally, labor mobility across regions is ruled out.

The national government’s objective function is:

$$W_1 = \max_{\tau_i^t, g_i^t} \left( V_A^t, V_B^t \right)$$  \hspace{1cm} (5)

where $V_A^t$ and $V_B^t$ are given by equation (4).$^8$ I assume that the initial stock of public debt is zero.$^9$ Thus, the intertemporal budget constraint of the national government is given by:

$$\sum_{t=1}^{2} \sum_{i=A,B} \beta^{t-1} g_i^t \leq \sum_{t=1}^{2} \sum_{i=A,B} \beta^{t-1} \tau_i^t \left( 1 - \tau_i^t \right) \left( \lambda^{t-1} \theta_i^t \right)^2$$  \hspace{1cm} (6)

where the left hand side is the present value of total public spending, and the right hand side is the present value of total receipts.$^{10}$

The optimization problem of the national government consists of choosing tax rates $\tau_i^t$, $t = 1, 2$, and $i = A, B$, and public spending levels $g_i^t$, $t = 1, 2$ and $i = A, B$, in order to maximize $W_1$, given by equation (5), subject to (6). The solution is characterized by the following equations:$^{11}$

$$\tau_A^1 = \tau_B^2 = \tau^t$$  \hspace{1cm} (7)

$$g_A^1 = \gamma g_B^1$$  \hspace{1cm} (8)

$$\frac{1 - \tau_i^t}{1 - 2 \tau_i^t} = (g_i^t)^{-0.5}$$  \hspace{1cm} (9)
\[
\frac{\partial W}{\partial V^A} (g^B_1)^{0.5} = \frac{\partial W}{\partial V^B} (g^A_1)^{0.5}
\]

\[
(g^A_1 + g^B_1) \frac{1 + \beta \gamma}{1 + \beta \lambda^2} = T^A_1 + T^B_1
\]

\[
= \tau^A (1 - \tau^A) (\theta^A)^2 + \tau^B (1 - \tau^B) (\theta^B)^2.
\]

Equation (7) reflects the tax-smoothing principle. In order to minimize distortions on labor supply, the national government chooses to equalize tax rates across periods. Equation (8) describes the optimal time profile of government spending. Equation (9) links taxation and government spending levels, in such a way that the marginal utility of public spending equals the marginal desutility of taxation. Equation (10) captures the redistributive aspect of fiscal policy. The distribution of government spending (and thus taxation) across regions depends on how the national government aggregates preferences. Finally, equation (11) is simply the intertemporal budget constraint.

We can now compute the level of public debt issued by the national government in the first period, \(B_1\). If we denote by \(T^t\) the aggregate level of taxes in period \(t\), and by \(G^t\) the aggregate level of public spending in period \(t\), from the intertemporal budget constraint (11) we get:

\[
B_1 = G_1 - T_1 = \frac{\beta (\lambda^2 - \gamma)}{1 + \beta \lambda^2} G_1 = \frac{\beta (\lambda^2 - \gamma)}{1 + \beta \gamma} T_1
\]

Notice that public debt comes from the discrepancy between the optimal time profile of taxes and expenditures. If \(\lambda = \gamma = 1\), then both tax revenues and expenditures are constant over time and there is no need to issue public debt. Thus, a necessary condition for the optimal fiscal policy to require a positive amount of borrowing is that \(\lambda^2 > \gamma\); i.e., the time profile of taxes must be steeper than the profile of expenditures. Such an assumption will be maintained throughout the paper.

4. A Characterization of the Implicit Inter-Regional Transfer

The fiscal policy of the national government, characterized by equations (7) to (11), clearly depends on regional equity considerations. A more egalitarian national government will tend to tax more heavily the rich region and spend relatively more resources in the poor region. The purpose of this section is to provide a reasonable definition of the inter-regional transfer implicit in the national fiscal policy and, specially, to characterize such a transfer exclusively in terms of the first period fiscal variables.

