Pricing ‘cyclical’ goods

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Abstract

Consumption of certain commodities causes transitory satiation, in the sense that potential instantaneous utility from an additional unit is very low immediately after a consumption episode, but it increases over time. Such cyclical pattern of preferences has important implications: (i) If the monopolist cannot commit to long-run prices, then some equilibria are Pareto dominated (both buyers and the seller would rather play a different equilibrium involving a lower price), (ii) introduction of loyalty-rewarding schemes may benefit both buyers and sellers, (iii) restrictions on the timing of purchases (purchase deadlines, sales, etc) are likely to hurt consumers and increase efficiency, and (iv) collusion may involve a price reduction.

Keywords: cyclical preferences, repeat purchases, monopoly pricing, loyalty-rewarding schemes

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1 Introduction

Consumption of certain commodities causes transitory satiation. The day after visiting an amusement park, the utility derived from another visit is likely to be very low, but tends to increase over time. Most people also experience a similar change in preferences after dining at an ethnic restaurant, getting a haircut, or attending a concert by their favorite pianist. One may even argue that transitory satiation is associated with the consumption of most types of goods, at least at some level of disaggregation and on different time scales.\(^1\) In this paper I focus on markets in which (i) consumer satiation is sufficiently persistent over time to the extent that can potentially interact with price dynamics, and (ii) sellers enjoy a significant amount of market power. Thus, amusement parks would be a typical example, but others will also be discussed.

I propose a very simple characterization of this type of preferences. I define a ‘cyclical’ good as a perishable good\(^2\) for which consumers have cyclical preferences, and a new cycle starts every time individuals consume the good.\(^3\) More specifically, consumers’ potential instantaneous utility, \(R\), measured in monetary units, increases monotonically with the time elapsed since the last consumption episode, \(s\), and falls discontinuously when consumption takes place. Thus, if a consumer pays a price \(p\) after \(s\) units of time since the last purchase, then she obtains an instantaneous net surplus of \(R(s) - p\). Such a representation does not take account of two potentially important issues. First, various types of random shocks may play a role in many real world situations, whereas the above characterization is purely deterministic. Second, preferences for some ‘cyclical’ goods may exhibit long-run decreasing marginal utility, in the sense that the function \(R(s)\) may shift downwards after every purchase. In contrast, I assume stationary preferences throughout

\(^1\)To the best of my knowledge Harmann (2004) is the only empirical paper that has approached this issue. In particular, he estimates a dynamic model of consumption decisions for rounds of golf, and concludes that it takes 32 days for the median consumer’s willingness to pay to return to pre-consumption levels. He shows that this fact has important implications for the estimation of the long-run own-price elasticity.

\(^2\)In Section 2 I discuss the analogies between (non-durable) cyclical goods and durable goods subject to depreciation.

\(^3\)Cyclical goods are quite different from ‘seasonal’ goods. In the latter case, the time pattern of preferences is exogenously given, and thus unrelated to the history of purchases. On the other hand, existing models of addiction and habit persistence capture the opposite phenomenon: current consumption raises future marginal utility.
the paper. Thus, the current approach should be interpreted as a first step in modelling cyclical preferences.

The main goal of this paper is to study the pricing of ‘cyclical’ goods. In the monopoly case, the main insights are the following. First, if the monopolist cannot commit to long-run prices, then some equilibria are Pareto dominated (buyers and the seller would rather play a different equilibrium involving a lower price). This result stands in sharp contrast to most monopoly models with standard preferences. Second, the model offers a natural justification for the introduction of loyalty-rewarding pricing schemes and restrictions on the timing of purchases (purchase deadlines, sales, etc.), which complements the existing literature. In the current set up, loyalty schemes may benefit both buyers and sellers, whilst restrictions on the timing of purchases are likely to hurt consumers and increase total surplus. Finally, in a competitive environment with product differentiation, and if transitory satiation is relatively stronger for the variety consumed, it is shown that equilibrium prices may above profit maximizing levels. In other words, collusion may involve price reductions.

A crucial implication of the cyclical pattern of preferences is that expected future prices influence current demand. When consumers choose the optimal timing of a purchase they balance the benefits of waiting, higher instantaneous utility, and the costs of waiting, delaying the realization of the net surplus associated with the next and all other future purchases. Thus, if buyers expect higher future prices (lower expected surpluses), then the costs of waiting are reduced and hence the next purchase is delayed. Consequently, equilibrium prices depend on the seller’s commitment capacity. In Section 3 I deal with the monopoly pricing problem when prices cannot depend on the history of purchases of individual consumers (anonymous markets). Whenever the seller commits to a constant price forever, then the price has a large effect on the timing of purchases because it affects not only the net surplus associated to the next purchase but also to the surplus associated to all subsequent purchases; and, as a result, the effect on the costs of waiting is large. In the opposite extreme, if consumers perceive the current price as being relevant only to the next purchase and believe that future prices are independent of the current price, then current demand is not very sensitive to the current price, since it only affects the surplus from the next purchase, i.e., the effect on the costs of waiting is small. In other words, in a game where the monopolist can only commit to the price of the next purchase, the equilibrium in Markov strategies features a higher price than in case the mo-
monopolist commits to a constant price forever. If we introduce the possibility of developing a reputation (by allowing for trigger strategies) then the range of prices than can be sustained as a subgame perfect equilibrium expands significantly. Moreover, some of these equilibria are Pareto dominated by others. Thus, players may find themselves stuck in a bad equilibrium with high prices, and instead they would rather be playing a different equilibrium with lower prices.

If sellers keep track of the history of purchases of individual consumers then they might also be able to commit to pricing policies that reward consumer loyalty. In the current set up if we allow the seller to commit to a sequence of prices (Section 4.1), with the $nth$ price corresponding to the $nth$ purchase, then the equilibrium policy includes a high price for the initial purchase and a price equal to marginal cost for the following ones. The introduction of such loyalty rewarding schemes obviously benefits the monopolist, in comparison to the case of commitment to a constant price, but its effect on consumer welfare is ambiguous.

Sellers of cyclical goods have strong incentives to restrict the timing of purchases. In some (properly defined) markets, products are available only occasionally. This is the case, for instance, of a live performance of an artist in a particular city. More often, sellers may find the way of restricting the availability of the product, or, more generally, of affecting the timing of purchases. This is the aim of some common marketing techniques, like subscriptions, occasional sales and purchase deadlines. In my set up, the seller always finds it optimal to restrict the timing of purchases (Section 4.2). Typically, consumers are worse off under restricted timing but total welfare is higher.

