Entry under Capacity Limitation and Vertical Differentiation: return of the Judo Economics

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Abstract

Shaked and Sutton (1982) and Gelman and Salop (1983) are best remembered for their neat conclusions: a limited quality or limited capacity is an effective tool to relax competition and facilitate entry in a market. We aim at comparing the respective merits of these two strategic commitments. We claim that capacity limitation is more effective than quality reduction, mainly because it acts directly upon the incumbent to reduce his aggressiveness in the final price competition whereas quality tools works indirectly trough consumer's willingness to pay.

Keywords: Entry, Quality, Differentiation, Bertrand-Edgeworth Competition

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1 Introduction

The fact that many industries feature one or few dominant firms and a fringe of small competitors has been nicely formalized by Gelman and Salop (1983): in order to relax price competition and make entry profitable, an entrant can use a carrot and stick strategy. She voluntarily limits her production capacity to guarantee a large residual demand for the incumbent; yet, she names a low price that would prove dear to undercut. In their discussion of possible means to achieve this credible commitment, the authors claim that “producing a product with limited consumer appeal is analogous to capacity limitation”.

It is indeed true that a similar strategic commitment is at work in the models of quality differentiation of Gabszewicz and Thisse (1979) and Shaked and Sutton (1982): the entrant optimally chooses a low quality and offers a substantial rebate on her product in order to induce the incumbent not to fight too aggressively in prices. The incumbent therefore prefers to accommodate entry although it is always possible for him to exclude the entrant from the market.

In this note we mix the two previous strand of literature by considering a game of entry where the entrant is allowed to choose the quality of its product and its production capacity. The question we raise is the following: does the entrant use product differentiation and capacity precommitment simultaneously? We show in Theorem 1 that under efficient rationing, quality imitation coupled with an optimal capacity limitation is more effective than having a large production capacity and a low quality. Even if differentiation occurs, it is limited. Furthermore, the product of quality by capacity remains equal to the optimal capacity limitation.

2 The model

We follow Mussa and Rosen (1978) and (Tirole, 1988, sec. 2.1) to model quality differentiation. A consumer with personal characteristic $x$ is willing to pay $xs$ for one unit of quality $s$ and nothing more for additional units. He maximizes surplus and when indifferent between two products, select his purchase randomly. Types
are uniformly distributed in \([0; 1]\).

In agreement with most observed real cases, the incumbent is committed to the best available quality (normalized to unity) before entrants get an opportunity to pick their own, without however the ability to leapfrog him. We also assume that quality is not costly for firms\(^1\) and that the marginal cost of production is nil (up to the capacity limit and equal to +\(\infty\) otherwise). These considerations lead us to study the following stage game\(^2\) \(G\):

- At \(t = 0\), an incumbent \(i\) enters the market and selects the top quality \(s_i = 1\) and a large capacity \(k_i = 1\).
- At \(t = 1\), an entrant \(e\) selects its quality \(s_e = s \leq 1\) and capacity \(k_e = k \leq 1\).
- At \(t = 2\), firms compete simultaneously in prices.

We denote \(G(s, k)\) the pricing game occurring at the last stage. Our solution concept for the game \(G\) is Subgame Perfect Nash Equilibrium. Observe that two classes of price subgames might be generated by choices made at \(t = 1\): either \(k = 1\) and we face a standard game of vertical differentiation or \(k < 1\) and we face a Bertrand-Edgeworth game with (possibly) product differentiation.

Consumers make their choice at the last stage by comparing the respective surpluses they derive when buying from the incumbent, the entrant or nobody i.e., \(x - p_i\), \(xs - p_e\) and 0. In the absence of differentiation \((s = 1)\), demands are as in the standard Bertrand game. In the presence of differentiation \((s < 1)\), it is a straightforward exercise to show that demands are given by

\[
D_i(p_i, p_e) = \begin{cases} 
0 & \text{if } p_e + 1 - s < p_i \\
1 - \frac{p_i - p_e}{s} & \text{if } \frac{p_e}{s} \leq p_i \leq p_e + (1 - s) \\
1 - \frac{p_i}{s} & \text{if } p_i \leq \frac{p_e}{s}
\end{cases}
\]  

\(1\) An upper bound on the admissible qualities is required to ensure that firms’ payoffs are bounded.

