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Family Restrictions at Work

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Abstract

This paper analizes the discrimination that individuals face at work due to their commitment to unpaid care work. The formal model presents a parametrization of the discrimination that affects the individual's optimal labor market participation. The welfare of individuals with commitment to family duties is reduced for two different reasons: for not being able to participate as much in the labor market and thus receive a lower labor income, and for not being able to contribute as much to their family commitments. We compare the results for the female and male sections of the society and we illustrate the observed gender gaps in terms of labor market participation, income levels, and overall utility obtained. We find that even though the gender wage gap may be alleviated with reductions of the cost associated to unpaid care work, the gender utility gap will persist.

Keywords: discrimination, labor market, unpaid care work.
JEL classification: J7, J31

1 Introduction

The under-representation of women with respect to men in many professions at all levels and in all professions at some levels is a fact. Given that women represent about 50 per cent of the population it is reasonable to describe this situation as gender unbalanced. The demand for a balanced proportion of women in all professions and at all levels has been raised and its support has been increasing over time. This demand can be based on the claim that a world in which males and females are found in equal shares in all professions and at all levels would be

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optimal. But it also can be based on an equity claim: female and male should have the same professional opportunities. This paper analyzes one of the causes of the current gender unbalanced situation and it aims at finding mechanisms that may induce a change from the current unbalanced situation to a world in which males and females are found in more equal shares in all professions and at all levels.

Across the EU, the gender employment gap (the difference between the employment rates of men and women of working age: 20-64 years) was 10.8 percentage points in 2021, meaning that the proportion of men of working age in employment exceeded that of women by 10.8 percentage points. Women tend to work less hours, they are more likely to engage in low paid and informal work, and in partial time jobs. Maternity leaves have a long run negative effect on the participation of women in the labor force (Bertrand 2020 and Isen *et al.* 2017).

The disproportionate representation of women in low paid and informal work also contributes to the observed gender earnings gap. The gender earnings gap measures the impact of the three combined factors (the average hourly earnings, the monthly average number of hours paid, and the employment rate) on the average earnings of all women of working age compared with men. In 2018, the gender overall earnings gap was 36.2 % in the EU. Across Member States, the gender overall earnings gap varied significantly (from 20.4 % in Lithuania and Portugal to 44.2 % in Austria).

The current gender unbalanced situation can be explained by causes that are related to specific gender conditions of the supply and demand in the labor market. Regarding the causes that produce a low demand for women in some professions it is important to refer to different kinds of discrimination originated in the decisions made by the employers that are biased in favor of men relative to women. Some kinds of discrimination are based on the fact that most employers are men and thus it is possible that the men are more likely to like men as employees or coworkers (taste-based discrimination). Given that in the past the female labor force has been disproportionately low with respect to the male's one, employers have had more experience with male employees and thus have more information about the characteristics of male labor force (statistical discrimination). In addition, the possibility of maternity leaves increases the risk and the expected costs associated with hiring women relative to those of hiring men. These larger costs also induce employers to hire men rather than women (Bertrand and Mullainathan 2014, Goldin 2014, and Bertrand and Duflo 2017). The smaller labor demand for women also produces a reduction in their labor supply: since women have a lower probability of being selected, their relative benefits of pursuing the job search are smaller.

Social norms may be considered as an additional cause of both lower supply and demand of females in the labor market. The fact that it is more naturally and more generally accepted that males participate in the labor market than females is one such social norm that affects negatively women's participation in the labor market because it induces employers (mostly men) to be more reluctant to hire women and at the same time it induces women to be more reluctant to search for jobs (Farré and Vella 2013, Bertrand 2019, and Fernandez et al. 2004).

Female labor supply is also affected by the differences between male and female preferences and attitudes (Bertrand et al. 2010, Gneezy et al. 2003, and Iriberri and Rey-Biel 2019) and especially by the female preference for maternity or by their preference to dedicate time and effort to non-professional activities related to unpaid care work: caring for household members and doing domestic chores (Azmat and Ferrer 2017 and Kleven et al. 2019).

This paper focuses only on the factors that affect the female labor supply that are originated on the individual's commitment to provide unpaid care work, that is "*All unpaid services provided by individuals within a household or community for the benefit of its members, including care of persons and domestic work. Common examples include cooking, cleaning, collecting water and fuel, and looking after children, older persons, and persons with illness or disabilities. Voluntary community work that supports personal or household care, such as community kitchens or childcare, are also forms of unpaid care work*" as defined by the United Nations (2022). The United Nations report also claims that the provision of unpaid care work is basically done by women: "*Women and girls have disproportionate responsibility for unpaid care and domestic work; globally they spend three times as much time on this work as do men and boys. Unpaid care work is one of the main barriers preventing women from moving into paid employment and better quality jobs.*"

Unpaid care work is absolutely necessary for the wellbeing of the society (Folbre 2001). The care, nurture and education of children is the basis for the growth of the economy and the evolution of the society. The care of elders is becoming more important over time. Maternity is unconditionally needed to the survival of our societies. Thus it is of great importance to study the role of unpaid care work in the economy.

Unpaid care work takes up a great amount of time and its responsibility falls disproportionately on women (Samman *et al.* 2016) and the individuals, whether male or female, that assume this responsibility will experience a restriction in the quantity and quality of their labor supply. This restriction can be thought in terms of the amount of time that they can offer to their professional activities.

The decision of how much time one should devote to unpaid care work relative to professional work is complicated for most people since both activities are considered as very relevant or even necessary (Folbre 2001). This paper aims at analyzing the implications of such decision. By making the economic costs and benefits of care provision more visible we might be able to change the current gender gaps related to the labor market.

This paper describes a formal model of individual choice in which the choice variable is the amount of time that an individual decides to devote to the labor market. This decision affects the total amount of income that an individual may obtain and also the total amount of cost that an individual has to bear depending on her or his previous commitments to unpaid care work. We assume that different individuals have different propensions to commit to family care, thus the society exhibits a variety of costs associated to them. However, in the real world it is evident that in general women are very commited to such activities while men are much less committed. Thus women are expected to be more affected by the costs originated by the family commitments.

The results offer a specific explanation of the lower labor market participation of women, the salary gap that is observed in the labor market due to unpaid care work commitments, and it also shows the reduction in overall welfare suffered by the female sector of the society due to both: the lower income levels and the higher costs from family commitments. The results obtained also show that the effects of unpaid care work commitments cannot be avoided. And since they are indispensable for the wellbeing of the society, the only possible way to diminish the discrimination against women and to attain a more gender balanced labor market is to induce men to commit to take family responsabilities.

The next section introduces the formal model. Section 3 describes the optimal individual choices of labor participation. Section 4 analyzes the effects of discrimination in the society. Section 5 compares the effects of discrimination between two sections of the society: female and male. Finally, section 6 contains some concluding remarks.

2 The model

This model considers the choice of an individual about her or his participation in the labor market. Let $s \in [0, 1]$ denote the corresponding choice variable and it is to be interpreted as follows: larger values of s denote higher levels of labor market participation, which could be thought as time devoted to work but may include other considerations such as the quality of the time devoted to work, or the quality of the jobs that may be attained. And of course, higher values of the choice variable correspond to higher values of the individual labor income.

Individuals are characterized by their type. The type of an individual is a measure of her or his level of commitment to activities that are related to the labor market relative to the individual's commitment to unpaid care work. Let $t \in [0, 1]$ denote the individual type and it is to be interpreted as the amount of time that an individual can devote to professional activities that is free of the cost derived from family commitments. This implies that lower values of t refer to individuals with strong family commitments, and higher values of t correspond to individuals with few family commitments. For an individual of type t devoting an amount of time larger than t to the labor market implies a reduction of her or his commitment to family activities, and it represents a reduction of her or his welfare. In particular, if an individual of type t chooses to dedicate and amount of time $s = t$ to the labor market, this individual does not suffer any additional cost. However, if an individual of type t chooses to dedicate and amount of time $s > t$ to the labor market, this individual will suffer a cost derived from her or his family restrictions. This cost can be thought of as the amount of income that has to be devoted to pay someone else for the provision of care, or it can be thought as the loss produced due to the diminished care on other family members.