In Section 5 I embed the cyclical pattern of preferences in a model of product differentiation. If transitory satiation is stronger for the last variety consumed, i.e., if consumers prefer a diversified consumption profile, then competition is relaxed. The reason is that firm-customer relations are only occasional and hence firms pay less attention to the effect of their prices on consumers’ future purchases. As a result, prices are determined by the net balance of two countervailing effects: the standard business stealing effect (static competition) and the frequency of purchases from the same supplier. It is shown that if static competition is not sufficiently intense then equilibrium prices might be above joint profit maximizing levels. In other words, collusion could lead to a reduction in prices.

The sellers’ ability to commit to future prices is also crucial in many other contexts. A well known example is the Coase conjecture. In the absence of
commitment, a durable goods monopolist may end up setting prices very close to marginal cost.\textsuperscript{4} In contrast to this literature, I am not so much interested in the (short-run) opportunistic behavior of the monopolist in reaction to endogenous changes in the distribution of consumers. Instead, I emphasize the effect of various pricing schemes on the long-run pattern of repeat purchases.

Under asymmetric information and consumer learning, prices have also been shown to follow various dynamic patterns.\textsuperscript{5} In the current model quality is common knowledge; the time dimension matters only because the preferences of individuals vary systematically over time according to the endogenous timing of consumption.

The literature on repeated games has improved our understanding of how tacit collusion can be sustained. In standard dynamic models of oligopolistic competition multiple equilibria arise and players disagree about their ranking. Firms prefer equilibria with high prices while consumers are better off in low price equilibria. Similar arguments have been applied in situations where the monopolist faces a time inconsistency problem.\textsuperscript{6} Reputation can replace commitment as a rent-extracting device. Once again, if we compare different equilibria an increase in consumer surplus is necessarily associated to a reduction in firms’ profits. In contrast, in the current model equilibria can be Pareto-ranked (players face a coordination problem).

In a model of experience goods Crémer (1984) showed that the monopolist would like to commit to a pricing scheme that includes prices equal to marginal costs for all repeat purchases. Thus, two apparently very different environments (experience and cyclical goods) deliver similar predictions regarding the monopoly pricing of repeat purchases. However, there is a

\textsuperscript{4}Bulow (1982) formalized these ideas in a finite horizon model, and Stockey (1981) and Gul et al. (1986) in an infinite-horizon model.

\textsuperscript{5}If consumers repeat purchases, monopolists set an introductory price to induce experimentation, and later increase the price as consumers become better informed (Milgrom and Roberts, 1986). When consumers purchase the good only occasionally, if the price is a positive signal of quality then a decreasing sequence of prices may be obtained in equilibrium (Bagwell and Riordan, 1991), although if consumers learn the quality of the good from market shares then the price sequence will, on average, increase (Caminal and Vives, 1996).

\textsuperscript{6}For instance, in the durable goods case, Ausubel and Deneckere (1989) have shown that competitive prices are not the only possible outcome. When the discount rate approaches zero all seller payoffs between zero and static monopoly profits can be obtained as the outcome of subgame perfect equilibria.
difference. In Crémer’s framework the price of the first purchase is equal to total gains from trade and, as a result, the seller’s commitment capacity hurts consumers, whereas in the case of cyclical goods the price of the first purchase must be relatively moderate in order to induce consumers to make their first purchase relatively soon. As a result, consumers may actually benefit from the introduction of such loyalty rewarding schemes.  

2 The baseline model

2.1 Description

Time is a continuous variable that runs from 0 to infinity. There is a single seller and an arbitrary number of consumers with identical preferences. In some of the cases considered in the paper prices are customer-specific and hence it will be a game between one buyer and one seller, where the former retains the power of unilaterally setting prices. The monopolist can instantaneously produce a homogeneous perishable good at zero cost. Immediately after a purchase, consumer’s potential instantaneous utility from an additional purchase is equal to \(-L < 0\) and evolves deterministically according to \(R(s)\), where \(s\) is the time elapsed since the last purchase. More specifically, \(R(s)\) is a three times continuously differentiable function, from \(\mathbb{R}_{++}\) into \(\mathbb{R}\), satisfying (See Figure 1 for an illustration):

A.1. \(R'(s) > 0, R''(s) < 0, R'''(s) > 0\).
A.2. \(\lim_{s \to 0} R(s) = -L < 0, \lim_{s \to \infty} R(s) = M\).
All agents discount the future at the rate \(r > 0\).

For simplicity, let us suppose that at time 0 the consumer has just purchased the good. If the buyer expects to pay a price \(p_n\) in the \(n\)th purchase, \(n = 1, 2, ...,\) and to spend \(s_n\) units of time between the \((n-1)\)th and the \(n\)th purchase, then the buyer’s expected payoff at time 0 is given by:

\[
U_0 = \sum_{n=1}^{\infty} [R(s_n) - p_n] e^{-r \sum_{j=1}^{n} s_j}
\]  

(1)

Similarly, the seller’s payoff is given by:

\[
\text{In oligopolistic markets with random consumer preferences, loyalty-rewarding schemes create consumer switching costs. Consumers tend to lose when sellers use coupons to reward loyalty (Banerjee and Summers, 1987), but they may gain if sellers commit to prices for repeat purchases (Caminal and Matutes, 1990).} \]
\[ \Pi_0 = \sum_{n=1}^{\infty} p_n e^{-r} \sum_{j=1}^{n} s_j \] (2)

Finally, total welfare is the sum of the consumer’s utility and the firm’s profits:

\[ W_0 = \sum_{n=1}^{\infty} R(s_n) e^{-r} \sum_{j=1}^{n} s_j \] (3)

### 2.2 Efficiency

The only variables that affect total welfare are the length of the time intervals between purchases. Thus, the efficient outcome is a sequence of time intervals with length \( \{s_n\}_{n=1}^{\infty} \) that maximize \( \Pi_0 \). We can set up the optimization problem as finding the optimal timing of the next purchase, \( s_1 \), that maximizes:

\[ W_0 = e^{-rs_1} [R(s_1) + W^*] \]

where \( W^* \) is the maximum surplus that can be obtained after the first purchase (which is independent of \( s_1 \)).

The solution is given by the first order condition:

\[ R'(s_1) - r[R(s_1) + W^*] = 0 \]

Thus, the optimal timing is obtained by balancing the gains from waiting, i.e., the increase in instantaneous utility, and the costs of waiting, i.e., the interest on the capitalized gains from trade. The latter is the sum of the instantaneous utility plus the net present value of future gains from trade. Note that the short-run optimal timing, \( s_1 \), depends on the long-run surplus, \( W^* \).