2 Recall that Gelman and Salop (1983)’s model is of the Stackelberg type where the entrant commits to capacity and price before the incumbent is able to respond in price.
Firms’ profits in the pricing game are

\[ \Pi_e(p_i, p_e) = p_e D_e(p_i, p_e) \quad \text{and} \quad \Pi_i(p_i, p_e) = p_i D_i(p_i, p_e) \quad (3) \]

When capacity is not an issue \((k = 1)\) and products are differentiated \((s < 1)\), Choi and Shin (1992) show that firms best replies are continuous and given by:

\[ \phi_i(p_e) = \begin{cases} \frac{p_e + 1 - s}{2} & \text{if } p_e \leq \frac{1 - s}{2 - s} s \\ \frac{p_e}{s} & \text{if } \frac{1 - s}{2 - s} s \leq p_e \leq \frac{s}{2} \\ \frac{1}{2} & \text{if } p_e \geq \frac{s}{2} \end{cases} \quad (4) \]

\[ \phi_e(p_i) = \begin{cases} \frac{p_i s}{2} & \text{if } p_i \leq \frac{2(1-s)}{2-s} \\ p_i - 1 + s & \text{if } \frac{2(1-s)}{2-s} \leq p_i \leq 1 - \frac{s}{2} \\ \frac{s}{2} & \text{if } p_i \geq 1 - \frac{s}{2} \end{cases} \quad (5) \]

These best replies are displayed on Figure 1 and the equilibrium is summarized in Lemma 1 below.
Lemma 1  For \( s < 1 \), the game \( G(s, 1) \) has a unique pure strategy equilibrium:

\[
p_i^* = \frac{2(1 - s)}{4 - s} \quad \text{and} \quad p_e^* = \frac{s(1 - s)}{4 - s}
\]  

(6)

Plugging (6) into (3), we obtain the entrant’s first stage payoff as a function of his quality: \( \Pi_e = \frac{s(1-s)}{(4-s)^2} \). Straightforward computations yield the following corollary.

Corollary 1  The optimal quality for the entrant in the class of pricing games \( \{G(s,1), s < 1\} \) is \( s^* = \frac{4}{7} \), yielding the profit \( \pi_e^* = \frac{1}{48} \).

Notice that the pricing game \( G(1,1) \) is a classical Bertrand game with linear demand \( D(p) = 1 - p \). In case of a price tie, demand is shared equally by the two firms.

2.1 Rationing and Sales

Whenever the entrant has unlimited capacity \( (k = 1) \), sales are equal to demand as characterized by equations (1) and (2). However, if the entrant has build a limited capacity \( (k < 1) \), there are prices leading up to more demand than can be served i.e., \( D_e(p_e, p_i) > k \). In such cases, some consumers will be rationed and possibly report their purchase on the incumbent. In order to characterize firms’ sales in that situation, we assume efficient rationing: rationed consumers are those exhibiting the lowest willingness to pay for the good. The limited \( k \) units sold by the entrant will be contested by potential buyers,\(^3\) the price \( p_e \) paid for them will rise to the level \( \rho_e \) where the excess demand vanishes. In the case of duopoly competition, we solve

\[
D_e(\rho_e, p_i) = \frac{p_i - \rho_e}{1-s} - \frac{\rho_e}{s} > k \iff p_e < \rho_e \equiv (p_i - k(1-s))s
\]  

(7)

while in the case of monopoly,

\[
D_e(p_e, p_i) = 1 - \frac{p_e}{s} > k \iff p_e < s(1-k)
\]  

(8)

\(^3\)We implicitely assume that a secondary market opens where consumers may take advantage of the arbitrage possibilities at no cost.
Using (7) and (8), the entrant is capacity constrained i.e., \( S_e(p_e, p_i) = k \) whenever
\[
p_e \leq \min\{\rho_e, s(1-k)\}
\] (9)

Now, using (1), we obtain the residual demand addressed to the incumbent firm as
\[
D_i^R(p_i) \equiv 1 - ks - p_i.
\] (10)