Individuals obtain their income from their participation in the labor market.

We assume that this income is increasing with the individual's level of participation in the labor market. The individual income is represented by $W(s) = \omega s$ where $\omega > 0$ denotes the maximal income that an individual may obtain when he or she decides a full time participation in the labor market, that is $s = 1$. When an individual participates in the labor market he or she may face a cost derived from a diminished dedication to the other important activities. We assume that this cost is represented by a convex function of the distance between the individual's type and the chosen level of dedication to labor market activities. Let $C(s) = \frac{\gamma}{2}(t - s)^2$ with $\gamma > 0$ denote the cost of labor market participation for an individual of type t. As explained before a choice of a labor market participation equal to the individual's type does not produce any extra welfare cost, however a deviation from it represents an increase in cost. This increase in cost is small for small deviations from the type, and it becomes increasingly large for larger deviations from the type.

The overall individual's welfare is measured by a utility function that combines the individual's income and the cost that he or she has to bear and it is represented by the following function:

$$U(s) = W(s) - C(s) = \omega s - \frac{\gamma}{2}(t - s)^2$$

Larger values of the cost parameter relative to the income parameter imply larger reductions of welfare. Thus $\frac{\gamma}{\omega}$ can be interpreted as a measure of the level of discrimination that an individual suffers due to her or his family commitments. Since the main tradeoff is determined by the relationship between the parameters ω and γ, without loss of generality we normalyze the wage to be $\omega = 1$.

Assumption 1: $\omega = 1$.

This implies that the interpretation of the parameter γ is the weight of the cost relative to the maximal wage and therefore we can consider γ as the *discrimination index* derived from unpaid care work commitments.

Definition 1: γ is the *discrimination index.*

We analyze the individual's optimal choice of labor market participacion as a function of its type and of the discrimination index γ. In particular we are interested in the effects that γ may have in the optimal individual choice and how it affects her or his labor income and total welfare. We also analyze the effects of the discrimination index on the overall society by considering the aggregate levels of labor market participation, income, and welfare that the society may obtain, and also the aggregate level of cost that the society has to bear. Finally, and most importantly we analyze these effects on two seggregated sections of the society: female and male. This distinction is important according to the empirical data since it is the female section of the society the one that bears most of costs derived from the family commitments or more generally from the unpaid care work. We thus compare the effects of discrimination between the female and male sections of the society.

3 The optimal individual choice

In order to find the optimal level of labor market participation for an individual of type $t \in [0, 1]$ we solve the maximization problem of her or his utility function with respect to the choice variable $s \in [0, 1]$. This result is stated in the next proposition.

Proposition 1: *The optimal level of labor market participation of an individual of type t is*

$$s^* (t, \gamma) = \begin{cases} \frac{1}{\gamma} + t & if \quad t \leq \frac{\gamma - 1}{\gamma} \\ 1 & if \quad t \geq \frac{\gamma - 1}{\gamma} \end{cases}$$

All proofs are relegated to the appendix. Notice that the optimal level of labor market participation for all types is positive, thus everyone chooses to devote some time to work. Individuals with larger types decide to devote more time to the labor market and only the highest types decide to fully participate in the labor market, that is $s^* (t, \gamma) = 1$.

Notice that $\frac{\gamma - 1}{\gamma}$ increases with γ. Thus, larger values of γ imply smaller proportions of individuals that choose a full labor market participation. For $0 < \gamma < 1$ we have that all individuals decide to fully participate in the labor market and obtain the maximal wage, and thus they decide to bear all the cost that is required for it. Even if in this case all types obtain the same income, they do not obtain the same utility, because each individual has to bear a different cost. Thus we also have female discrimination. However, full female participation in the labor market is not what we observe that happens in the real world.

Since we aim at explaining the existing gender unbalanced features of the real labor market for most of the paper we have to consider that $1 < \gamma$. In this case, we have that only some individuals, those with higher types, decide to fully participate in the labor market and obtain the maximal wage. Instead individuals with lower types opt for a partial labor market participation, which is exactly what the empirical results show. For most of the paper we assume that $1 < \gamma$ which implies that the commitment to unpaid work is considered as very important to all individuals relative to the labor income. This assumption is relaxed towards the end of the paper, when we consider possible policies that may alleviate the existing gender gaps in the real labor market.

Assumption 2: $\gamma > 1$

Proposition 1 highlights the twofold relevance of the discrimination in the individual's optimal decision about labor market participation: it determines which part of the society decides to participate full time in the labor market and it also explains the distribution of the part time jobs derived from the individuals' optimal choice.

On the one hand, the ratio $\frac{\gamma-1}{\gamma}$ characterizes all the individual types whose decision on how much to participate in the labor market is negatively affected by the cost bearing of family commitments. All types with values below $\frac{\gamma-1}{\gamma}$ opt for a partial labor market participacion and thus suffer from the effects of the discrimination because they are not able to obtain the full income. While all types with values above $\frac{\gamma-1}{\gamma}$ opt for a full labor market participacion and obtain the full income from it. Notice that larger levels of discrimination imply larger sets of types that decide on a partial labor market participacion.

On the other hand, the discrimination index also affects the level of participation in the labor market for those individuals that opt for partial labor market participation: their optimal choice of labor market participation decreases with γ. Thus, larger values of γ imply that most individuals decide to work less.

The gains and losses produced on the individuals' total welfare are represented by the individual's income, cost, and utility computed at the optimal level of labor market participation given by proposition 1 and they are stated in the next proposition.

Proposition 2: *The optimal income obtained by an individual of type t is*

$$W^*(t,\gamma) = \begin{cases} \frac{1}{\gamma} + t & if \quad t \leq \frac{\gamma-1}{\gamma} \\ 1 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

The optimal cost supported by an individual of type t is

$$C^*(t,\gamma) = \begin{cases} \frac{1}{2\gamma} & if \quad t \leq \frac{\gamma-1}{\gamma} \\ \frac{\gamma}{2}(1-t)^2 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

The optimal utility obtained by an individual of type t is

$$U^*(t,\gamma) = \begin{cases} \frac{1}{2\gamma} + t & if \quad t \leq \frac{\gamma-1}{\gamma} \\ 1 - \frac{\gamma}{2}(1-t)^2 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

Observe that the individual income and the individual utility received by all types are positive and they both increase with the type for those individuals that opt for a partial labor market participation, who in turn obtain only a fraction of the full income. Individuals with a larger type $\left(t \geq \frac{\gamma-1}{\gamma}\right)$ decide to fully participate in the labor market, they obtain the maximal wage, and their individual utility increases with their type. However this increase becomes smaller for larger values of t.

The individual cost supported by all types is also positive. The cost supported by the individuals that opt for a full labor market participation decreases with her or his type, and this reduction becomes smaller for larger types. And all those types that opt for a partial labor market participation end up supporting the exact same total amount of individual cost (see figure 1).

FIGURE 1 ABOUT HERE

Increases in the discrimination index γ produce decreases in the individual income, in the individual cost, and in the individual utility for those types that choose a part time. Notice that on the one hand, they decide to work less when the discrimination index increases, and that is why they obtain a lower income. However their individual cost also increases and overall it produces a large reduction in individual utility. The individual income for the larger types, those that choose a full time participation in the labor market, is not affected by increases in the cost parameter. However their individual cost increases and thus their individual utility also decreases. Therefore, increases in then value of γ imply lower utility for all types. The formal description of the comparative statics is included in the proof of the proposition contained in the appendix.

4 Discrimination in the society

Suppose that the individual types in the society are distributed according to a uniform probability distribution function over the support $[0,1]$. We compute the labor market participation of the society $T^*(\gamma)$, total income of the society $TW^*(\gamma)$, the total cost derived from labor participation $TC^*(\gamma)$, and the total utility obtained $TU^*(\gamma)$ as stated in the next proposition. Notice that since we have normalized the maximal wage to be 1 we now have that the measure of the labor market participation is equal to the measure of the total income.