Since the optimization problem is stationary, the optimal time intervals are constant and the maximum surplus after a purchase is given by:

\[ W^* = e^{-rs_0} 1 - e^{-rs_0} R(s^o) \]

where \( s^o \) is given by:

\[ R'(s^o) - r R(s^o) 1 - e^{-rs^o} = 0. \] (4)

Note that \( s^o \) increases with \( r \) and is invariant to multiplicative transformations of \( R(s) \).
2.3 Analogies with durable goods

The above characterization is meant to represent the case of non-durable goods with transitory satiation. Nevertheless, there are close analogies to the case of durable goods subject to depreciation.\(^8\)

Let us consider a durable good that generates a flow of services equal to \(q e^{-\delta s}\), where \(s\) is the age of the good and \(\delta > 0\) is the rate of depreciation. Suppose that the durable good can be produced under a constant returns to scale technology with no capacity limits. Let \(c\) denote the unit cost of producing the durable good. For simplicity, suppose that at time 0 consumers have just purchased a new unit. If consumers expect to pay a price \(p_n\) in their \(n^{th}\) purchase, then their payoff function can be written as:\(^9\)

\[
U_0 = \frac{q}{\delta + r} \left[1 - e^{-(r+\delta)s_1}\right] + \sum_{n=1}^{\infty} \left\{ \frac{q}{\delta + r} \left[1 - e^{-(r+\delta)s_{n+1}}\right] - p_n \right\} e^{-r \sum_{j=1}^{n} s_j}
\]

Similarly, the seller’s payoff can be written as:

\[
\Pi_0 = \sum_{n=1}^{\infty} (p_n - c) e^{-r \sum_{j=1}^{n} s_j}
\]

In this case I cannot normalize variable costs to zero, otherwise in the efficient allocation consumers would continuously buy new units. In the above formulation of non-durable cyclical goods, continuous purchases were ruled out by the assumption that potential utility right after each purchase fell below zero (marginal cost). Thus, we must keep in mind that \(p\) represents the price-cost margin in one case, but the absolute price in the other. Except for this, ?? and ?? are identical. The analogies between ?? and ?? are a bit less obvious. However, we can define:

\(^8\)The analogy can be easily grasped by considering one of the examples mentioned in the introduction. A hair cut can perhaps be interpreted as an instantaneous service that produces a level of utility which depends on the length of time since the last hair cut. However, a hair cut is better characterized as a durable service that deteriorates over time.

\(^9\)A model with these characteristics has been analyzed in a companion paper (Caminal, 2004). In that paper I study the incentives to innovate and the rationale behind pricing policies that aim at affecting the timing of purchases, like trade-in allowances, which are contingent on the age of the old unit.
\[
R(s) = \frac{q}{\delta + r} \left[ 1 - e^{-(r+\delta)s} \right]
\]

(7)

and note that this function satisfies assumptions A.1 and A.2, where \( L = c \) and \( M = \frac{c}{\delta r} - c \). If we plug \( ?? \) into \( ?? \) then the only difference with respect to \( ?? \) is that in the latter case \( R(s_n) \) is enjoyed by consumers at the time of the \( nth \) purchase, while in the former is enjoyed at the time of the \( (n-1)th \) purchase. It turns out that such a difference has no economic significance.

Thus, the pricing of some non-durable goods (like a visit to an amusement park) is in fact subject to similar considerations than the pricing of durable goods that depreciate over time (like automobiles). Nevertheless, physical characteristics do play a role in some cases. For instance, in what Fudenberg and Tirole (1988) call semianonymous markets, sellers of durable goods can offer discounts to those buyers that trade in their old units. Obviously, this is not possible in markets for (non-durable) cyclical goods.

Another important consideration is that in most durable goods markets (hardware, software) technological innovations are crucial. In such highly non-stationary (and stochastic) environment is difficult to think about commitment to future prices, or about contracting on the frequency of purchases. In contrast, many (non-durable) cyclical goods markets are fairly stationary and complex intertemporal pricing policies are more likely to be feasible.

For all these reasons in the rest of the paper I stick to the non-durable cyclical good interpretation.

3 Monopoly pricing and commitment capacity

In this section I consider the case in which the seller cannot keep track of the history of purchases of individual consumers (anonymous market). At the end of the section I will discuss the problems involved in handling posted prices and asynchronized consumers. For the moment, I consider a simple set up that illustrates very clearly the role of the seller’s commitment capacity on monopoly prices. First, I present the benchmark case in which the seller can

\footnote{Some of the issues analyzed in this paper resemble those studied by Fishman and Rob (2000). In particular, both papers attempt to characterize the equilibrium timing of purchases under monopoly. However, they focus on product innovation and consumers’ adoption decisions are trivial: consumers purchase the good as soon as it becomes available.}
commit to a constant price forever. Second, I consider the consequences of
time-limited commitment power. In particular, I assume that the seller sets
customer-specific prices and commits to maintain the announced price until
the buyer makes the next purchase. I start characterizing the equilibrium
in Markov strategies and next I consider more general strategies (to con-
sider reputation effects). I also discuss the case of intermediate commitment
capacities and alternative modelling approaches.

3.1 Commitment to a constant price

Suppose the monopolist commits to a constant price forever. At time 0 the
seller sets a price \( p \) and consumers choose the timing of purchases. For a
given price \( p \), the consumer chooses \( s_1, s_2, ... \) in order to maximize \( \pi \). The
first order condition that characterizes the optimal \( s_1 \) is given by:

\[
R'(s_1) - r[R(s_1) - p + U^*] = 0 \tag{8}
\]

where \( U^* \) is the consumer’s continuation value. Thus, the consumer’s
short-run optimal timing depends not only on the current price but also on
the long-run surplus. However, in this subsection the seller’s price is constant
and hence \( U^* \) also depends on \( p \). More specifically, given the stationarity of
the problem, the buyer’s continuation value can be written as:

\[
U^* = \frac{e^{-rs}}{1 - e^{-rs}} [R(s) - p]
\]

where \( s = s_1 \) is also given by equation (8). Thus, the relationship between
the optimal length of interpurchase time period and the constant price, \( \hat{s}(p) \),
is implicitly given by:

\[
R'(\hat{s}) - r[R(\hat{s}) - p]1 - e^{-r\hat{s}} = 0 \tag{9}
\]

From the above expression we can compute the sensitivity of \( \hat{s} \) to changes
in the (constant) price. In particular:

\[
d\hat{s}dp = G(\hat{s})1 - e^{-r\hat{s}}
\]

where

\[
G(s) = \frac{r}{-R''(s) + rR'(s)} > 0
\]
Thus, a higher price increases the length of the time intervals between purchases (decreases frequency). Also note that $G'(s) > 0$, and that $\hat{s}(0) = s^0$.