The expressions for the sales functions are therefore:

\[
S_e(p_i, p_e) = \begin{cases} 
0 & \text{if } p_e \geq p_i s \\
\frac{p_i - p_e}{1-s} - \frac{p_e}{s} & \text{if } p_e \in [\max\{p_i - (1-s), \rho_e\}; p_i s] \\
1 - \frac{p_e}{s} & \text{if } p_e \in [s(1-k); p_i - (1-s)] \\
k & \text{if } p_e \leq \min\{\rho_e, s(1-k)\}
\end{cases}
\] (11)

\[
S_i(p_i, p_e) = \begin{cases} 
0 & \text{if } p_i \geq p_e + 1-s \\
1 - ks - p_i & \text{if } p_i \in \left[\frac{p_e}{s} + k(1-s); p_e + 1-s\right] \\
1 - \frac{p_i - p_e}{1-s} & \text{if } p_i \in \left[\frac{p_e}{s}; \frac{p_e}{s} + k(1-s)\right] \\
1 - \frac{p_i}{s_i} & \text{if } p_i \leq \frac{p_e}{s}
\end{cases}
\] (12)

where branch (11:c) is void if \( p_i < 1 - ks \).

### 2.2 Price Best Responses

Whenever \( k < 1 \), the analysis of \( G(k, s) \) must take into account the possibility that firms sales are respectively given by equations (12:b) and (11:d) where the entrant’s capacity is binding and the incumbent recovers all rationed consumers. It is immediate to see that the best she can do is to sell her capacity at the highest price, which is \( \rho_e \). On the other hand, whenever the incumbent plays along segment (12:b), he maximizes profits by trying to set \( \bar{p}_i \equiv 1 - ks \), and, if successful, obtains a minmax profit equal to \( \bar{\pi}_i \equiv \frac{(1-ks)^2}{4} \).

Given the incumbent’s price \( p_i \), the entrant’s payoff function remains concave in own prices (over the domain where \( D_e(.) \geq 0 \)). The best response function is
now given by

\[ BR_e(p_i, k) = \begin{cases} 
\frac{p_i s}{2} & \text{if } p_i \leq 2k(1-s) \\
\rho_e & \text{if } 2k(1-s) \leq p_i \leq \min\{1-\frac{s}{2}, 1-ks\} \\
p_i - 1 + s & \text{if } 1-ks \leq p_i \leq 1-\frac{s}{2} \\
\max\{\frac{s}{2}, s(1-k)\} & \text{if } p_i \geq \min\{1-\frac{s}{2}, 1-ks\} 
\end{cases} \]  

(13)

On Figure 2 we illustrate the case \( k > \frac{1}{2} \) (in the other case, the third branch of (13) vanishes).

As should appear from the inspection of \( S_i(p_e) \), the payoff of the incumbent is likely to be non-concave when his sales switch from segment (12:b) to (12:c). Accordingly, the best response to \( p_e \) might be non-unique. Solving \( \pi_i\left(\frac{p_e+1-s}{2}, p_e\right) = \bar{\pi}_i \) for \( p_e \), we obtain:

\[ \hat{p}_e(s, k) \equiv \sqrt{1-s} \left(1 - ks - \sqrt{1-s}\right) \]  

(14)

which is represented on Figure 2. Yet, if \( \bar{\pi}_i > \pi_i\left(\frac{p_e+1-s}{2}, p_e\right) \) over the whole domain where \( \phi_i(p_e) = \frac{p_e+1-s}{2} \) is defined by equation (4:a), we must compute the incum-
bent’s payoff along segment (4:b). Solving \( \frac{p_e}{s} \left( 1 - \frac{p_e}{s} \right) = \pi_i \) for \( p_e \), we obtain:

\[
\hat{p}_e(s, k) \equiv \frac{s}{2} \left( 1 - \sqrt{k s (2 - k s)} \right)
\]

(15)

Last, to know which case applies, we solve \( \hat{p}_e = \hat{p}_e \) to obtain:

\[
h(s) \equiv \frac{1}{s} \left( 1 - \frac{2 \sqrt{1 - s}}{2 - s} \right)
\]

(16)

Depending on the value of the capacity \( k \), we might therefore obtain two different shapes for the best response of the incumbent firm in the pricing game:

- if \( k \geq h(s) \), then

\[
BR_i(p_e) = \begin{cases} 
\frac{1 - k s}{2} & \text{if } p_e \leq \hat{p}_e \\
\frac{p_e + 1 - s}{2} & \text{if } \hat{p}_e < p_e \leq \frac{1 - s}{2 - s} \\
\frac{p_e}{s} & \text{if } \frac{1 - s}{2 - s} \leq p_e \leq \frac{s}{2} \\
\frac{1}{2} & \text{if } p_e \geq \frac{s}{2}
\end{cases}
\]