Proposition 3: *The labor market participation, total income, total cost, and total utility for the society are:*

$$T^*(\gamma) = TW^*(\gamma) = \frac{\gamma^2 + 2\gamma - 1}{2\gamma^2}$$

$$TC^*(\gamma) = \frac{3\gamma - 2}{6\gamma^2}$$

$$TU^*(\gamma) = \frac{3\gamma^2 + 3\gamma - 1}{6\gamma^2}$$

We find that the labor market participation and total income decrease with γ because lower types choose to dedicate less time to work when its associated cost becomes more expensive. Total utility decreases with γ because an increase in the cost parameter reduces the individual utility for all types: lower types work less and higher types pay a higher cost.

The total cost may increase or decrease with γ depending on the value of the discrimination. Recall that the individual cost for lower types decreases with the cost parameter while the individual cost for higher types increases with the cost parameter. Overall we have that the total cost increases with γ if the discrimination is very low $\left(1 < \gamma < \frac{4}{3}\right)$ because in this case most individuals decide to fully participate in the labor market and thus their individual cost increases with γ. However the total cost decreases with γ if the discrimination is more severe $\left(\gamma > \frac{4}{3}\right)$ because in this case more individuals opt for a part time participation in the labor market and they reduce their participation when the cost parameter increases. This implies that the reduction in cost due to the reduction in part time labor market participation compensates the increase in cost that suffer the larger types. The reason is two fold: the cost that higher types have to pay is relatively small compared to that of lower types, and for large values of the discrimination index the proportion of types that decide to work full time is smaller.

5 Female and male types

Suppose that the types in the society are distributed according to a uniform probability distribution function over the support $[0, 1]$ as before. We assume that the female types are represented by those t such that $0 \leq t \leq \frac{1}{2}$ and the male types are represented by those t such that $\frac{1}{2} \leq t \leq 1$. The reason is that it is a fact that women are much more committed to undpaid care work than men.

We compute the labor market participation, the total income, the total cost, and the total utility evaluated at the optimal individual choice for female types ($T^{F*}(\gamma)$, $TW^{F*}(\gamma)$, $TC^{F*}(\gamma)$, $TU^{F*}(\gamma)$) and for male types ($T^{M*}(\gamma)$, $TW^{M*}(\gamma)$, $TC^{M*}(\gamma)$, $TU^{M*}(\gamma)$) separately. We analyze how these variables are affected by changes in the discrimination index γ. Then we compare the results obtained for each section of society in order to evaluate the extend of the effect of the gender discrimination in the labor market over participation, income, cost, and utility. The next proposition shows the results obtained for the aggregated economic variables corresponding to the female types. Notice that for this analysis we have to consider two cases depending on the value of the parameter γ. For $\gamma > 2$ we have all female types decide to work part time while for $\gamma < 2$ we have that some female types decide to work part time and some of them decide to work full time.

Proposition 4: *The labor participacion, total income, total cost, and total utility for female types are:*

$$T^{F*}(\gamma) = TW^{F*}(\gamma) = \begin{cases} \frac{2\gamma-1}{2\gamma^2} & if \quad \gamma \leq 2 \\ \frac{4+\gamma}{8\gamma} & if \quad \gamma \geq 2 \end{cases}$$

$$TC^{F*}(\gamma) = \begin{cases} \frac{24\gamma-16-\gamma^3}{48\gamma^2} & if \quad \gamma \leq 2 \\ \frac{1}{4\gamma} & if \quad \gamma \geq 2 \end{cases}$$

$$TU^{F*}(\gamma) = \begin{cases} \frac{\gamma^3+24\gamma-8}{48\gamma^2} & if \quad \gamma \leq 2 \\ \frac{2+\gamma}{8\gamma} & if \quad \gamma \geq 2 \end{cases}$$

Recall that for $\gamma \geq 2$ all female types work part time, while for $\gamma \leq 2$ some of them decide to work full time. In all cases, the total income and the total utility for female types decrease with the discrimination index, as we found in the overall society. For $\gamma \geq 2$ we have that the total cost for female types decreases with γ, because as we have seen before part time workers decrease their labor market participation when the cost parameter increases, thus in this case the total cost is reduced. For $\gamma \leq 2$ we have that some female types work full time. In this case the comparative statics for the female types resembles very much that of the society overall: the total cost for female types decreases with γ for relatively small values of the discrimination index ($\gamma < \overline{\gamma} < \frac{4}{3}$) and it increases with γ for larger values of the discrimination index. The only difference is that for values of γ such that $\overline{\gamma} < \gamma < \frac{4}{3}$ the total cost of the female types increases with γ while the total cost of the society decreases with γ. The reason is that for values of γ such that $\overline{\gamma} < \gamma < \frac{4}{3}$ the proportion of female types that decide to work full time relative to the female population is not large enough, and thus the total cost for the female types decreases with the cost of the majority because part time workers that decide to work less. While for the same parameter values, the proportion of full time workers in the society is large enough relative to the total population, and thus the total cost of the society increases.

Now we replicate the previous analysis for the section of society of male types. Again, for this analysis we have to consider two cases depending on the value of the parameter γ. For $\gamma < 2$ we have all male types decide to work full time while for $\gamma > 2$ we have that some male types decide to work part time and some of them decide to work full time.

Proposition 5: *The labor participacion, total income, total cost, and total utility for male types are:*

$$T^{M*}(\gamma) = TW^{M*}(\gamma) = \begin{cases} \frac{1}{2} & if \quad \gamma \leq 2 \\ \frac{3\gamma^2+4\gamma-4}{8\gamma^2} & if \quad \gamma \geq 2 \end{cases}$$

$$TC^{M*}(\gamma) = \begin{cases} \frac{\gamma}{48} & if \quad \gamma \leq 2 \\ \frac{3\gamma-4}{12\gamma^2} & if \quad \gamma \geq 2 \end{cases}$$

$$TU^{M*}(\gamma) = \begin{cases} \frac{24-\gamma}{48} & if \quad \gamma \leq 2 \\ \frac{9\gamma^2+6\gamma-4}{24\gamma^2} & if \quad \gamma \geq 2 \end{cases}$$

In this case we also have to consider two different situations. When discrimination is low ($\gamma \leq 2$) all male types decide to work full time. In this case the total income is not affected by changes in γ. Total cost increases with γ because all male types are working full time, and therefore their total utility decreases with γ. Of course, increases of γ beyond 2 imply that some male types decide to reduce their labor market participation.

For higher values of the discrimination index ($\gamma > 2$) some male types decide to work part time and in this case we have that total income and total utility for male types decrease with γ while total cost only increases with γ for large values of the discrimination index. The only difference with respect to the comparative statics of the society is found for values of γ such that $\frac{4}{3} < \gamma < \frac{8}{3}$. For these values the total cost of the male types increases with γ while the total cost of the society decreases with γ. The reason is that for values of γ such that $\frac{4}{3} < \gamma < \frac{8}{3}$ the proportion of male types that decide to work full time relative to the male population is large enough, and thus the total cost for the male types increases with the cost of the majority because full time workers have to pay a larger cost. While for the same parameter values, the proportion of full time workers in the society is not large enough relative to the total population, and thus the total cost of the society decreases.

Now we compare the aggregated economic variables found before for the two sections of society and the next proposition illustrates the extend of the discrimination suffered by female types overall in the labor market by stating the shares of total income, total cost, and total utility that correspond to the female types relative to the male types.

Proposition 6: *The shares of labor market participation, total income, total cost, and total utility for female types relative to male types decrease with* γ *and they are bound by:*

$$\frac{T^{F*}(\gamma)}{T^{M*}(\gamma)} = \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} \in \left[\frac{1}{3}, 1\right]$$

$$\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} \in [1, 7]$$

$$\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} \in \left[\frac{1}{3}, \frac{17}{23}\right]$$

Total income and total utility for the female types are always smaller than those of the male types because the ratios are always smaller than 1. Labor market participation and total income for the female types approaches those of the male types when discrimination becomes very low ($\gamma \to 1$). This implies that a reduction of the discrimination index produces a reduction on the gender wage gap and a more gender balanced labor market participation (see figure 2). The gender gap may even vanish completely if the discrimination index is low enough. However, with respect to the gender utility gap the implications are not as optimistic. Even though this gap is reduced with decreases of the discrimination index, it is not possible to eliminate it completely. That is, as long as there are costs associated to the unpaid care jobs, even if they are very small, there will be a gap in the utility obtained by the female and male sections of society. In fact the share of total utility for female types is always smaller than $\frac{3}{4}$ of the total utility for then males types. This is due to the fact that the total cost associated to the unpaid care jobs that is mostly beared by the female types represents a magnitude of many times that of the total cost beared by the male types for all values of the discrimination index (see figure 3).