Let us consider the fiscal policy determined by a regional government that cares only about the welfare of the representative consumer of its own region. Suppose that the regional government can freely choose tax rates and expenditure levels but, perhaps because of the existence of a federal constitution, has to transfer to the rest of the nation an amount of resources \(R\) every period. Our definition of the inter-regional transfer implicit in the national budget relies on the comparison between the fiscal policies chosen by the
national government and such an independent regional government (subject to a compulsory transfer). The idea is to measure the opportunity cost of being part of the national fiscal policy for the representative consumer of the region (its willingness to pay for achieving fiscal independence).

**Definition.** The transfer implicit in the national fiscal policy is the level of $R$, denoted by $\tilde{R}$, that leaves the representative consumer of the region indifferent between the national and the regional policies.\(^{13}\)

Thus, $\tilde{R}$ measures the contribution of each region to the national budget. In the absence of gains and losses out of fiscal coordination, this is a suitable definition of the regional fiscal transfer since, as we will see below, the sum of $\tilde{R}$'s is equal to zero. Hence, in this case we can refer to $\tilde{R}$ as a pure transfer. In a more general framework, we should face the issue of how to separate regional redistribution from aggregate efficiency considerations.

The optimization problem of the government of region $i$ consists of choosing tax rates $(\tau'_1, \tau'_2)$ and expenditure levels $(g'_1, g'_2)$ in order to maximize $V^i$, given by equation (4), subject to the following intertemporal budget constraint:

$$\sum_{i=1}^{2} \beta^{t-1} g'_i + (1 + \beta) R^i \leq \sum_{i=1}^{2} \beta^{t-1} \tau'_i (1 - \tau'_i) (\theta'_i)^2$$  \hspace{1cm} (13)

Provided $R$ is not too large, the solution to the optimization problem of the regional government is characterized by the following equations:\(^{14}\)

$$\tau'_i = \tau'_2 \equiv \tau^f$$  \hspace{1cm} (14)

$$g'_2 = \gamma g'_1$$  \hspace{1cm} (15)

$$\frac{1 - \tau^f}{1 - 2 \tau^f} = (g'_1)^{-0.5}$$  \hspace{1cm} (16)

$$g'_1 (1 + \beta \gamma) + (1 + \beta) R^i = \tau^i (1 - \tau^i) (\theta'_i)^2 (1 + \beta \lambda^2)$$  \hspace{1cm} (17)

Notice that equations (14) to (16) coincide with equations (7) to (9). Both the national government and the regional government choose the same intertemporal pattern of taxes and expenditures and equalize the marginal utility of spending to the marginal desutility of taxation in each region. The only difference between these two fiscal plans will be in the regional distribution of expenditures\(^{15},\) which depends on the objective function of the national government (equation (10)).

Thus, comparing the solution to the national government’s optimization problem (equations (7) to (11)) and the regional government’s (equations (14) to (17)) it is straightforward
to show (See Appendix) that any policy chosen by the national government can be rationalized in terms of a pure inter-regional transfer, i.e.,

Result 1. For any objective function of the national government, there exists a unique pure transfer. More explicitly, for any national fiscal policy, there exists a unique pair \((R^A, R^B)\), that satisfies \(R^A + R^B = 0\), such that the national and regional fiscal policies coincide.

The next step is to characterize such an implicit transfer in terms of first period fiscal variables. From equations (12) and (17) we have that:

\[
\tilde{R}^i = (T_i - g_i) + \left[ \frac{T_i}{G_i} + (1 - \alpha) \frac{g_i}{G_i} \right] (G_i - T_i) \tag{18}
\]

where

\[
\alpha = \frac{(\lambda^2 - 1)(1 + \beta \gamma)}{(\lambda^2 - \gamma)(1 + \beta)}
\]

In words, the implicit inter-regional transfer can be computed by adding the primary balance of the national government in the region (the first term) and an imputation of the national budget deficit (the second term), which depends on a linear combination of the region’s share of tax revenue and the region’s share of expenditures.