(17)

- if \( k \leq h(s) \), then

\[
BR_i(p_e) = \begin{cases} 
\frac{1 - k s}{2} & \text{if } p_e \leq \tilde{p}_e \\
\frac{p_e}{s} & \text{if } \tilde{p}_e < p_e \leq \frac{s}{2} \\
\frac{1}{2} & \text{if } p_e \geq \frac{s}{2}
\end{cases}
\]

(18)

The critical values \( \hat{p}_e \) and \( \tilde{p}_e \) therefore identify the price level at which firm \( i \) is indifferent between naming the security price \( \bar{p}_i = \frac{1 - k s}{2} \) or naming a lower price which ensures a larger market share. The resulting discontinuity is likely to destroy the existence of a pure strategy equilibrium.

2.3 Price Equilibrium

We analyze the Nash equilibria for each price subgame \( G(s, k) \). Let us first deal with imitation whereby the entrant chooses top quality \( (s = 1) \). In this case, the vertical differentiation model degenerates into a Bertrand-Edgeworth competition for an homogenous product. Levitan and Shubik (1972) analyze this game
under the efficient rationing hypothesis and derive the following result whose proof is given in Appendix A. Notice that applying Gelman and Salop (1983)’s Stackelberg sequentiality to the current demand yields exactly the same optimal capacity (cf. Appendix B).

**Lemma 2** $G(1, k)$ has a unique price equilibrium in which the entrant earns exactly $k\hat{p}_e(1, k)$. Furthermore the maximum of this payoff is $\pi^+_e \equiv \frac{3}{4} - \frac{1}{\sqrt{2}} \approx 0.043$ and is reached for $k^+ \equiv 1 - \frac{1}{\sqrt{2}} \approx 0.293$.

When products are differentiated and one firm faces a capacity constraint, the existence of a price equilibrium is not problematic since payoffs are continuous (the Nash existence theorem applies). Besides, there exists quality-capacity constellations where a pure strategy equilibrium exists. More precisely, the pure strategy equilibrium prevailing in the limiting case where $k = 1$ is preserved. Let us define $g(s) \equiv 1 - \frac{4\sqrt{1-s}}{4-s} > h(s)$.

**Lemma 3** For $s < 1$, $p_i^* = \frac{2(1-s)}{4-s}$ and $p_e^* = \frac{s(1-s)}{4-s}$ is a pure strategy equilibrium of $G(s, k)$ whenever $k \geq g(s)$.

**Proof** The candidate equilibrium is $(p_i^*, p_i^*)$ characterized in Lemma 1 (cf. eq. (6) and Figure 1). The price $p_i^*$ remains a best response to $p_e^*$ only if $p_e^* \geq \hat{p}_e$; straightforward computations yield the condition $k \geq g(s)$ and since $g(s) > h(s)$, we check that $\hat{p}_e$ was indeed the benchmark to use.

Whenever $k < g(s)$, a pure strategy equilibrium fails to exist. For intermediate capacities, it is easy to identify a particular equilibrium in which the incumbent randomizes over two atoms while the entrant plays the pure strategy $\hat{p}_e$. However, there also exists a domain of small capacities where even this equilibrium fails to exist. When this is the case, both firms use non-degenerate mixed strategy in equilibrium. The equilibrium strategy used by firm $j = i, e$ in equilibrium of

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4Since $h(1) = 1$, the relevant benchmark is $\hat{p}_e$.

5Indeed, $g(s) > h(s) \Leftrightarrow 16s^2 (1 - s) + s^4 (3 + s) > 0$ which is always true (over the relevant domain $0 \leq s \leq 1$).
\( G(k, s) \) is denoted \( F_j \); the lower bound and upper bound of the support of \( F_j \) are denoted respectively by \( p_j^- \) and \( p_j^+ \).

**Lemma 4** Let \( k < g(s) \) and \( s < 1 \). In equilibrium of \( G(k, s) \), \( p_i^+ \leq \frac{1-ks}{2} \) and \( p_e^+ \leq BR_e \left( \frac{1-ks}{2} \right) \).