FIGURES 2 AND 3 ABOUT HERE

Higher discrimination implies lower ratios of total income and total utility for the female types, but they are always above 1/3. At the same time the ratio of total cost beared by the female types decreases with the discrimination index, because lower types decide to work less when γ increases. The total cost beared by female and male are equal (and equal to zero) when the discrimination index reaches its maximum. However at this point the inequality in terms of total utility is maximal. This is due to the fact that maximal discrimination implies maximal inequality in terms of income because when the salary is so low relative to the cost everybody has an incentive to offer exactly its own type and pay no cost. This implies of course, that all male types obtain a higher income than female types.

Until now we have assumed that $\gamma > 1$, that is, the commitment to activities not related to the labor market is considered as very important to all individuals relative to the salary that they may obtain in the labor market. This assumption has allowed us to obtain significant results in terms of being able to explain the gender variations observed in the labor market. In particular we have characterized the section of the society that decide to work part time and those that decide to work full time. If instead we consider that the discrimination index is much lower, such that $0 < \gamma < 1$, we have that all individuals prefer to bear all the cost that allows them to obtain the maximal income. Therefore, they all decide a full participation in the labor market and we have that $s^* (t, \gamma) = 1$ for all types. This is not what we observe that happens in the real world, and this is the reason we have assumed $\gamma > 1$ for our main results that aim at a description of the real world facts. However, if instead of trying to explain the observed gender variation of the labor market participation in the real world we want to see how much of that gender variation can be reduced through a reduction in the discrimination index, we have to consider values of γ that go below 1. The next proposition shows that if we allow for lower indices of discrimination we find that the inequality between female and male types does not disappear.

Proposition 7: *If* $0 < \gamma < 1$ *we have that* $\frac{T^{F*}(\gamma)}{T^{M*}(\gamma)} = \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} = 1$, $\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} = 7$, and $\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = \frac{24-7\gamma}{24-\gamma} < 1$ *with* $\frac{\partial \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)}}{\partial \gamma} < 0$ *and* $\frac{TU^{F*}(0)}{TU^{M*}(0)} = 1$.

This proposition states that for lower values of the discrimination index we have that the gender wage gap disapears and the labor market participation becomes gender balanced. The gender utility gap decreases when γ becomes smaller, however it only disappears when $\gamma = 0$. Indeed, the burden of the personal cost derived from family commitments and other unpaid care jobs cannot be avoided, and even though when we push down the discrimination index we manage to balance the labor market participation and the salary gap, the utility difference between female and male remains significantly different. This is because our discrimination index relates the magnitudes of the cost derived from commitments to unpaid care jobs and the labor market salary. And as long as we have a positive cost we must have a positive discrimination.

6 Concluding remarks

This paper has analyzed an individual choice model about the labor market participation that includes a specific cost that individuals have to bear if they are commited to unpaid care work or other types of non professional activities. This analysis shows that the presence of this cost produces a great distorsion on the supply of labor in the society which in turn produces a wage gap and a much larger utility gap between the types that are committed to unpaid care work and those that are not.

The general conclusion is that since women devote more time to unpaid care work and thus they have to devote less time to professional activities, that implies that they obtain lower wages, more partial time jobs, less promotions, and so on. Thus the commitment to unpaid care work is one of the clear causes of the observed gender *wage gap* and *glass ceiling*.

Since it is a fact that unpaid care work is of vital importance for the proper development of the society, the discrimination of the individual types that are committed to them has to be considered a very relevant social problem. The aim of this paper is to make the economic costs and benefits of unpaid care provision more visible so that we might be able to change the currently gender unbalanced labor market.

The present paper has shown that there does not exist a solution that solves this problem completely, but we can think of ways to alleviate it that imply the reduction of the cost that individuals that are committed to family care have to bear. In particular, the increase in child benefits and the reduction of pre-primary school costs are expected to produce a positive effect. Gammage *et al.* (2019) show that increasing the government expenditure on pre-primary education by 1 percentage point of GDP can reduce the labor force participation gap by about 10 points. The introduction of flexible work schedules (Goldin 2014) and the provision of care by the public sector would also reduce the current effects of discrimination. The solutions proposed by the OECD for developing countries are in the line of increasing the offer of public services, infrastructures, social protection policies, and of promoting the shared responsibility within the household (OECD 2019).

However, it is not clear that these measures will completely correct the currently unbalanced gender labor market. In particular, extended maternity leave mandates have been found to increase female labor force participation at the cost of lower wages, less presence of women in high-profile ocupations, and they induce a more traditional division of tasks within the family (Farré 2016).

Thus it is very important to find a different way to compensate women for the costs induced by maternity issues.

Another proposal is based on the effects of increasing the incentives to share the jobs related to reproduction. Paternity leave has been implemented in many countries (Patnaik 2019) with the intention of inducing the share of the jobs related to reproduction but again the effects have been found to be from negligible to a small positive impact (Olivetti and Petrongolo 2017 and Gonzalez and Zoaby 2021). In particular this measure did not affect the parents' longer-term leave taking but only delayed higher-order births (Farré and González 2019). Thus its effectiveness to increase the long term involvement of fathers in the childcare and the household work has not been confirmed.

Explicit education about the relevance of the care and nurture issues are important to try to change the existing social values and social norms. Bertrand (2019) finds that childhood exposure to a nontraditional family (a working married mother, a married mother that is the primary breadwinner, or a non married mother) affects gender role attitudes in young adulthood. And regarding the transmission of values Farré and Vella (2013) show that a mother's attitudes have a statistically significant effect on those of her children, while Fernandez *et al.* (2004) indicate that wives of men whose mothers worked are themselves significantly more likely to work.

The analysis proposed in this paper can be extended in several ways. One important way is to include in the model some factors that affect the demand of labor (some of them are mentioned in the introduction) that produce an additional gender discrimination. Another important extension is to introduce imperfect information at hiring that affects the individual decision to look for a job and that has a negative effect specially on women in the labor market because of their lower expected probability of being hired. The present model can also be extended to include not only the individual choice, but also the family choice. In particular, the analysis of the family choices about the labor market could help us to better understand the possible effects of the paternity leave over time. These extensions would complement the current analysis and offer a more complete analysis of the gender discrimination of the labor market. Finally, the current model and its possible extensions could be replicated over time. With a repeated version of the basic model it would be possible to figure out the dynamics associated with the discrimination in the labor market.

Additional possible extensions are related to the generalization of some of the main assumptions of the present model. If instead of a linear income function we consider a more general income function of the time or effort devoted to work our results may be affected quantitatively but not qualitatively. Including a fixed cost in the model may induce some individual types to decide not to participate in the labor market, and this is a feature that may be desirable because it is what we observe in the real world. Adding a fixed wage in the current model may reduce the labor market participation and also the total cost beared by the society overall.

Finally, in order to obtain the aggregated economic variables we have assumed a uniform distribution of types. The main reason behind this choice is that it is not clear what kind of distribution of costs related to family commitments exist in the real world. If the empirical literature could obtain a specific shape for such a distribution it would be interesting to apply it to our model to obtain more accurate predictions for the effects of the discrimination in the labor market. Up to our knowlegde at this point there are some studies that have investigated this question but the results are still not conclusive (Beneria 1999).