If \(\lambda = 1\) and \(\gamma < 1\), then receipts are constant over time and public debt is used exclusively to finance the extra expenditures of the first period. In this case \(\alpha = 0\) and the national primary deficit must be allocated according to the share of the region in total public spending. However, if \(\gamma = 1\) and \(\lambda > 1\), then public debt reflects exclusively the fact that tax revenues increase over time, while expenditures are constant. In this case \(\alpha = 1\) and the national primary deficit must be allocated according to the share of the region in total tax revenue.

In general, the regional imputation of the national deficit involves a linear combination of the region’s share of tax revenue and expenditures. The relative weight placed on the receipts and expenditures shares is positively related to the relative weight of receipts and expenditures in generating national deficits. Unfortunately, in the general case the precise computation of the implicit transfer would require detailed information on certain parameter values, and not only first period fiscal variables. All these conclusions can be summarized as follows:

Result 2. 1) The implicit transfer of a region can be computed by adding the primary balance (receipts minus expenditures) of the national government in the region and a certain allocation of the national primary deficit (total expenditures minus total receipts) to the region.

2.a) If receipts are constant over time, then the national primary deficit is allocated according to the region’s share of expenditures.

2.b) If expenditures are constant over time, then the national primary deficit is allocated according to the region’s share of receipts.
2.c) For all other parameter values the national primary deficit is allocated according to a linear combination of the region’s share of receipts and the region’s share of expenditures, and requires specific information about parameter values.

We could interpret the above results pessimistically, since exact measures of implicit transfers require much more information about parameter values than it is usually available. We can also see the glass half full. In empirical work we could construct the following indicators of the implicit transfer of a region:

\[(A) \quad R^i(T) = (T^i_t - g^i_t) + \frac{T^i_t}{T^i_t}(G_t - T_t)\]

\[(B) \quad R^i(g) = (T^i_t - g^i_t) + \frac{g^i_t}{G_t}(G_t - T_t)\]

In both cases the transfer of region \(i\) to the rest of the nation is the sum of two terms. The first term is the region’s primary surplus, and the second is an imputation of the national government’s primary deficit. In terms of our model, \(R^i(T)\) coincides with \(N^R\) in the case of constant expenditures (\(\gamma = 1\)), and \(R^i(g)\) coincides with \(\bar{R}\) in the case of constant receipts (\(\lambda = 1\)). In the absence of specific information about parameter values, these two indicators provide some kind of confidence interval of the right implicit transfer.

5. Concluding Remarks

The use of specific functional forms has considerably simplified the analysis and enhanced transparency. The characterization of the implicit transfer is robust to certain changes in functional forms as long as optimal receipts and expenditures are linear with respect to the time variable. If linearity breaks up, then national primary deficits will also be allocated according to a non-linear function of the regional shares of receipts and expenditures.

More substantially, in our framework the activity of the national government is purely redistributive. However, in the presence of inter-regional externalities, a centralized decision procedure would enhance efficiency, and the sum of \(\bar{R}\)’s would be negative. On the contrary, if regional governments are better informed than the national government about consumer preferences, then decentralization is welfare increasing and the sum of \(\bar{R}\)’s would be positive. Thus, in a more general framework a proper definition of fiscal transfers should be able to disentangle the aggregate efficiency and regional redistribution aspects. Nevertheless, there are no reasons to expect any significant changes in the regional allocation of national deficits.

The two-period model considered above, plus the assumption of zero initial public debt, was useful in highlighting the main ingredients in the characterization of inter-regional transfers. In the working paper version (Caminal, 1998) I extend the analysis by considering the case of an infinite horizon economy. It is shown that as long as the initial stock of national public debt is null, the characterization of the implicit transfer provided in the two-period generalizes to the infinite horizon case. If the initial stock of debt is positive then the
characterization of the implicit transfer is similar, although the exact coefficients used to allocate the national primary balance change over time.\footnote{204}