**Proof:** The proof proceeds by iteration; Figure 2 is helpful to follow the argument. Observe firstly that \( p_i^+ \leq \frac{1}{2} \), the monopoly price because at any \( p_i > \frac{1}{2} \), \( \pi_i(p_i, p_e) \) is decreasing in \( p_i \), thus the average \( \pi_i(p_i, F_e) \) is also decreasing in \( p_i \) which proves that such a price cannot belong to the support of \( F_i \). Next, since \( BR_e(p_i) \) is increasing and \( p_i^+ \leq \frac{1}{2} \), \( BR_e \left( \frac{1}{2} \right) \) is the largest best reply for the entrant to consider. This means that for \( p_e > BR_e \left( \frac{1}{2} \right) \), \( \pi_e(p_i, p_e) \) is decreasing in \( p_e \) whatever \( p_i \leq \frac{1}{2} \), thus the average \( \pi_e(p_e, F_i) \) is also decreasing in \( p_e \) which proves that \( p_e^+ \leq BR_e \left( \frac{1}{2} \right) \).

Referring to Figure 2, one observes that because \( BR_i(p_e) \) for \( p_e > \rho_e \) and \( BR_e(p_i) \) are both increasing, they cannot cross. Reiterating the previous reasoning, we can sequentially reduce the upper price played by each firm in a Nash equilibrium. This tendency to lower prices comes to a stop at \( \bar{p}_i = \frac{1-ks}{2} \) because there is no reason to exclude the incumbent from putting mass on that price. We thus end up with \( p_i^+ \leq \frac{1-ks}{2} \) and \( p_e^+ \leq BR_e \left( \frac{1-ks}{2} \right) \).

**Lemma 5** Let \( k < g(s) \). In equilibrium of \( G(k, s) \), \( p_i^+ = \frac{1-ks}{2} \) and the equilibrium payoff is the minimax \( \pi_i \).

**Proof** We may check by algebra that when \( k < g(s) \), it is true that \( 2k(1-s) < \bar{p}_i = \frac{1-ks}{2} \). This implies that \( BR_e(\bar{p}_i) = \rho_e \) and by the previous lemma, that \( p_e^+ \leq \rho_e \). Hence, for \( p_i \) in a neighborhood of \( \bar{p}_i \), the incumbent's sales are the residual ones \( D_i' \) so that we have \( \pi_i(p_i, F_e) = p_i(1 - ks - p_i) \).

If \( 2k(1-s) \leq p_i^+ < \bar{p}_i \), then \( \pi_i(p_i, F_e) \) is strictly increasing over \( \left| p_i^+ ; \bar{p}_i \right| \) which implies that \( p_i^+ \) cannot be part of an equilibrium strategy for the incumbent.

If, on the contrary, \( p_i^+ < 2k(1-s) \), then the previous argument does not apply because the incumbent’s sales might vary. However, if this case occurs then
the entrant’s demand, when facing \( F_i \), is always of the duopolistic kind without capacity constraint, hence his best reply is the pure strategy \( \phi_e \) computed at the average of \( p_i \). Since the pure strategy equilibrium does not exists over the present domain, the incumbent must be playing a mixed strategy and the only candidate when the entrant plays a pure strategy involves playing the security price \( \tilde{p}_i \), a contradiction with \( p_i^+ < \tilde{p}_i \).

We have thus shown that \( p_i^+ = \frac{1-ks}{2} \) and since the equilibrium payoff can be computed at any price in the support of \( F_i \), we have \( \pi_i(p_i^+, F_e) = p_i^+(1 - ks - p_i^+) = \frac{(1-ks)^2}{4} = \pi_i \). ■

**Lemma 6** Let \( k < g(s) \). In equilibrium of \( G(k, s) \), \( p_e^- \leq \hat{p}_e \) if \( k \geq h(s) \) and \( p_e^- \leq \hat{p}_e \) if \( k \leq h(s) \). The entrant’s equilibrium payoff is bounded from above by \( k \hat{p}_e(s, k) \) if \( k \geq h(s) \) and by \( k \check{p}_e(s, k) \) if \( k \leq h(s) \).