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8 Appendix

Proof of Proposition 1:

The optimization of $U(s) = s - \frac{\gamma}{2}(t-s)^2$ with respect to $s \in [0,1]$ produces a first order condition given by $\frac{\partial U(s)}{\partial s} = 1 + \gamma(t-s) = 0$ which implies that the optimal value of s is given by $s^*(t,\gamma) = \frac{1}{\gamma} + t$. Notice that the second order condition $\frac{\partial^2 U(s)}{\partial s^2} = -\gamma < 0$ is always satisfied.

We have that $s^*(t,\gamma) = \frac{1}{\gamma} + t > 0$ for all types and for all parameter values. And we also have that $s^*(t,\gamma) = \frac{1}{\gamma} + t \leq 1$ if and only if $t \leq \frac{\gamma-1}{\gamma}$. This implies that for $t > \frac{\gamma-1}{\gamma}$ we must have $s^*(t,\gamma) = 1$.

Thus

$$s^*(t,\gamma) = \begin{cases} \frac{1}{\gamma} + t & if \quad t \leq \frac{\gamma-1}{\gamma} \\ 1 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases} . \blacktriangledown$$

Proof of proposition 2:

Given $s^*(t,\gamma) = \begin{cases} \frac{1}{\gamma} + t & if \quad t \leq \frac{\gamma-1}{\gamma} \\ 1 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$

we have that

$$W^*(t,\gamma) = \begin{cases} \frac{1}{\gamma} + t & if \quad t \leq \frac{\gamma-1}{\gamma} \\ 1 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$C^*(t,\gamma) = \begin{cases} \frac{1}{2\gamma} & if \quad t \leq \frac{\gamma-1}{\gamma} \\ \frac{\gamma}{2}(1-t)^2 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$U^*(t,\gamma) = \begin{cases} \frac{1}{2\gamma} + t & if \quad t \leq \frac{\gamma-1}{\gamma} \\ 1 - \frac{\gamma}{2}(1-t)^2 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

Comparative statics:

$$\frac{\partial W^*(t,\gamma)}{\partial t} = \begin{cases} 1 & if \quad t \leq \frac{\gamma-1}{\gamma} \\ 0 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$\frac{\partial C^*(t,\gamma)}{\partial t} = \begin{cases} 0 & if \quad t \leq \frac{\gamma-1}{\gamma} \\ -\gamma(1-t) & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$\frac{\partial U^*(t,\gamma)}{\partial t} = \begin{cases} 1 & if \quad t \leq \frac{\gamma-1}{\gamma} \\ \gamma(1-t) & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$\frac{\partial W^*(t,\gamma)}{\partial \gamma} = \begin{cases} -\frac{1}{\gamma^2} & if \quad t \leq \frac{\gamma-1}{\gamma} \\ 0 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$\frac{\partial C^*(t,\gamma)}{\partial \gamma} = \begin{cases} -\frac{1}{2\gamma^2} & if \quad t \leq \frac{\gamma-1}{\gamma} \\ \frac{1}{2}(1-t)^2 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases}$$

$$\frac{\partial U^*(t,\gamma)}{\partial \gamma} = \begin{cases} -\frac{1}{2\gamma^2} & if \quad t \leq \frac{\gamma-1}{\gamma} \\ -\frac{1}{2}(1-t)^2 & if \quad t \geq \frac{\gamma-1}{\gamma} \end{cases} . \blacktriangledown$$

Proof of Proposition 3:

$$T^*(\gamma) = TW^*(\gamma) = \int_0^1 W^*(t;\gamma)\,dt = \int_0^{\frac{\gamma-1}{\gamma}} \frac{1}{\gamma} + tdt + \int_{\frac{\gamma-1}{\gamma}}^1 dt =$$

$$\left[\frac{1}{\gamma}t + \frac{t^2}{2}\right]_0^{\frac{\gamma-1}{\gamma}} + [t]_{\frac{\gamma-1}{\gamma}}^1 = \frac{\gamma^2+2\gamma-1}{2\gamma^2}$$

$$TC^*(\gamma) = \int_0^1 C^*(t;\gamma)\,dt = \int_0^{\frac{\gamma-1}{\gamma}} \frac{1}{2\gamma}dt + \int_{\frac{\gamma-1}{\gamma}}^1 \frac{\gamma}{2}(1-t)^2\,dt =$$

$$\left[\frac{1}{2\gamma}t\right]_0^{\frac{\gamma-1}{\gamma}} + \left[-\frac{\gamma}{6}(1-t)^3\right]_{\frac{\gamma-1}{\gamma}}^1 = \frac{3\gamma-2}{6\gamma^2}$$

$$TU^*(\gamma) = 1 - \frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2 - \frac{1}{2\gamma^2}\left(\gamma - \frac{2}{3}\right) = \frac{3\gamma^2+3\gamma-1}{6\gamma^2}$$

Comparative statics:

$$\frac{\partial TW^*(\gamma)}{\partial \gamma} = \frac{1}{2}\frac{(2\gamma+2)2\gamma - 4\left(\gamma^2-1+2\gamma\right)}{\gamma^3} = \frac{-2(\gamma-1)}{\gamma^3} < 0$$

$$\frac{\partial^2 TW^*(\gamma)}{\partial \gamma^2} = \frac{4\gamma-6}{\gamma^4} > 0 \text{ iff } \gamma > \frac{3}{2}$$

$$\lim_{\gamma\to\infty} TW^*(\gamma) = \frac{1}{2}$$

$$\frac{\partial TC^*(\gamma)}{\partial \gamma} = \frac{1}{6}\frac{3\gamma - 2(3\gamma-2)}{\gamma^3} = \frac{4-3\gamma}{6\gamma^3} > 0 \text{ iff } \gamma < \frac{4}{3}$$

$\frac{\partial^2 TC^*(\gamma)}{\partial\gamma^2} = \frac{\gamma-2}{\gamma^4} > 0$ iff $\gamma > 2$
$\lim_{\gamma\to\infty} TC^*(\gamma) = 0$
$\frac{\partial TU^*(\gamma)}{\partial\gamma} = \frac{2-3\gamma}{6\gamma^3} < 0$ iff $\frac{2}{3} < \gamma$ which always holds since $\gamma > 1$.
$\frac{\partial^2 TU^*(\gamma)}{\partial\gamma^2} = \frac{\gamma-1}{\gamma^4} > 0$
$\lim_{\gamma\to\infty} TU^*(\gamma) = \frac{1}{2}$.▼

Proof of Proposition 4:
We consider two separate cases depending on whether $\gamma \gtrless 2$.
Case 1: For $\gamma \leq 2$ we have $\frac{\gamma-1}{\gamma} \leq \frac{1}{2}$ and:

$$T^{F*}(\gamma) = TW^{F*}(\gamma) = \int_0^{\frac{1}{2}} W^{F*}(t;\gamma) = \int_0^{\frac{\gamma-1}{\gamma}} \left(\frac{1}{\gamma} + t\right) dt + \int_{\frac{\gamma-1}{\gamma}}^{\frac{1}{2}} dt = \left[\frac{1}{\gamma}t + \frac{t^2}{2}\right]_0^{\frac{\gamma-1}{\gamma}} + [t]_{\frac{\gamma-1}{\gamma}}^{\frac{1}{2}} = \frac{2\gamma-1}{2\gamma^2}$$

$$TC^{F*}(\gamma) = \int_0^{\frac{1}{2}} C^{F*}(t;\gamma) = \int_0^{\frac{\gamma-1}{\gamma}} \frac{1}{2\gamma} dt + \int_{\frac{\gamma-1}{\gamma}}^{\frac{1}{2}} \frac{\gamma}{2}(1-t)^2 dt = \left[\frac{1}{2\gamma}t\right]_0^{\frac{\gamma-1}{\gamma}} + \left[-\frac{\gamma}{6}(1-t)^3\right]_{\frac{\gamma-1}{\gamma}}^{\frac{1}{2}} = \frac{24\gamma-16-\gamma^3}{48\gamma^2}$$

$$TU^{F*}(\gamma) = \frac{1}{2} - \frac{1}{2}\left(\frac{\gamma-1}{\gamma}\right)^2 - \frac{3\gamma-2}{6\gamma^2} + \frac{\gamma}{48} = \frac{\gamma^3+24\gamma-8}{48\gamma^2}$$