The monopolist anticipates that consumers’ behavior is given by $\hat{s}(p)$ and chooses $p$ in order to maximize:

$$\Pi_0 = \frac{e^{-r\hat{s}(p)}}{1 - e^{-r\hat{s}(p)}} p$$

The first order condition characterizes the equilibrium price:

$$1 - rp1 - e^{-rs\hat{s}}d\hat{s}dp = 0 \quad (10)$$

Thus, equation (10) shows the trade-off faced by the monopolist: a higher price increases the margin but reduces the frequency of purchases. The size of the latter effect depends on $\frac{ds}{dp}$. Combining equations (10) and (11) we can characterize the equilibrium value of $s$, denoted by $s^e$ (c stands for commitment):

$$R'(s^e) - rR(s^e)1 - e^{-rs^e} + 1 - e^{-rs^e}G(s^e) = 0 \quad (11)$$

Second order conditions imply that the left hand side of equation (11) decreases with $s$. Also, from equation (10), we know that the left hand side, evaluated at $s^o$, is positive. Hence, we obtain the following (straightforward) result.

Under commitment to a constant price, interpurchase time periods are inefficiently long: $s^o < s^e$.

The intuition is straightforward. A monopolist charges a price above marginal cost, which reduces the consumer’s costs of waiting, and as a result the frequency of purchases decreases.

Finally, I denote the equilibrium price as $p^e$, i.e., the value of $p$ such that $\hat{s}(p^e) = s^e$.\footnote{Note that the equilibrium price is independent of the initial distribution of consumers, provided no consumer starts with an $s$ higher than $s^e$.}

### 3.2 Short-run commitment

Suppose now that at time 0 the monopolist sets customer-specific prices and can only commit to keeping those prices unchanged until the next purchase.
Immediately after the consumer purchases the good then the seller can set a different price. The idea is to rule out short-run pricing policies that restrict de facto consumers’ timing of purchases, while allowing for some discretionary power in the medium and long-run.

In this case a Markov strategy for the seller is simply a price, since every time the seller sets a new price all payoff relevant variables take the same value. A Markov strategy for the buyer can be expressed as a reservation price, which depends on the current state of preferences, \( \mathcal{p}(s) \); or, more conveniently, as a choice of the timing of next purchase as a function of the current price, \( s(p) \). The consumer’s optimization problem is similar to that of the previous section and thus \( s(p) \) is also given by equation \( ?? \). The crucial difference is that now her continuation value, \( U^* \), does not depend on the current price but only on expected future prices. In fact, the relationship between the timing of the first purchase and the current price, \( s(p) \), is different than in the previous subsection. In particular:

\[
d\hat{s}dp = G(s)
\]

Hence, in this case, \( s \) is less sensitive to \( p \) than in the case of commitment to a constant price. The reason is that in the latter case a change in the price is expected to be permanent, while in the Markov Equilibrium of the current game any deviation from the equilibrium price is expected to be transitory.

Given consumer behavior, the seller’s best response is the value of \( p \) which maximizes:

\[
\Pi_0 = e^{r\hat{s}(p)\left[p + \Pi^\ast \right]}
\]

where \( \Pi^\ast \) is independent of \( p \). Thus, the first order condition can be written as:

\[
1 - r(p + \Pi^\ast)d\hat{s}dp = 0 \quad (12)
\]

Since the game is stationary, from equations \( ?? \) and \( ?? \) the equilibrium timing of purchases, \( s^d \) (\( d \) stands for discretion), is given by:

\[
R'(s^d) - rR(s^d)1 - e^{-rs^d} + 1G(s^d) = 0 \quad (13)
\]

Also, from equation \( ?? \) we have that the equilibrium price, \( p^d \), is such that \( s^d = \hat{s}(p^d) \). Next, let us compare equations \( ?? \) and \( ?? \).
In the Markov Equilibrium of the short-run commitment game the price is higher and the average interpurchase time interval longer than under commitment to a constant price, i.e., $p^d > p^c$, and $s^d > s^c$.

The driving force of this result is the effect of commitment power on consumers’ expectations. In the Markov equilibrium of the short-run commitment game any deviation from the equilibrium price is interpreted by consumers as a transitory deviation and hence the impact on the timing of the next purchase is relatively small. In contrast, whenever the seller can commit to a constant price, a deviation is perceived as permanent, which has a larger effect on the frequency of purchases. As a result, in the Markov equilibrium of the short-run commitment game the monopolist has incentives to charge a higher price than under long-run commitment.\footnote{Again, the equilibrium price is independent of initial conditions, provided the initial value of $s$ is lower than $s^d$.}

3.3 Discussion

3.3.1 Reputation

Suppose expectations about future prices are formed according to the following rule: If past and current prices have been equal to $q$ then consumers expect future prices to be equal to $q$, otherwise they expect future prices to be equal to $p^d$. This is reminiscent of trigger strategies, in the sense that if the seller deviates from the prescribed price then the punishment consists on consumers playing their strategy in the Markov equilibrium from then onwards. In the Appendix it is shown that any price in the interval $[p_l, p^d]$, where $p_l < p^c < p^d$ can be supported as a subgame perfect equilibrium. If we compare the players’ payoffs across these equilibria then a higher price in the interval $[p_l, p^d]$ is associated with lower consumer surplus and higher firm profits. However, a higher price in the interval $[p^c, p^d]$ is associated with lower payoffs for both the buyer and the seller (See Figure 2.) In other words, there exist equilibria that are Pareto dominated: both buyers and sellers could benefit from switching to a different equilibrium with lower prices.

3.3.2 Intermediate commitment capacity

Suppose that the seller can commit to a (constant) price for the next $N$ purchases. For each $N$ we obtain a price and a length of interpurchase time peri-
ods \( \{p_N, s_N\} \) prevailing in the Markov equilibrium. It is immediate (although the algebra is quite messy) that as the degree of commitment, \( N \), increases the sensitivity of demand to a price cut also increases, which implies that the equilibrium price falls and the frequency of purchases increases. More specifically, \( p_N < p_{N-1}, s_N < s_{N-1} \). Also, \( p_1 = p^d, s_1 = s^d, \lim_{N \to \infty} p_N = p^c, \lim_{N \to \infty} s_N = s^c \).

In fact, for any finite \( N \) we can talk about a double margin. For instance, if \( N = 1 \) we can split the Markov equilibrium margin, \( p^d \), into the margin caused by monopoly power, \( p^c \), and the margin associated to the lack of commitment, \( p^d - p^c \).

As the degree of commitment increases both buyer and seller payoffs increase. This result suggests that since sellers benefit from any increase in commitment capacity they could be willing to invest in various commitment devices. In the current model, if the seller can choose the degree of price commitment at the beginning of the game then he will choose to commit to a fixed price forever. However, in a richer model sellers might face a trade-off between the benefits from price commitment analyzed in this paper and the costs of price rigidity. For instance, marginal costs may be random. If the variance of these costs is sufficiently large then the costs of price rigidity may overcome the benefits from commitment. Thus, higher cost volatility would be associated with more price flexibility and higher average margins.

### 3.3.3 Modelling short-run commitment

The game studied in Section 3.2 is highly stylized, but nevertheless it provides some useful insights on the effects of commitment on equilibrium prices through consumers’ price sensitivity. Two assumptions seem particularly controversial: customer-specific prices and the seller’s open-ended commitment capacity (price is maintained until the buyer shows up).