**Proof:** Let us consider first the case \( k < h(s) \). If \( p_e^- > \tilde{p}_e \) then for any \( p_i < \frac{p_e^-}{s} \), the incumbent’s demand is monopolistic whatever \( p_e \geq p_e^- \). Hence, \( \pi_i(p_i, F_e) = p_i(1 - p_i) \) is strictly increasing, which means the lowest price of the mixed strategy \( F_i \) cannot belong to this area. We have thus show that \( p_i^- \geq \frac{p_e^-}{s} \) holds true. If \( p_i^- = \frac{p_e^-}{s} \), then at \( p_i^- \), the incumbent is a monopoly whatever \( p_e \geq p_e^- \), thus \( \pi_i(p_i^-, F_e) = p_i^-(1 - p_i^-) = \frac{p_e^-}{s} \left( 1 - \frac{p_e^-}{s} \right) > \frac{p_e^-}{s} \left( 1 - \frac{p_e^-}{s} \right) = \frac{(1-ks)^2}{4} = \pi_i \) by definition of \( \tilde{p}_e \) and by the previous lemma. This inequality is a contradiction with \( p_i^- \) being in the support of \( F_i \). The last case is thus \( p_i^- > \frac{p_e^-}{s} \). Then, \( \pi_i(p_i^-, F_e) \geq \pi_i \left( \frac{p_e^-}{s}, F_e \right) \) since \( p_i^- \) is an optimal price and \( \frac{p_e^-}{s} \) is not; observing that \( \pi_i \left( \frac{p_e^-}{s}, F_e \right) = \frac{p_e^-}{s} \left( 1 - \frac{p_e^-}{s} \right) \), the previous argument applies and we obtain again a contradiction. This proves \( p_e^- > \tilde{p}_e \) is not true i.e., our claim.

The second claim is a simple consequence of the fact that the equilibrium payoff can be computed at any price in the support of \( F_e \), hence

\[
\pi_e(p_e^-, F_i) = p_e^- \int S_e(p_e^-, p_i) dF_i(p_i) \leq kp_e^- \leq k\check{p}_e
\]

since sales are bounded by the capacity. The case for \( k \geq h(s) \) is identical since the benchmarks \( \hat{p}_e \) and \( \tilde{p}_e \) play a symmetric role. ■
Although we do not have a full characterization of the mixed strategy equilibrium in all possible subgames, we have derived enough to state:

**Proposition 1** An optimal quality-capacity pair is $s = 1$ and $k = k^\dagger$. Other optimal pairs necessarily satisfy $s \geq \bar{s} \equiv 2(\sqrt{2} - 1) \approx 0.83$ and $sk = k^\dagger$.

**Proof** For $k < h(s)$, $\pi_e(F_e, F_i) \leq k\hat{p}_e(s, k) = \frac{ks}{2} \left(1 - \sqrt{ks(2 - ks)}\right)$ which is a function of the product $x = ks$, whose maximum is reached for $x = k^\dagger$ and yields an overall maximum $\pi_e^\dagger$. It then remains to observe that this is precisely the optimal quality and the maximum entrant’s payoff for $s = 1$ and $k = k^\dagger$ as shown in Lemma 2. The pair $(1, k^\dagger)$ is shown as a diamond on Figure 3. The maximum payoff over the domain $s < 1$ and $k < h(s)$ is therefore dominated by that in $G(1, k^\dagger)$.

![Figure 3: Strategy Space](image)

A likewise analysis applies for $s < 1$ and $h(s) \leq k \leq g(s)$. The upper bound $k\hat{p}_e(s, k) = k\sqrt{1-s}(1 - ks - \sqrt{1-s})$ reaches its maximum for $k = \frac{1-\sqrt{1-s}}{2s}$. Replacing by the optimal value and simplifying, the objective is now $\frac{\sqrt{1-s(1-\sqrt{1-s})^2}}{4s}$. The maximum is achieved at $\bar{s}$ (previously defined) and leads to the optimal capacity $k^\dagger / \bar{s} \approx 0.35$ and profit $\pi_e^\dagger$. The pair $\left(\bar{s}, \frac{k^\dagger}{\bar{s}}\right)$ satisfies $k = h(s)$ and is shown as a dot on Figure 3. We have thus shown that the entrant’s profit for $h(s) \leq k \leq g(s)$ is lesser than a function whose maximum is $\pi_e^\dagger$.

