The Dynamic Provision of Product Diversity: under Duopoly

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Abstract

This paper builds a dynamic duopoly model to examine the provision of new varieties over time. Consumers experience temporary satiation, and hence higher consumption of the current variety lowers demand for future varieties. The equilibrium can be characterized by a combination of monopolistic pricing and nearly zero profits (competitive timing). In particular, if the cost of producing a new variety is not too low then firms tend to avoid head-to-head competition and set the short-run profit maximizing price. However, firms tend to introduce new varieties as soon as demand has grown sufficiently to cover costs. From a second best perspective, the equilibrium may exhibit excessive product diversity. However, if firms coordinate their frequency of new product introductions, then consumers are likely to be harmed. It is also shown that equilibrium prices are moderated by two factors. First, consumers' option value of waiting reduces their willingness to pay. Second, competition reduces firms' incentives to engage in intertemporal price discrimination.

Key words: temporary satiation, product diversity, dynamic duopoly, repeat purchases, endogenous timing.

JEL codes: L13

1 Introduction

There is an extensive literature on the effect of the market structure on the provision of product diversity. Most of these studies are built around the idea that a larger number of varieties available at a point in time allows for a better match with the preferences of heterogeneous consumers.[1] However, much less attention has been devoted to the temporal dimension of variety. In particular, in many product categories individual consumers have a taste for variety that can only be satisfied over time. This is the case, for example, of leisure goods -such as films, books, music recordings, plays, live musical performances, computer games, etc. In these markets, a consumer tends to purchase only one unit of a particular variety at a certain date, but engage in repeat purchases in the same product category as new varieties become available (*dynamic product diversity*). These markets usually exhibit two important characteristics. First, the rate at which consumers can absorb new varieties is clearly limited, and closely related to the amount of time they need to recover from a previous consumption episode (*temporary satiation*). Second, there is a high degree of synchronization between commercialization and consumption, as most purchases are typically made immediately following the release of a new variety.[2]

Temporary satiation is relevant in a broad range of markets that includes, but is not restricted to, the leisure goods mentioned above. The rate of recovery may vary quite markedly across consumers. Some people may be willing to attend a live performance of a rock band or go see a romantic comedy film every week, whereas others may prefer to wait much longer until they feel ready for a new experience of that sort. Indeed, observed differences in the frequencies of repeat purchases across consumers tend to reflect heterogeneous degree of exposure to temporary satiation (See, for instance, Hartmann and Viard, 2008).[3] Such *durability of consumption* has been shown to generate elasticity patterns similar to those observed in storable and durable goods, as higher current consumption implies lower future demand (Hartmann, 2006). Unfortunately, hard empirical evidence on the effect of temporary satiation in the markets of interest (in which new varieties of a

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[1] This principle is clearly reflected in some of the most popular "spacial" models; including Hotelling (1929), Salop (1979) or Chen and Riordan (2007). Other models, like Spence (1976) or Dixit-Stiglitz (1977), postulate a "representative consumer" with a preference for diversity. However, this is interpreted as an aggregation device rather than as a literal representation of individual behavior.

[2] The marketing literature calls the products exhibiting such synchronization "short life-cycle" products (See, for instance, Calantone et al.(2010))

[3] This paper shows that low frequency consumers can hardly take advantage of loyalty programs, because of the existing deadlines to redeem a reward. This indicates that the timing of purchases is largely determined by temporary satiation.

leisure good are introduced over time) does not abound. One exception is Einav (2010), who shows that box office revenues in the US would increase if film distributors did not cluster their releases so much. Thus, in a market with essentially fixed consumer prices, such a relation between aggregate revenues and the timing of releases clearly manifests the presence of temporary satiation.