A natural alternative would be a game where the seller posts a price that can only be occasionally changed. In this case the length of price rigidity would parametrize the degree of commitment. More specifically, prices could be fixed for a time interval of length \( T \) (the length of the ‘period’) but trade can take place at any time within that interval. In such a set up we could even think of dealing with asynchronized consumers. Unfortunately, such a game involves formidable analytical challenges, even in the case of a single consumer. First of all, stationary equilibria do not generically exist. The reason is that the number of purchases in a given ‘period’ will tend to
fluctuate along the equilibrium path, since generically $T$ will not be a multiple of the time between two purchases. As a result, prices will also fluctuate. In particular, the larger the number of purchases in a given period, the lower the equilibrium price. The intuition is analogous to the one discussed in the previous subsection.

A possible solution to the non-stationarity problem would be to restrict ourselves to those values of $T$ that generate a stationary pattern of purchases and prices. For instance, the case $T = s^d$ would appear to be a particularly interesting case, since we may hope that in such a case the equilibrium price might be $p^d$, which in turn would induce a stationary pattern of purchases. Unfortunately, even in this particular example the characterization of equilibria with Markov strategies is rather cumbersome because of the existence of a deadline effect. Consumers’ willingness to pay discontinuously increases right at the end of the period when a higher price is expected to replace the current one. Thus, for some initial conditions, the seller might have incentives to deviate and set a price below $p^d$ in order to induce the buyer to purchase twice over the period (the second purchase right before a new price is quoted). Therefore, the players’ continuation value at the time of setting a new price will in general depend on the time elapsed since the last purchase. As a result, any attempt to obtain a tractable characterization of stationary strategies looks hopeless.

In spite of these analytical complications it is not clear at all that such an alternative model could provide substantial additional insights. In particular, it is very unlikely that such a deadline effect could offset the driving force behind the main result of section 3.2. In the alternative game where the price is posted for $T$ units of time, consumers’ price sensitivity will also depend on the number of purchases to be made at the current price and, hence, it seems reasonable to conjecture that average prices along a Markov equilibrium will also decrease with $T$.

4 Alternative pricing schemes

So far I have focused on the case in which the seller cannot keep track of the history of purchases of individual consumers and cannot restrict the timing of purchases. However, in some cyclical goods markets it might be feasible to keep records of individual transactions or at least to discriminate between old customers and newcomers, through coupons and similar devices. Similarly,
sellers might be able to commit to supply some cyclical goods only at specific points in time. In this section I consider first the case where the seller can set prices conditional to the number of previous purchases but, as above, cannot directly restrict the timing of those purchases. Next, I consider the opposite case: the seller can choose in advance the timing of the next purchase but cannot condition the price on previous transactions. Finally, I briefly discuss the possibility of writing long-run contracts specifying both the price and the frequency of purchases, like in the case of subscriptions to magazines.

4.1 Commitment to a sequence of prices

Suppose that the seller can keep track of the individual history of purchases and is able to commit to a sequence of prices \( \{p_n\} \) where \( n \) refers to the order of purchases, \( n = 1, 2, \ldots \). Now the seller can reward or penalize consumer loyalty by setting a decreasing or an increasing price sequence. Given the sequence \( \{p_n\} \), consumers choose the timing of purchases \( \{s_n\} \) in order to maximize \( U_0 \) (equation ??). The optimality condition for the timing of the first purchase is well known by now and given by equation ??.

Thus, as in the Markov Equilibrium of the game of Section 3.2, the effect of \( p_1 \) on \( s_1 \) is given by:

\[
\frac{\partial s_1}{\partial p_1} = G(s_1)
\]

However, the monopolist can influence \( s_1 \) not only through \( p_1 \) but also through \( p_2, p_3, \ldots \). By the envelope theorem we have that for all \( n > 1 \):

\[
\frac{\partial s_1}{\partial p_n} = G(s_1) \frac{\partial U^*_1}{\partial p_n} = G(s_1) e^{-r\sum_{j=2}^{n} s_j}
\]

Note that a change in \( p_1 \) has the same effect on \( s_1 \) as a change of the same size in the present value of \( p_n \). However, the change in \( p_n \) has additional effects on \( (s_2, \ldots, s_n) \).

The monopolist chooses \( \{p_n\} \) in order to maximize ??, anticipating how the timing of purchases is affected by the price sequence. The cumulative effect of future prices drives the following result:

The equilibrium price sequence includes a positive margin in the first purchase and zero margin in the following purchases, i.e., \( p_1 > 0, p_n = 0 \) for all \( n > 1 \). As a result, \( s_1 > s^o, s_n = s^o \) for all \( n > 1 \).
For the proof see the Appendix. The intuition goes as follows. The first price of the sequence only has an effect on the timing of the first purchase. However, successive prices affect not only the timing of the corresponding purchases but also the timing of the previous ones. Consider a sequence of prices that involves a positive margin in the \( n \)th purchase. The monopolist can make higher profits by raising the first price and lowering the \( n \)th price in such a way that the present value of prices (evaluated at the timing of purchases associated with the original price sequence) remains unchanged. The reason is that the new price sequence does not have any first round effect on the timing of the first purchase, but it does bring forward the second, third, ..., and \( n \)th purchases.

Next, I characterize \( p_1 \) and \( s_1 \). Since, the consumer appropriates all the surplus after the first purchase the optimality condition for \( s_1 \) is an adaptation of equation ??:

\[
R'(s_1) - r[R(s_1) - p_1 + \frac{e^{-rs_1}}{1 - e^{-rs_1}}R(s^o)] = 0
\]

Since the monopolist makes zero profits after the first purchase, the optimality condition for \( p_1 \) is an adaptation of equation ??:

\[
1 - rp_1 ds_1 dp_1 = 1 - rp_1 G(s_1) = 0
\]

Combining equations ?? and ?? we obtain:

\[
R'(s_1) - r[R(s_1) + \frac{e^{-rs_1}}{1 - e^{-rs_1}}R(s^o)] + \frac{1}{G(s_1)} = 0
\]

Thus, the optimal pricing policy rewards consumer loyalty. In fact, the result of marginal cost pricing after the first purchase is analogous to that of Crémer (1984) in a different context (consumer learning in a two-period model). The mechanism behind such a result is different although both models share the same principle. In both cases the first best can be implemented through a two-part tariff, and the monopolist can capture the entire surplus. In Crémer’s two period model, the first period price is analogous to an up-front fee. In my model if the monopolist can charge a fee upfront (before the game starts and thus unrelated to any purchase) and a price per purchase then the profit maximizing policy also includes a price equal to marginal cost in all purchases and a fee equal to the present value of total surplus. In most cases payment of an upfront fee is not feasible. Whenever seller and buyer
are ready to sign a contract it is very likely that the buyer’s potential utility changes over time. In this case, the buyer is willing to pay the upfront fee only at the moment of the first purchase. Hence, the price of the first purchase is the instrument that the monopolist uses to collect rents, although the size of these rents is moderated by the incentives to induce consumers to make the first purchase relatively soon.