Finally, for $s < 1$ and $k \geq g(s)$, the optimum strategy is to differentiate with $s^* = \frac{4}{7}$ to earn $\pi_e^* = \frac{1}{48} \approx 0.021 < \pi_e^\dagger \approx 0.043$. Overall, the pair $(1, k^\dagger)$ is an optimal
strategy; there might other optimal strategies satisfying \( ks = k^\dagger \) but they all give the same final payoff. ■

Appendix

A: Proof of Lemma 2

Let \( F_e \) and \( F_i \) be the equilibrium cumulative distributions, assuming no mass except at the end points. Due to the nature of demand, the entrant gets all demand if her price \( p \) is the lowest i.e., with probability \( 1 - F_i(p) \), her payoff is thus \( \pi_e = p(1 - F_i(p)) \min \{ k, 1 - p \} \). Likewise the incumbent’s is \( \pi_i = p \left( 1 - p - F_e(p) \min \{ k, 1 - p \} \right) \).

Bottom prices have to be the same because otherwise one profit would be strictly increasing in between (all prices are lesser than the monopoly one) and this would contradict the equilibrium definition.

At the common bottom price \( p_l \), \( F_i = 0 \) and \( 1 - p_l > k \), thus \( \pi_e = kp_l \). The entrant’s top price cannot be greater than the incumbent’s one because \( \pi_e \) would be zero, hence at the top price \( p_h \), \( F_e = 1 \). If there was no rationing at \( p_h \) then \( \pi_i \) would be zero, thus \( 1 - p_h > k \) and \( \pi_i = p_h \left( 1 - p_h - k \right) \). Furthermore the right derivative must be negative to make sure than no other greater price is better, hence \( p_h \geq \frac{1-k}{2} \). We also have \( F_e(p) = \frac{1-p-\pi_i/p}{k} \) (recall that \( 1 - p > k \) over the whole interval) thus the density must be \( f_e(p) = \frac{1}{k} \left( \pi_i / p^2 - 1 \right) \). Being positive, we derive \( p^2 \leq \pi_i = p_h \left( 1 - p_h - k \right) \) and applying this inequality at the top price, we get \( p_h \leq \frac{1-k}{2} \). Combining with the reverse inequality, we obtain \( p_h = \frac{1-k}{2} \), so that \( \pi_i = \frac{(1-k)^2}{4} \). Now, at the bottom price \( \pi_i = p_l(1 - p_l) \), thus \( p_l = \frac{1}{2} \left( 1 - \sqrt{k(2-k)} \right) \) which is \( \tilde{p}_e(1, k) \) so that \( \pi_e = k\tilde{p}_e(1, k) \) as claimed. ■

B: Gelman and Salop (1983)’s optimal capacity

In Gelman and Salop (1983)’s setting, the challenger enters with capacity \( k \) and committed price \( p_e \) to which the incumbent later responds with \( p_i \). The incumbent’s payoff with the aggressive price-cutting strategy is \( (1 - p_e)p_e \). By accommodating and serving the residual demand, his profit is \( (1 - k - p_i)p_i \). The optimal price is \( p_i^* = \frac{1-k}{2} \) yielding profit \( \frac{(1-k)^2}{4} \).
Playing on the possibility of inducing accommodation, the entrant can maximize her profit \( kp_e \) under the constraints \( p_i^* > p_e \) (undercut the incumbent) and 
\[
(1 - p_e) p_e \leq \frac{(1-k)^2}{4}
\]
(leave the incumbent happy). We obtain two conditions on the entrant’s capacity that must be satisfied simultaneously i.e.,
\[
k \leq \min \left\{ 1 - 2 p_e, 1 - 2 \sqrt{(1 - p_e) p_e} \right\} \Leftrightarrow k \leq \min \left\{ \frac{1}{2}, 1 - 2 \sqrt{(1 - p_e) p_e} \right\}
\]
Since the entrant’s profit is increasing with capacity, he will choose a value that saturates the constraint i.e., \( k = 1 - 2 \sqrt{(1 - p_e) p_e} \). The profit of the entrant is thus
\[
p_e \left( 1 - 2 \sqrt{(1 - p_e) p_e} \right)
\]
and is maximum for \( p_e^* = \frac{1}{2} - \frac{1}{2 \sqrt{2}} \approx 0.15 \), leading to \( k^* = 1 - \frac{1}{\sqrt{2}} = k^\dagger \).

References