Comparative statics:
$\frac{\partial TW^{F*}(\gamma)}{\partial\gamma} = -\frac{\gamma-1}{\gamma^3} < 0$
$\frac{\partial^2 TW^{F*}(\gamma)}{\partial\gamma^2} = \frac{2\gamma-3}{\gamma^4} > 0$ iff $\gamma > \frac{3}{2}$
$\frac{\partial TC^{F*}(\gamma)}{\partial\gamma} = \frac{4-3\gamma}{6\gamma^3} - \frac{1}{48} > 0$ iff $\gamma < \overline{\gamma} < \frac{4}{3}$ because
if $\frac{4}{3} < \gamma$ we have that $\frac{\partial TC^{F*}(\gamma)}{\partial\gamma} = \frac{4-3\gamma}{6\gamma^3} - \frac{1}{48} < 0$
if $\frac{4}{3} > \gamma$ we have that $\frac{\partial TC^{F*}(\gamma)}{\partial\gamma} = \frac{4-3\gamma}{6\gamma^3} - \frac{1}{48} < 0$ iff $0 < \gamma^3 + 24\gamma - 32$
thus there is a $1 < \overline{\gamma} < \frac{4}{3}$ such that $\frac{\partial TC^{F*}(\gamma)}{\partial\gamma} = \frac{1}{2\gamma^2}\left(\frac{4}{3\gamma} - 1\right) - \frac{1}{48} < 0$ iff $\gamma > \overline{\gamma}$
$\frac{\partial^2 TC^{F*}(\gamma)}{\partial\gamma^2} = \frac{-12}{6\gamma^4} < 0$
$\frac{\partial TU^{F*}(\gamma)}{\partial\gamma} = \frac{2-3\gamma}{6\gamma^3} + \frac{1}{48} < 0$ iff $\gamma^3 - 24\gamma + 16 < 0$
since $\gamma^3 - 24\gamma + 16$ is a decreasing function of γ and it holds for $\gamma = 1$ then it also holds for all $1 < \gamma < 2$.
$\frac{\partial^2 TU^{F*}(\gamma)}{\partial\gamma^2} = \frac{\gamma-1}{\gamma^4} < 0$.

Case 2: For $\gamma \geq 2$ we have $\frac{\gamma-1}{\gamma} \geq \frac{1}{2}$ and:

$$TW^{F*}(\gamma) = \int_0^{\frac{1}{2}} W^{F*}(t;\gamma) = \int_0^{\frac{1}{2}} \left(\frac{1}{\gamma} + t\right) dt = \left[\frac{1}{\gamma}t + \frac{t^2}{2}\right]_0^{\frac{1}{2}} = \frac{1}{2}\left(\frac{1}{\gamma} + \frac{1}{4}\right) = \frac{4+\gamma}{8\gamma}$$

$$TC^{F*}(\gamma) = \int_0^{\frac{1}{2}} C^{F*}(t;\gamma) = \int_0^{\frac{1}{2}} \frac{1}{2\gamma} dt = \left[\frac{1}{2\gamma}t\right]_0^{\frac{1}{2}} = \frac{1}{4\gamma}$$

$$TU^{F*}(\gamma) = \frac{1}{2}\left(\frac{1}{\gamma} + \frac{1}{4}\right) - \frac{1}{4\gamma} = \frac{1}{4}\left(\frac{1}{2} + \frac{1}{\gamma}\right) = \frac{2+\gamma}{8\gamma}$$

Comparative statics:

$\frac{\partial TW^{F*}(\gamma)}{\partial \gamma} = -\frac{1}{2\gamma^2} < 0$
$\frac{\partial^2 TW^{F*}(\gamma)}{\partial \gamma^2} = \frac{1}{\gamma^3} > 0$
$\lim_{\gamma \to \infty} TW^{F*}(\gamma) = \frac{1}{8}$
$\frac{\partial TC^{F*}(\gamma)}{\partial \gamma} = -\frac{1}{4\gamma^2} < 0$
$\frac{\partial^2 TC^{F*}(\gamma)}{\partial \gamma^2} = \frac{1}{2\gamma^3} > 0$
$\lim_{\gamma \to \infty} TC^{F*}(\gamma) = 0$
$\frac{\partial TU^{F*}(\gamma)}{\partial \gamma} = -\frac{1}{4\gamma^2} < 0$
$\frac{\partial TU^{F*}(\gamma)}{\partial \gamma} = \frac{1}{2\gamma^3} > 0$
$\lim_{\gamma \to \infty} TU^{F*}(\gamma) = \frac{1}{8}$.

Overall we have that:
$T^{F*}(\gamma) = TW^{F*}(\gamma) = \begin{cases} \frac{2\gamma-1}{2\gamma^2} & if \quad \gamma \leq 2 \\ \frac{4+\gamma}{8\gamma} & if \quad \gamma \geq 2 \end{cases}$
$TC^{F*}(\gamma) = \begin{cases} \frac{24\gamma-16-\gamma^3}{48\gamma^2} & if \quad \gamma \leq 2 \\ \frac{1}{4\gamma} & if \quad \gamma \geq 2 \end{cases}$
$TU^{F*}(\gamma) = \begin{cases} \frac{\gamma^3+24\gamma-8}{48\gamma^2} & if \quad \gamma \leq 2 \\ \frac{2+\gamma}{8\gamma} & if \quad \gamma \geq 2 \end{cases}$
$\frac{\partial TW^{F*}(\gamma)}{\partial \gamma} < 0$
$\frac{\partial TC^{F*}(\gamma)}{\partial \gamma} < 0$ iff $\gamma > \overline{\gamma}$ for some $1 < \overline{\gamma} < \frac{4}{3}$
$\frac{\partial TU^{F*}(\gamma)}{\partial \gamma} < 0$.▼

Proof of Proposition 5:
We consider two separate cases depending on whether $\gamma \gtrless 2$.
Case 1: For $\gamma \leq 2$ we have $\frac{\gamma-1}{\gamma} \leq \frac{1}{2}$ and:
$T^{M*}(\gamma) = TW^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} W^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} dt = [t]_{\frac{1}{2}}^{1} = 1 - \frac{1}{2} = \frac{1}{2}$
$TC^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} C^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} \frac{\gamma}{2}(1-t)^2 dt = \left[-\frac{\gamma}{6}(1-t)^3\right]_{\frac{1}{2}}^{1} = \frac{\gamma}{48}$
$TU^{M*}(\gamma) = \frac{1}{2} - \frac{\gamma}{48} = \frac{24-\gamma}{48}$
Comparative statics:
$\frac{\partial TW^{M*}(\omega,\gamma)}{\partial \gamma} = 0$
$\frac{\partial TC^{M*}(\omega,\gamma)}{\partial \gamma} = \frac{1}{48} > 0$
$\frac{\partial TU^{M*}(\omega,\gamma)}{\partial \gamma} = -\frac{1}{48} < 0.$

Case 2: For $\gamma \geq 2$ we have $\frac{\gamma-1}{\gamma} \geq \frac{1}{2}$ and:
$TW^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} W^{M*}(\gamma) = \int_{\frac{1}{2}}^{\frac{\gamma-1}{\gamma}} \left(\frac{1}{\gamma} + t\right) dt + \int_{\frac{\gamma-1}{\gamma}}^{1} dt =$
$\left[\frac{1}{\gamma}t + \frac{t^2}{2}\right]_{\frac{1}{2}}^{\frac{\gamma-1}{\gamma}} + [t]_{\frac{\gamma-1}{\gamma}}^{1} = \frac{3\gamma^2+4\gamma-4}{8\gamma^2}$

$TC^{M*}(\gamma) = \int_{\frac{1}{2}}^{1} C^{M*}(\gamma) = \int_{\frac{1}{2}}^{\frac{\gamma-1}{\gamma}} \frac{1}{2\gamma} dt + \int_{\frac{\gamma-1}{\gamma}}^{1} \frac{\gamma}{2}(1-t)^2 dt =$

$\left[\frac{1}{2\gamma}t\right]_{\frac{1}{2}}^{\frac{\gamma-1}{\gamma}} + \left[-\frac{\gamma}{6}(1-t)^3\right]_{\frac{\gamma-1}{\gamma}}^{1} = \frac{3\gamma-4}{12\gamma^2}$

$TU^{M*}(\gamma) = \frac{3}{8} + \frac{\gamma-1}{2\gamma^2} - \frac{3\gamma-4}{12\gamma^2} = \frac{9\gamma^2+6\gamma-4}{24\gamma^2}$

Comparative statics:

$\frac{\partial TW^{M*}(\gamma)}{\partial\gamma} = \frac{2\gamma^2-4\gamma(\gamma-1)}{2\gamma^2} = \frac{2-\gamma}{2\gamma^3} < 0$ iff $\gamma > 2$ which always holds in this case.