The equilibrium policy characterized in this section may look somewhat unrealistic. First, consumers could be liquidity constrained and unable to pay at the first purchase an amount equivalent to a significant fraction of the present value of all future gains from trade. Second, the monopolist’s incentives to default on her promises are very powerful and therefore her commitment capacity must also be very strong. If we assume that consumers are unable to pay at a single purchase a price above a certain threshold, and/or that the monopolist is only subject to a finite (and relatively small) penalty if he defaults on the pricing policy announced at time 0, then the slope of the time profile of equilibrium prices is reduced, although the main qualitative features remain.

Do consumers benefit from such loyalty rewarding policies? Let us compare consumer payoffs in the equilibrium where the monopolist commits to a constant price \((p^c, s^c)\) with the equilibrium where the monopolist can commit to a (decreasing) price sequence. In principle, there are two countervailing effects. In the latter case, on the one hand, the price charged after the first purchase is lower but, on the other hand, the price of the first purchase is higher than in the constant price equilibrium. The examples discussed in the Appendix suggest that consumers may actually lose or gain from loyalty rewarding policies, depending on parameter values. In particular, in Example 1 consumers lose if the monopolist can commit to a variable price policy, and in Example 2 consumers gain. Thus, the introduction of loyalty rewarding schemes in cyclical goods markets may be a Pareto improvement, which contrasts with the results of Crémer (1984).

4.2 Restricting the timing of purchases

Sellers may be able to restrict the actual timing of purchases. For instance, they might credibly announce a very high regular price with occasional and

13For instance, when a new variety is introduced consumers’ potential utility is likely to be affected by the time period elapsed since the last purchase of a different variety.
predetermined periods of ‘sales’. Similarly, sellers could restrict the length of the time period for which new varieties of the same product are available (purchase deadlines). Finally, sellers could offer long-term contracts that include the price and the frequency of purchases. Real world examples of such practices are not hard to find. For instance, Disney video tapes are usually marketed under purchase deadlines, and subscriptions to magazines include a price and a frequency. Moreover, some products can only be offered occasionally at a particular location. For instance, a live concert of Bruce Springsteen is available in Barcelona only from time to time.

4.2.1 Occasional purchasing periods

Suppose that the monopolist wishes to induce consumers to purchase every \( s \) units of time. In principle he could do that either by making the product available only at time \( s, 2s, ... \), or by setting a very high price for purchases made at other points in time. Suppose that the monopolist cannot refuse to serve a consumer at time \( ns \) just because she has not purchased at time \( (n-1)s \). In this case, the monopolist will be able to implement a price, \( p \), and a time interval between purchases, \( s \), provided:

\[
R(s) - p + U^* \geq e^{-rs} [R(2s) - p + U^*] \tag{17}
\]

In other words, the consumer purchases at time \( s \) only if it is not worthwhile to wait until the next trading period, \( 2s \). The gains from waiting have to do with the increase in the instantaneous utility, and the costs are due to the discounting. Since I only consider stationary policies, the continuation utility, \( U^* \), is given by:

\[
U^* = \frac{e^{-rs}}{1 - e^{-rs}} [R(s) - p] \tag{18}
\]

Plugging equation \( \text{??} \) into condition \( \text{??} \) we obtain the highest price that the monopolist can charge for a given frequency:

\[
p = \left(1 + e^{-rs}\right) R(s) - e^{-rs} R(2s) \tag{19}
\]

Thus, the optimal policy consists of choosing a pair \((p, s)\) that maximizes \( \text{??} \) subject to constraint \( \text{??} \). By restricting the timing of purchases the monopolist faces a more favorable trade-off between prices and frequency. The optimal value of \( s \), denoted by \( s^r \), is given by:
\[ r R(s^r) - \left(1 - e^{-rs^r}\right) R'(s^r) = e^{-rs^r} \left(1 - e^{-rs^r}\right) \left[2R'(2s^r) - R'(s^r)\right] + (20) \]

\[ +re^{-rs^r} \left(2 - e^{-rs^r}\right) \left[R(2s^r) - R(s^r)\right] \]

and the optimal price is given by condition ?? evaluated at \(s^r\).

In order to compare the outcome of the current game with the case in which the monopolists sets either a constant price (Section 3.1) and a sequence of prices (Section 4.1) we must turn to a particular example. Consider the following functional form:

\[ R(s) = M \left(1 - \frac{e^{-rs}}{1 - z}\right) \]

In this case we can actually compute the payoffs under the various pricing policies (See Appendix for details). The following table reports the results for the limiting case of \(z = 0\), and \(M = 100\).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(U_0)</td>
<td>25</td>
<td>25</td>
<td>16.1</td>
</tr>
<tr>
<td>(\Pi_0)</td>
<td>25</td>
<td>50</td>
<td>38.3</td>
</tr>
<tr>
<td>(W_0)</td>
<td>50</td>
<td>75</td>
<td>54.4</td>
</tr>
</tbody>
</table>

Columns (1) and (2) contain the payoffs under a constant price and a sequence of prices, respectively. Column (3) presents the payoffs under the stationary policy with restricted timing analyzed in this section. Comparison between columns (1) and (2) illustrates the discussion of the previous subsection: Setting a price sequence significantly increases the seller’s pay-off without necessarily hurting consumers. However, the seller’s ability to restrict the timing of purchases increases firm profits in comparison with the case of commitment to a constant price (columns (1) and (3)), but it hurts consumers. Nevertheless, total surplus is higher. This is because by restricting the timing of purchases the seller is able to induce more frequent consumption at a higher price. That is, \(s^o < s^r < s^c\). Finally, if we compare columns (2) and (3) we realize that restricting the timing of purchases

---

\(^{14}\)Payoffs turn out to be proportional to \(M\), thus setting \(M = 100\) is only a normalization. However, the choice of \(z\) is not at all irrelevant.
is Pareto dominated by the commitment to a sequence of prices. In other words, the seller’s ability to commit to trading exclusively at certain periods of time has only a modest impact on total surplus and the seller’s ability to appropriate rents. This is because the seller cannot refuse consumers that did not purchase the good in the previous trading period. Therefore, the price cannot be too high otherwise consumers will find it optimal to wait until the next trading period.