The high degree of synchronization between commercialization and consumption can also be easily illustrated. For example, many artists and performers often present their new work in a specific location in a single event, or in several events taking place in consecutive days. Thus, consumption happens at the same time the new variety is released. Of course, this is an extreme example. In most industries, such synchronization is not perfect, but still very high. In the film industry, between 40 and 50 per cent of US box-office revenues are taken during a movie's first week and very few movies generate significant revenue beyond the sixth week.[4,5] In a similar vein, two thirds of the purchases of video games are made during the first three months after release.[6]

A natural question is whether different market structures provide a socially optimal level of dynamic product diversity at reasonable prices. In Caminal (2016) I analyzed a model in which a monopolist sequentially provides different varieties of a non-durable good to a customer base whose preferences are subject to temporary satiation. The welfare results depend on the balance between two opposing effects. On the one hand, if varieties are introduced very frequently then they become imperfect substitutes and the firm has incentives to engage in intertemporal price discrimination: raise prices above the short-run profit maximizing level, and sell each variety only to consumers with very high valuations (better preference matching). On the other hand, higher frequency also generates market expansion. I showed that under strong temporary satiation, better preference matching dominates (consumer surplus decreases) and the frequency of new product introductions is socially excessive.

Clearly, most of the markets for leisure goods, no matter how narrowly the product category is defined, are characterized by intense competition. Indeed, given the high degree of synchronization between commercialization and consumption and the relevance of temporary satiation, firms use the timing of their new product introductions as a crucial

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[4]www.boxofficemojo.com

[5]See also Corts [2001], Krider and Weinberg [1998], and Einav [2007] for stylized facts and common practices in the motion picture industry.

[6]This is a summary statistic privately provided by Ricard Gil, from the dataset used in (Gil and Warzynski, 2014).

strategic variable. It has been observed by the popular media that large Hollywood studios and publishing houses often play around with the timing of their releases, as a response to new information about the rival's moves.

In the current paper I study the effect of competition on both prices and the timing of new product introductions in markets characterized by temporary satiation and perfect synchronization between commercialization and consumption.[7] The model aims at capturing the main features of a specific segment of a leisure good; like, horror movies, historical novels, classical music concerts, etc.[8] More specifically, I consider a dynamic duopoly model in which two symmetric firms sequentially introduce new varieties of a non-durable good. After a consumption episode individual consumers stay out of the market for a random number of periods until they become active again. Hence, current demand depends negatively on past consumption. An important feature of the model is that consumers are ex-ante identical but they differ in their valuations of specific varieties. In such a framework it is possible to study product diversity and durability of consumption decisions and yet avoid Coasian price dynamics.

The model considers both dimensions of product differentiation, static and dynamic. The static dimension is represented by Hotelling's linear city model. From a dynamic point of view, and because of temporary satiation, two consecutive varieties become imperfect substitutes and, as a result, firms may have incentives to engage in intertemporal price discrimination.

Some of the results depend on the length of the time period, relative to other parameters of the model. If we were to consider a continuous time framework, then any new product introduction would be characterized by its release date and also by the date such variety ceases to be available. As such a time interval expands, the chances that varieties supplied by rival firms overlap increase, and hence consumers are more likely to choose among varieties that are simultaneously available (head-to-head competition). The length of the period in a discrete time framework (although taken as exogenous) can also be interpreted as the amount of time that suppliers need in order to take the new variety to the hands of the potential customers. Such amount of time is likely to vary across different industries.

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[7]Thus, firms do not have to worry about the potential cannibalization of existing varieties.

[8]As far as I know, the only paper with a similar goal is Krider and Weinberg (1998). They study the timing of movie releases in a duopoly model in which aggregate demand changes deterministically (there is peak season) and individual demands decay after the release. It is a one-shot timing game, as each firm only introduces a single variety.