Notes

1. See, for instance, Sala-i-Martin and Sachs (1992) or Asdrubali et al. (1996).
2. This study also sketches the history of this line of research and provides many useful references.
3. On the revenue side, the deficit is allocated according to the tax contributions of the regions. On the expenditure side, debt repayment is allocated differently between the flow and the benefit calculations. For the benefit approach, debt payments are allocated according to the regional distribution of expenditures. For the flow approach, debt payments are allocated according to the regional distribution of savers or shareholders.
5. See Barro (1986).
6. See, for instance, a recent survey by Alesina and Perotti (1995), where they discuss the role of different political and institutional factors.
7. The analysis is unaffected if we assume instead that initial savings also differ across regions.
8. We assume that \( W \) satisfies the usual properties that are required to guarantee a unique interior solution to the optimization problem.
9. See Section 5 for a discussion of this issue.
10. It is important to notice that we allow tax rates to differ across regions.
11. Notice that, provided \( \frac{\partial g_i}{\partial R} \) is strictly positive in the relevant range, the non-negativity constraints \( g_i^t \geq 0 \) and \( 1 - \tau_i^t \geq 0 \) are not binding. The reason is that the marginal utility of public expenditure goes to infinity as the variable goes to zero. Thus, the national government always chooses strictly positive levels of public expenditure in both regions. Also, equation (9) indicates that as \( g \) goes to zero, \( \tau \) goes to one-half (the peak of the Laffer curve). The intuition is that even if the central government puts little weight on the welfare of a particular region and provides a negligible amount of expenditure to that region, it will set a tax rate that approximately maximizes total revenue.
12. As long as governments can borrow or lend at the same exogenous interest rate, the only variable relevant in the optimization problem of the regional government is the present value of transfers not its time distribution. For expositional convenience, we focus on a flow variable (the transfer per period, which is considered to be constant over time) rather than on a stock variable (the present value of transfers).
13. This is essentially an Equivalent Variation or Compensating Variation measure.
14. Given that government spending can not take negative values and that there is an upper bound on government receipts, the maximum feasible transfer is given by equation (17) with \( g = 0 \) and \( \tau = 0.5 \). On the contrary, if \( R \) is sufficiently low then the tax rate can be negative (can become a subsidy).
15. Or, equivalently, on the regional distribution of receipts.
16. Notice that \( \alpha \) could be higher than 1 and also negative.
17. Similarly, if the national government can not set different tax rates across regions then decentralization is welfare increasing.
18. There I also discuss how public debt can be apportioned in the event of secession, in a distributionally neutral fashion, and how to allocate purchases of national public debt by residents of a particular region in constructing the region’s capital account.

References

Appendix

**Proof of Result 1:** Given \((g^A, g^B)\), equations (7) to (9) uniquely determine the values of the rest of variables in the optimal plan of the national government. Consider the solution to the national government optimization problem \((\bar{n}_i^g, \bar{\bar{n}}_i^g)\). Since \(\bar{n}_i^g > 0\) (see footnote 14 in the text) then if we define:

\[
\chi \equiv \bar{n}_i^g (1 + \beta \gamma) - \bar{\bar{n}}_i^g (1 - \bar{\bar{n}}_i^g) \left( \theta_1^A \right)^2 \left( 1 + \beta \lambda^2 \right)
\]

from equation (11) we have that:

\[
-\chi \equiv \bar{n}_i^B (1 + \beta \gamma) - \bar{\bar{n}}_i^B (1 - \bar{\bar{n}}_i^B) \left( \theta_1^B \right)^2 \left( 1 + \beta \lambda^2 \right)
\]

Similarly, given \(g_i^r\), equations (14) to (16) uniquely determine the values of the rest of the variables in the optimal plan of the regional government. Moreover, these equations coincide with equations (7) to (9). Also, notice that for all \(R^i\) feasible, there is a unique solution to equations (14) to (17), and that \(\frac{\partial c}{\partial R^i} < 0\).

Finally, from equation (17) the solution to the regional government’s optimization problem coincides with \((\bar{\bar{n}}_i^r, \bar{n}_i^r)\) if and only if:

\[
\bar{\bar{n}}_i^A = \frac{\chi}{1 + \beta} = -\bar{n}_i^B.
\]