4.2.2 Contracting price and frequency

Clearly, the monopolist could implement the first best and appropriate the entire surplus if he can contract ex-ante both the price and the frequency of purchases. Subscription to magazines is one example of this type of contract. Other services such as house cleaning, maintenance of equipment, and so on, are sometimes marketed under contracts that include a price and a frequency, although usually the arrangement can be breached at no pecuniary cost. In particular, if the monopolist can commit to serving only those consumers that buy a contract that includes \((p, s)\), then the optimal contract consists of \(p = R(s^0)\) and \(s = s^0\). Notice that if breaching the contract involves no cost, consumers will prefer to purchase at the price \(R(s^0)\) at a lower frequency, and hence we are back to the case analyzed in the previous subsection.

5 Competition

So far I have only considered an homogeneous good produced by a single firm. Introducing product differentiation in a cyclical goods framework involve non-trivial modelling choices. In particular, the potential utility derived from the consumption of a particular variety may depend not only on the time elapsed since the last consumption episode, but also on which varieties have been consumed recently. More specifically, the relative valuation of two particular varieties may change after consuming one of them. For instance, after dining at the local Chinese restaurant a consumer may value relatively more an additional meal at the local Italian restaurant vis à vis the Chinese. That is, temporary satiation of the good may in fact be stronger for the variety that was actually consumed. This implies that consumers may pursue a diversified consumption pattern. If different varieties are produced by independent firms, then diversification is likely to have important implications
for their optimal pricing strategies.

In order to consider these issues let us embed the cyclical pattern of preferences into Salop’s circular market model. There are \( n \) equally distant locations in the unit circumference. In each location there is a single firm. Consumers are uniformly distributed over these locations (no consumer is located between two firms). Thus, this is a model of \( n \) cities scattered in a circumference. If a consumer purchases from the local firm at price \( p \) then she obtains an additional utility equal to \( R(s) - p \), where \( s \) is the time elapsed since the last consumption episode. Instead if she purchases from the clockwise neighboring firm she obtains \( R(s) - p - t \), where \( t \) can be interpreted the ‘transportation’ cost. Consuming from neighboring firms located counterclockwise\(^{15}\) or more distant locations is assumed to be prohibitively costly. Finally, consumers are heterogeneous with respect to the transportation costs. More specifically, \( t \) is uniformly distributed in the interval \([0, T]\).

We consider two extreme patterns of preference dynamics. In the benchmark case, consumer preferences are fixed. In other words, the relative valuation does not change with the history of purchases. In the second case, consumers randomly reallocate after each purchase. The interpretation is that consumers start up with certain preferences over all possible varieties. After each consumption episode, satiation is stronger for the variety that has been recently consumed and as well as for other similar varieties. As a result, consumers will only consider relatively distant varieties in the next purchase.

### 5.1 Fixed relative preferences

Let us first consider the case in which consumer location remains fixed over time. In order to focus on the effects of competition it seems reasonable to disregard the problems associated with limited commitment capacity and let firms to commit to a constant price.\(^{16}\)

In a symmetric equilibrium consumers purchase always from their local suppliers. Thus, their behavior can be summarized by \( \hat{s}(p) \), which is given by equation ??.

\(^{15}\)If we allow consumers to buy from both neighbouring firms then individual demand functions are convex and have a kink, and no symmetric price equilibria exist. Alternatively, we could have considered the two-firm Hotelling set up. The message would have been very similar, but the presentation would have become less transparent.

\(^{16}\)Also, for simplicity, at time 0 all consumers have just purchased one of the varieties, i.e., they have initially \( s = 0 \).
Let us denote by $p^*$ the symmetric equilibrium price. If a particular firm sets $p > p^*$, then it will sell only to those local consumers with $t \geq p - p^*$. Similarly, if $p \leq p^*$ then it attracts those consumers in the clockwise neighboring location with $t \leq p^* - p$. Therefore, given that other firms are playing $p^*$ the payoff function of a particular firm that charges $p$ can be written as:

$$
\Pi_0 = \frac{pe^{-r\hat{s}(p)}}{1 - e^{-r\hat{\alpha}(p)}} \left( 1 - \frac{p - p^*}{\bar{t}} \right) \text{if } p \geq p^*
$$

$$
= \frac{pe^{-r\hat{s}(p)}}{1 - e^{-r\hat{s}(p)}} + \frac{1}{\bar{t}} \int_0^{p^* - p} \frac{pe^{-r\hat{s}(p+t)}}{1 - e^{-r\hat{s}(p+t)}} dt \text{otherwise}
$$

In a symmetric equilibrium, the first order condition evaluated at $p = p^*$ must be zero:

$$
1 - \frac{p^*}{\bar{t}} - r p^* 1 - e^{-r\hat{s}(p^*)} \hat{s} dp = 0 \quad (21)
$$

If we compare (20) with equation (21) then it is clear that as $\bar{t}$ goes to infinity then $p^*$ goes to $p^c$ (the monopoly price under long-run commitment). Also, if $\bar{t}$ is equal to zero then $p^* = 0$. Finally, using second order conditions, $p^*$ increases with $t$. Hence, as usual, competition reduces prices below the joint profit maximizing level, $p^c$, because of the business stealing effect.

### 5.2 Variable relative preferences

Let us now consider the other extreme case. Suppose that after consuming variety $i$ consumer switches location and the probability of every other location is the same. This assumption captures the idea that relatively satiation is stronger for the variety consumed. Under variable relative preferences, competition has two different effects on prices: (a) business stealing, as firms may have incentives to undercut their neighbors’ prices, and (b) less frequent purchases from the same supplier, which implies that firms pay less attention to the effect of their prices on the long-run behavior of their current customers.

It is convenient to assume that $n$ is arbitrarily large so that we can disregard the probability that a consumer revisits the current supplier. In this
case, the effect (b) is magnified, as firms completely disregard the effect of their prices on the long-run behavior of their current customers.

Since consumers switch suppliers after each purchase, their behavior is given by \( \tilde{s}(p) \) as defined in Subsection 3.2, since they expect the current price to be effective in their next purchase only. Also, the firm takes as exogenous the long-run behavior of consumers in any location. Hence, given that other firms are playing \( p^* \) the payoff function of a particular firm that charges \( p \) can be written as:

\[
\Pi_0 = \frac{pe^{-r\tilde{s}(p)}}{1 - e^{-r\tilde{s}(p^*)}} \left( 1 - \frac{p - p^*}{\bar{t}} \right) \text{if } p \geq p^*
\]

\[
\Pi_0 = \frac{pe^{-r\tilde{s}(p)}}{1 - e^{-r\tilde{s}(p^*)}} + \frac{2}{\bar{t}} \int_0^{p^*-p} \frac{pe^{-r\tilde{s}(p+t)}}{1 - e^{-r\tilde{s}(p^*+t)}} dt \text{otherwise}
\]

The first order condition at \( p = p^* \) is given by:

\[
1 - \frac{2p^*}{\bar{t}} - rp^* \frac{d\tilde{s}(p^*)}{dp} = 0 \quad (22)
\]

Now the comparison between the equilibrium price and the joint profit maximizing price is less straightforward. Once again if \( \bar{t} = 0 \) then \( p^* = 0 \). Also, using second order conditions \( p^* \) increases with \( \bar{t} \). Finally, as \( \bar{t} \) goes to infinity then \( p^* \) goes to \( p^h \). By comparing equations ?? and ?? then we have that \( p^h > p^d > p^e \). In words, if the business stealing effect is not present (\( \bar{t} \) equal to infinity) then the only effect of competition comes from shortening firm-customer relationships and as a result, demand is less sensitive to prices and firms have lower incentives to cut prices in order to bring purchases forward. In this case, the equilibrium price is above the joint profit maximizing level (collusion would involve a price cut).