I examine two extreme scenarios. First, I consider the case of sufficiently long periods, relative to the rate of demand recovery. In this case new product introductions are frequent, as at least one new variety is introduced every period. The analysis focuses mostly on the firms' incentives to synchronize or stagger their new product introductions, as well as the associated pricing policies. Unfortunately, such a large "discrete-time" friction blurs any insights on the determinants of the frequency of new product introductions. More specifically, I show that if the fixed cost of producing a new variety is sufficiently low then both firms introduce new products every period and prices are essentially determined by the degree of static product differentiation. Alternatively, if the fixed cost is not so low then firms have incentives to alternate their new product introductions. As a result, firms are temporary monopolists and, under certain conditions, they set the short-run profit maximizing price and sell their varieties to all active consumers (covered market). Consequently, if firms succeed at avoiding head-to-head competition, then prices tend to be higher. Nevertheless, firms' ability to extract surplus from consumers is limited by two factors. First, forward-looking consumers anticipate that current consumption reduces their expected future surplus, since they might be out of the market when future varieties are introduced. Hence, they require a sufficiently high current surplus (a sufficiently low price) in order to compensate for the option value of waiting. More specifically, the short-run profit maximizing price decreases with the intensity of temporary satiation and with the discount factor. Second, competition reduces firms' incentives to engage in intertemporal price discrimination. As mentioned above, a monopolist may have incentives to raise the price above the short-run profit maximizing level, in order to boost future demand and sell the next variety to these marginal consumers at higher prices. Under duopoly, and provided that firms alternate, it would the rival firm who would mostly benefit from such an increase in future demand, and hence firms tend to stick to the short-run profit maximizing price.

In the opposite extreme case, periods are arbitrarily short and new product introductions are infrequent, in the sense that the introduction of two consecutive varieties is separated by a large number of periods. For this case I build an equilibrium in which a new product is introduced every T periods, and the length of the cycle is endogenously determined. Firms are again temporary monopolists who set the short-run profit maximizing price, and the market is covered. Like before, their ability to extract surplus is also limited by the same two factors discussed above. The new important feature that

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appears in this environment is that profits are nearly zero. In this equilibrium, there is always a firm ready to introduce a new variety as soon as demand has grown enough to cover costs. Such a behavior is fuelled by the rival firm's threat to respond immediately to the failure to introduce a new variety by introducing its own in the following period, a magnified business-stealing effect. Thus, every cycle looks like a race in which the winner takes (almost) nothing. Consequently, when the-length-of-the-period is small the equilibrium is characterized by the unusual combination of monopolistic prices and (nearly) zero profits. Abusing the language, it could perhaps be argued that the Bertrand and Diamond paradoxes can be reconciled in this type of market. On the one hand, two is enough for competition (zero profits), like in the Bertrand paradox. On the other hand, competition between two (ex-ante) identical firms with constant marginal cost result in monopoly prices, like in the Diamond paradox.

From a total welfare point of view, the equilibrium may be inefficient and dynamic product diversity may be socially excessive. In particular, I consider a second best scenario in which the social planner can dictate the timing of new product introductions as well as prices, with the aim of maximizing total surplus, under the constraint that firms make non-negative profits (and consumers maximize their utility). Higher frequencies are not feasible, since they would result in lower consumption of each variety and hence negative profits. But the social planner may consider lower frequencies. In this case, there will be more consumption per variety (more time for demand to recover), but lower number of varieties per unit of time. If the discount rate is sufficiently low relative to the rate of demand recovery, then the first effect dominates and a reduction in the frequency of new product introductions raises total welfare.[9]

Independently of whether the equilibrium is or is not efficient, firms always have incentives to coordinate the timing of their new product introductions. By increasing the distance between two consecutive varieties, firms can raise consumers' willingness to pay (by lowering their option value of waiting), as well as the fraction of active consumers who are ready to purchase each variety. Thus, firms would set higher prices and sell larger quantities.[10] However, consumers are likely to be harmed. First, total welfare may decrease, and hence consumer surplus would fall even further (as profits increase). Second,

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[9]Notice that in the current framework the monopolistic equilibrium prices are not distortionary, since all active consumers purchase the new variety.

[10]The lower frequency of new product introductions is irrelevant, since profits per variety are almost zero in the absence of coordination.

even if total surplus increases as a result of less frequent new product introductions, consumers can still loose because of the higher prices (profits may increase more than total welfare). Thus, private and social interests are not aligned, and hence competition authorities should be concerned about any attempt by rival firms to coordinate the timing of their releases; even more so, if the authorities put a higher weight on consumer surplus.