$\frac{\partial^2 TW^{M*}(\gamma)}{\partial\gamma^2} = \frac{\gamma-3}{\gamma^4} > 0$ iff $\gamma > 3$

$\lim_{\gamma\to\infty} TW^{M*}(\gamma) = \frac{3}{8}$

$\frac{\partial TC^{M*}(\gamma)}{\partial\gamma} = \frac{3\gamma-2(3\gamma-4)}{12\gamma^3} = \frac{8-3\gamma}{12\gamma^3} < 0$ iff $\gamma > \frac{8}{3}$

$\frac{\partial^2 TC^{M*}(\gamma)}{\partial\gamma^2} = \frac{\gamma-4}{2\gamma^4} > 0$ iff $\gamma > 4$

$\lim_{\gamma\to\infty} TC^{M*}(\gamma) = 0$

$\frac{\partial TU^{M*}(\gamma)}{\partial\gamma} = \frac{4-3\gamma}{12\gamma^3} < 0$ iff $\gamma > \frac{4}{3}$ which always holds in this case.

$\frac{\partial^2 TU^{M*}(\gamma)}{\partial\gamma^2} = \frac{\gamma-2}{2\gamma^4} > 0$ iff $\gamma > 2$

$\lim_{\gamma\to\infty} TU^{M*}(\gamma) = \frac{3}{8}$

Overall we have that:

$$T^{M*}(\gamma) = TW^{M*}(\gamma) = \begin{cases} \frac{1}{2} & if \quad \gamma \le 2 \\ \frac{3\gamma^2+4\gamma-4}{8\gamma^2} & if \quad \gamma \ge 2 \end{cases}$$

$$TC^{M*}(\gamma) = \begin{cases} \frac{\gamma}{48} & if \quad \gamma \le 2 \\ \frac{3\gamma-4}{12\gamma^2} & if \quad \gamma \ge 2 \end{cases}$$

$$TU^{M*}(\gamma) = \begin{cases} \frac{24-\gamma}{48} & if \quad \gamma \le 2 \\ \frac{9\gamma^2+6\gamma-4}{24\gamma^2} & if \quad \gamma \ge 2 \end{cases}$$

and

$\frac{\partial TW^{M*}(\gamma)}{\partial\gamma} \lesseqqgtr 0$ if $\gamma \gtrless 2$

$\frac{\partial TC^{M*}(\gamma)}{\partial\gamma} < 0$ iff $\gamma > \frac{8}{3}$

$\frac{\partial TU^{M*}(\gamma)}{\partial\gamma} < 0$ for all γ.▼

Proof of Proposition 6:

We consider two separate cases depending on whether $\gamma \gtrless 2$.

Case 1: For $\gamma \le 2$ we have:

$\frac{T^{F*}(\gamma)}{T^{M*}(\gamma)} = \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} = \frac{\frac{2\gamma-1}{2\gamma^2}}{\frac{1}{2}} = \frac{2\gamma-1}{\gamma^2} \in \left[\frac{3}{4}, 1\right] < 1$

$\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} = \frac{\frac{24\gamma-16-\gamma^3}{48\gamma^2}}{\frac{\gamma}{48}} = \frac{24\gamma-16-\gamma^3}{\gamma^3} \in [3, 7]$

$\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = \frac{\frac{24\gamma-8+\gamma^3}{48\gamma^2}}{\frac{24-\gamma}{48}} = \frac{24\gamma-8+\gamma^3}{24\gamma^2-\gamma^3} \in \left[\frac{6}{11}, \frac{17}{23}\right]$

and

$\frac{\partial \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)}}{\partial\gamma} = \frac{2(1-\gamma)}{\gamma^3} < 0; \frac{TW^{F*}(1)}{TW^{M*}(1)} = 1; \frac{TW^{F*}(2)}{TW^{M*}(2)} = \frac{3}{4}$

$\frac{\partial \frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)}}{\partial\gamma} = \frac{48(1-\gamma)}{\gamma^4} < 0; \frac{TC^{F*}(1)}{TC^{M*}(1)} = 7; \frac{TC^{F*}(2)}{TC^{M*}(2)} = 3$

$\frac{TU^{F*}(1)}{TU^{M*}(1)} = \frac{17}{23}; \frac{TC^{F*}(2)}{TC^{M*}(2)} = \frac{6}{11}$

$\frac{\partial \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)}}{\partial \gamma} = 24\gamma \frac{\gamma^3+2\gamma^2-25\gamma+16}{(24\gamma^2-\gamma^3)^2} < 0$ iff $\gamma^3 + 2\gamma^2 - 25\gamma + 16 < 0$

Notice that for $\gamma = 1$ we have that $\gamma^3 + 2\gamma^2 - 25\gamma + 16 = -6 < 0$;
for $\gamma = 2$ we have that $\gamma^3 + 2\gamma^2 - 25\gamma + 16 = -18 < 0$
and in addition we have that $\gamma^3 + 2\gamma^2 - 25\gamma + 16$ decreases with γ for $1 < \gamma < 2$.

Case 2: For $\gamma \geq 2$ we have:

$\frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} = \frac{\frac{4+\gamma}{8\gamma}}{\frac{3\gamma^2+4\gamma-4}{8\gamma^2}} = \frac{4\gamma+\gamma^2}{3\gamma^2+4\gamma-4} \in \left[\frac{1}{3}, \frac{3}{4}\right]$

$\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} = \frac{\frac{1}{4\gamma}}{\frac{3\gamma-4}{12\gamma^2}} = \frac{3\gamma}{3\gamma-4} \in [1, 3]$

$\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = \frac{\frac{2+\gamma}{8\gamma}}{\frac{9\gamma^2+6\gamma-4}{24\gamma^2}} = \frac{6\gamma+3\gamma^2}{9\gamma^2+6\gamma-4} \in \left[\frac{1}{3}, \frac{6}{11}\right]$

and

$\frac{\partial \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)}}{\partial \gamma} = -4\frac{2+\gamma+\gamma^2}{(3\gamma^2+4\gamma-4)^2} < 0 < 0; \frac{TW^{F*}(2)}{TW^{M*}(2)} = \frac{3}{4}; \lim_{\gamma\to\infty} \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} = \frac{1}{3}$

$\frac{\partial \frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)}}{\partial \gamma} = \frac{-12}{(3\gamma-4)^2} < 0; \frac{TC^{F*}(2)}{TC^{M*}(2)} = 3; \lim_{\gamma\to\infty} \frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} = 1$

$\frac{\partial \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)}}{\partial \gamma} = -2\frac{2+2\gamma+3\gamma^2}{(9\gamma^2+6\gamma-4)^2} < 0; \frac{TU^{F*}(2)}{TU^{M*}(2)} = \frac{6}{11}; \lim_{\gamma\to\infty} \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = \frac{1}{3}$

Overall we have:

$\frac{T^{F*}(\gamma)}{T^{M*}(\gamma)} = \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} \in \left[\frac{1}{3}, 1\right]$

$\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} \in [1, 7]$

$\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} \in \left[\frac{1}{3}, \frac{17}{23}\right]$

and

$\frac{\partial \frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)}}{\partial \gamma} < 0; \frac{\partial \frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)}}{\partial \gamma} < 0; \frac{\partial \frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)}}{\partial \gamma} < 0.$▼

Proof of Proposition 7:

For $\gamma \leq 1$ we have that $s^*(t, \gamma) = 1$ and

$W^*(t, \gamma) = 1$

$C^*(t, \gamma) = \frac{\gamma}{2}(1-t)^2$

$U^*(t, \gamma) = 1 - \frac{\gamma}{2}(1-t)^2$.