Thus, in general, the sign of \( (p^* - p^c) \) is ambiguous. If \( \bar{t} \) is very small, then the static competition effect dominates and \( p^* \) is below \( p^c \). However, if \( \bar{t} \) is relatively large, then again the shortening of the firm-customer relationship dominates and \( p^* \) is above \( p^c \).

6 References


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7 Appendix

7.1 Reputation equilibria

Let us consider the game of Section 3.2. Suppose expectations about future prices are formed according to the following rule: If $p_1 = q$ then consumers expect future prices to be equal to $q$, otherwise they expect future prices to be equal to $p^d$. 

25
If the seller conforms to expected behavior then the optimal timing is given as usual by equation ?? in the text:

$$R'(\hat{s}) - r[R(\hat{s}) - q]1 - e^{-rs} = 0$$

and hence consumers purchase the good according to $\hat{s}(q)$. If the firm deviates and sets $p \neq q$, then $s$ varies with $p$ according to:

$$dsdp = M(s)$$

Clearly, if the firm deviates then it faces the same incentives as in the Markov equilibrium, and hence it will optimally set $p_1 = p^d$. As a result, a price $q$ can be sustained in equilibrium if and only if:

$$\Pi(q) \equiv e^{-rs(q)}1 - e^{-rs}q \geq e^{-rs^d}1 - e^{-rs^d}p^d.$$  

Since $\Pi(q)$ is a continuous function that increases if $q < p^c$ and decreases if $q > p^c$ and given that $p^c < p^d$ (Proposition 2), then there exists a price, $p^l$, $0 < p^l < p^c$, such that any price $q$ in the interval $[p^l, p^d]$ can be sustained as a subgame perfect equilibrium.

### 7.2 Proof of Proposition 3

The monopolist chooses the price sequence $\{p_n\}$, $n = 1, 2, \ldots$ in order to maximize ??.. The first order condition with respect to $p_1$ is given by:

$$\frac{\partial \Pi_0}{\partial p_1} = e^{rs_1} [1 - r (p_1 + \Pi_1^*) G(s_1)] = 0$$

where $\Pi_1^*$ is the present value of profits after the first purchase, evaluated at the optimal solution. Similarly, the first order condition with respect to $p_2$ is given by:

$$\frac{\partial \Pi_0}{\partial p_2} = e^{r(s_1 + s_2)} [1 - r (p_1 + \Pi_1^*) G(s_1) - r (p_2 + \Pi_2^*) G(s_2)] = 0$$

Combining the first order conditions with respect to $p_1$ and $p_2$ we get:

$$p_2 + \Pi_2^* = p_2 + e^{-rs_3} (p_3 + \Pi_3^*) = 0$$

From the first order condition with respect to $p_3$ we derive that $p_3 + \Pi_3^* = 0$, and hence $p_2 = 0$. If we repeat the procedure with the other first order conditions we can show that $p_n = 0$, for all $n > 1$.  

26
7.3 Consumer welfare under various pricing policies

I wish to compare consumers’ utility when the monopolist commits to a constant price and when the monopolist commits to a sequence of prices. Plugging the first order condition into the payoff function, then consumers’ utility is a decreasing function of the optimal value of $s_1$:

$$U_0 = e^{-rs_1} \frac{R'(s_1)}{r}$$

Thus, consumers prefer the constant price policy over the variable price policy if and only if they find it optimal to make the first purchase sooner under the first pricing policy than under the second. Let us consider two examples. In the first, the constant price policy is strictly preferred to the variable price policy, while this result is reversed in the second example.

7.3.1 Example 1

Let us take $R(s)$ to be given by:

$$R(s) = M \left( 1 - \frac{e^{-rs}}{1 - z} \right)$$

where $1 > z > 0$. Notice that all the assumptions I made regarding $R(s)$ are satisfied (in particular, $L = \frac{zM}{1-z}$). Under the constant price policy, $s_1$ is given by equation. Thus, if we denote by $g^c = e^{-rs^c}$, then we can write equation as:

$$(1 - g^c)^2 (1 - 2g^c) = z \tag{23}$$

Similarly, under the variable price policy $s_1$ is given by equation. Let us denote by $z^o = e^{-rs^o}$ and $z^v = e^{-rs^v}$ where $s^v$ represents the timing of the first purchase under commitment to a price sequence. In this case we can write equation as follows:

$$g^v = \frac{1}{4} \left[ 1 - z + \left( 1 - \sqrt{z} \right)^2 \right] \tag{24}$$

We can immediately see from equations and that as $z$ goes to zero then both $g^c$ and $g^v$ go to one half, and as $z$ goes to one then both $g^c$ and $g^v$ go to zero. Moreover, for any $z \in (0, 1)$, $z^c > z^v$, i.e., consumers prefer the constant price over the variable price policy.
7.3.2 Example 2

Let us take $R(s)$ to be given by:

$$R(s) = M \left( 1 - e^{-2rs} \right)$$

where $1 > z > 0$. Notice that once again all the assumptions I made regarding $R(s)$ are satisfied.

In this case, using the same notation as in Example 1, equation ?? can be written as:

$$\left[ 1 - (g^c)^2 \right] - 2 (g^c)^2 (1 - g^c)(4 - 3g^c) = z \quad (25)$$

Taking the limit as $z$ goes to zero, then $g^c$ is given by:

$$\Psi(g^c) = 0$$

where

$$\Psi(g) \equiv 1 + g - 8g^2 + 6g^3$$

Notice that $\Psi(1) = 0$, but $g^c = 1$ is not the limit of any meaningful solution to equation ???. It can quickly be confirmed that there is a single solution to $\Psi(g^c) = 0$ such that $0 < g^c < 1$, and that $\Psi(g) > 0$ for all $g < g^c$.

Similarly, from equation ???

$$\lim_{z \to 0} g^v = \frac{1}{\sqrt{3}}$$

Finally, $\Psi\left(\frac{1}{\sqrt{3}}\right) > 0$. Thus, by continuity, if $z$ is not too high $g^v > g^c$, i.e., consumers prefer the variable over the constant pricing policy.