As mentioned above, the present model captures some of the characteristics attributable to durable goods. It would not be hard to rephrase some of the elements of the model and describe every purchase as an investment in a stock that provides a flow of services that decreases over time. However, several features of the model seem more suitable for representing leisure goods rather than standard durable goods. In particular, different varieties introduced over time have independent characteristics in the present model, whereas new varieties of durable goods typically introduce quality improvements. Also, in the present model consumers have idiosyncratic preferences over individual varieties whereas consumers' willingness to pay for quality improvements are likely to be highly correlated over time. Indeed, the literature on durable goods has emphasized how the cumulative nature of innovation affects the frequency of new product introductions (Fishman and Rob, 2000), underinvestment in the durability of the good (Waldman, 1993), compatibility between old and new models (Ellison and Fudenberg, 2000) and, of course, pricing policies in the absence of commitment (Bulow, 1982; Stokey, 1982). In contrast, the present paper focuses on horizontal rather than vertical product differentiation and abstracts from Coasian price dynamics.

The next section presents the model. Section 3 examines the case of long time periods (frequent introduction of new products), whereas Section 4 focuses on short-time periods (infrequent new product introductions).

2   The model

Consider a market in which two firms sequentially provide different varieties of a non-durable good to a mass one of consumers. Time is discrete, and indexed by $t, t = 0, 1, 2, ...$, and horizon is infinite. All agents are forward looking and discount the future using the same discount factor, $\delta \in (0, 1)$.

In each period the two firms, A and B, choose whether or not to introduce a new variety. If they do, they incur a fixed cost γ but can produce any arbitrary amount of the variety at zero variable cost. Innovating firms announce their price p_t^j, $j = A, B$. I assume perfect

synchronization between commercialization and consumption; that is, the new variety is only available during the introductory period and is immediately consumed. In the introduction I mentioned some empirical observations that motivate such an assumption.

Consumers are ex ante identical but they differ in their valuation of specific varieties. Moreover, they are subject to temporary satiation. More specifically, a consumer can be either active or inactive in a particular period. An inactive consumer does not have a taste for the new variety(ies) and hence does not participate in the market. In contrast, if consumer i is active in a period t, a period in which either one or two new varieties are introduced, then she learns her valuation of the new variety(ies), $v_{it}^j, j = A, B$, that are negatively correlated: $v_{it}^A = R + 1 - z_{it}$, and $v_{it}^B = R + z_{it}$, where R is an exogenous parameter, and z_{it} is an independent realization of a random variable, uniformly distributed on $[0, 1]$. A consumer purchases at most one unit of a single variety. Thus, the preference representation corresponds to the standard Hotelling model with unit transportation cost equal to 1. If consumer j purchases one unit of the variety introduced by firm j then she obtains a payoff equal to $v_{it}^j - p_t^j$, and zero if she does not make any purchase.

The transition between the two states (active and inactive) are governed by the following probabilities. An active consumer who chooses not to purchase at t stays active at $t + 1$. Instead, if she purchases at t then she will be active at $t + 1$ with probability μ_1, and inactive with the complementary probability. If a consumer is inactive in period $t + s$ (s periods after the last consumption episode, $s > 0$) then she becomes active in period $t + s + 1$ with probability μ_2. I assume that $0 < \mu_1 \leq \mu_2 \leq 1$.

The timing of decisions is the following. At the beginning of period t, with full knowledge of the history of purchases, firms simultaneously choose whether or not to introduce a new variety. After observing new product introductions, firms simultaneously quote prices. Next, consumers observe these prices and whether or not they are active. If they are, then they learn their valuations and choose which variety to purchase, if any.

Thus, consumers are heterogeneous with respect to the current varieties, although the future looks exactly the same for all active consumers, on the one hand, and for all inactive consumers, on the other.[11] Hence, it is possible to analyze the effect of competition under static product differentiation and intertemporal substitutability, and yet avoid Coasian price dynamics.