For females types we have:

$T^{F*}(\gamma) = TW^{F*}(\gamma) = \int_0^{\frac{1}{2}} W^{F*}(t; \gamma) = \int_0^{\frac{1}{2}} dt = [t]_0^{\frac{1}{2}} = \frac{1}{2}$

$TC^{F*}(\gamma) = \int_0^{\frac{1}{2}} C^{F*}(t; \gamma) = \int_0^{\frac{1}{2}} \frac{\gamma}{2}(1-t)^2 dt = \left[-\frac{\gamma}{6}(1-t)^3\right]_0^{\frac{1}{2}} = \frac{7\gamma}{48}$

$TU^{F*}(\gamma) = \frac{1}{2} - \frac{7\gamma}{48} = \frac{24-7\gamma}{48}$

For male types we have:

$T^{M*}(\gamma) = TW^{M*}(\gamma) = \int_{\frac{1}{2}}^1 W^{M*}(\gamma) = \int_{\frac{1}{2}}^1 dt = [t]_{\frac{1}{2}}^1 = \frac{1}{2}$

$TC^{M*}(\gamma) = \int_{\frac{1}{2}}^1 C^{M*}(\gamma) = \int_{\frac{1}{2}}^1 \frac{\gamma}{2}(1-t)^2 dt = \left[-\frac{\gamma}{6}(1-t)^3\right]_{\frac{1}{2}}^1 = \frac{\gamma}{48}$

$TU^{M*}(\gamma) = \frac{1}{2} - \frac{\gamma}{48} = \frac{24-\gamma}{48}$
Comparing female and male types we obtain:
$\frac{TW^{F*}(\gamma)}{TW^{M*}(\gamma)} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$
$\frac{TC^{F*}(\gamma)}{TC^{M*}(\gamma)} = \frac{\frac{7\gamma}{48}}{\frac{\gamma}{48}} = 7$
$\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)} = \frac{\frac{24-7\gamma}{48}}{\frac{24-\gamma}{48}} = \frac{24-7\gamma}{24-\gamma}$
with $\frac{\frac{TU^{F*}(\gamma)}{TU^{M*}(\gamma)}}{\gamma} = \frac{-144}{(24-\gamma)^2} < 0$, $\frac{TU^{F*}(1)}{TU^{M*}(1)} = \frac{17}{23}$, and $\frac{TU^{F*}(0)}{TU^{M*}(0)} = 1$.▼

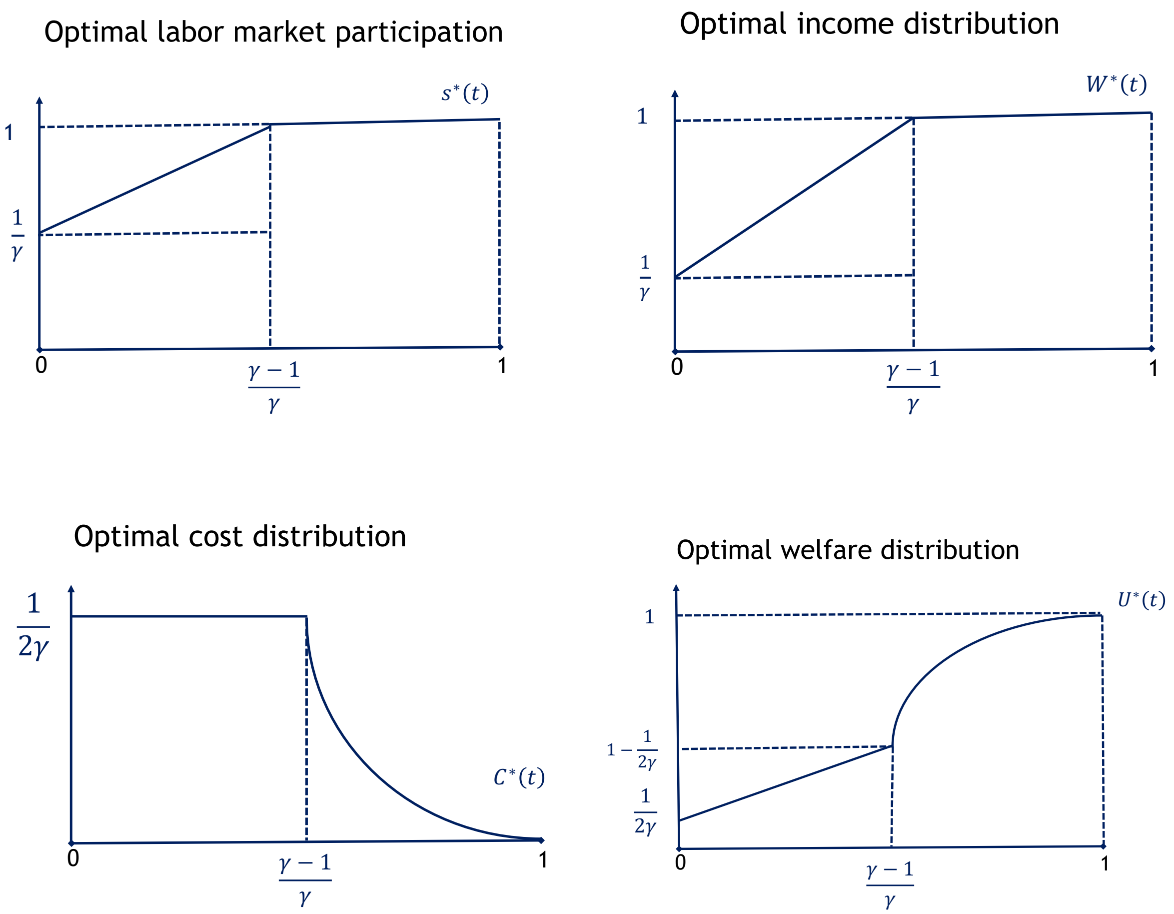

Figure 1: Optimal individual decisions.

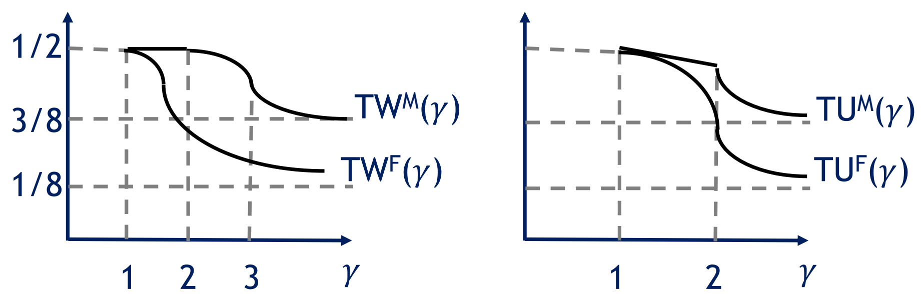

Figure 2: Comparing total income and total utility for female and male types.

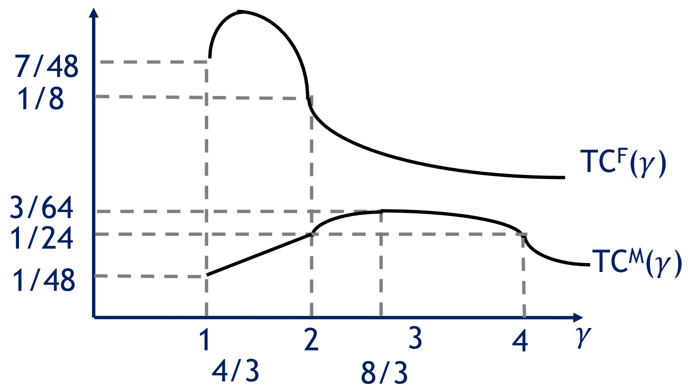

Figure 3: Comparing total cost for female and male types.