The intensity of temporary satiation is reflected in parameters (μ_1, μ_2), with lower

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[11]Thus, the model overlooks brand-specific preferences.

values of these parameters indicating more persistent satiation. In different sections I will place alternative, additional restrictions on the values of μ_1, μ_2 in order to enhance tractability. Obviously, the impact of temporary satiation depends on how much agents discount the future (δ). The remaining parameters of the model are R (that measures consumers' average willingness to pay), γ (the fixed cost of introducing a new variety), and x_0, the fraction of active consumers in the initial period $(t = 0)$.

The goal of this paper is to study the effect of competition and hence I disregard any form of tacit collusion. Thus, it would be natural to focus on Markov strategies: prices and production decisions should in principle only depend on the unique state variable: the fraction of active consumers. However, it also seems reasonable to build equilibria in which firms alternate in the introduction of new products. Thus, I allow firms to use strategies that depend on the state variable as well as on the identity of the firm that introduced the last variety.

3 Long time periods

In this section I examine the case in which time periods are long enough so that at least one new product is introduced every period. Tractability will be enhanced by focusing on the case $\mu_2 = 1$. To simplify notation I let $\mu_1 = \mu$. Thus, consumers' temporary satiation lasts for at most one period, and the value of μ reflects the intensity of satiation in the period after a consumption episode. I will mostly focus on two types of equilibrium configurations. In the first one, both firms introduce a new variety every period (head-to-head competition). In the second one, only one variety is introduced every period, and firms alternate. At the end of the section I will also consider mixed equilibrium configurations.

3.1 Head-to-head competition

In this subsection I build an equilibrium in which, from period 1 onwards, both firms introduce a new product every period and charge prices $p_t^A = p_t^B = 1$. Moreover, all active consumers always make a purchase, and since prices are identical firms share the market equally. Thus, if we let x_t be the fraction of active consumers at the beginning of period t, each firm makes profits per period equal to $\pi_t^j = \frac{1}{2}x_t - \gamma$. Moreover, the law of

motion of the fraction of active consumers is:

$$x_{t+1} = 1 - (1 - \mu) \, x_t \tag{1}$$

That is, all inactive consumers in period t, $1 - x_t$, plus a fraction μ of all the consumers who purchased in t, μx_t, will be active at the beginning of period $t + 1$. Notice that the sequence $\{x_t\}_{t=0}^{\infty}$ converges to $\frac{1}{2-\mu}$, and oscillates around such steady state. In particular, if $x_t < \frac{1}{2-\mu}$, then $x_{t+1} > \frac{1}{2-\mu}$, and the other way around, for all $t > 0$. Also, for any initial condition, x_0, $x_t \geq \mu$, for all $t > 0$. See Figure 1 for an illustration.

An equilibrium with two varieties per period, for $t > 0$, is only feasible if the cost of introducing a new variety is sufficiently low. In particular, I consider Assumption $A1 : \gamma \leq \frac{\mu}{2}$. Since, for all $t > 0$, $x_t \geq \mu$, then Assumption $A1$ implies $\pi_t^j \geq 0$ for all $t > 0$. If we define $\overline{x}^d$ as the level of demand that generates zero profits, $\frac{1}{2}\overline{x}^d - \gamma = 0$, then assumption $A1$ is equivalent to $\overline{x}^d \leq \mu$.

Consumer choices. If consumers expect to purchase every period at a price equal to 1 (provided they are active), then the continuation value of an active consumer at the beginning of a period, before learning the realization of z_i, is equal to $U^* = R - \frac{1}{4} + \mu\delta U^* + (1 - \mu)\,\delta^2 U^*$. That is, $R - \frac{1}{4}$ is the current expected surplus. Also, with probability μ the consumer will be active again in period $t + 1$ and hence will enjoy a discounted continuation value of δU^*, and with probability $1 - \mu$ will be active in period $t + 2$ and thus obtain a discounted continuation value of $\delta^2 U^*$. Solving for U^*:

$$U^* = \frac{R - \frac{1}{4}}{1 - \mu\delta - (1 - \mu)\,\delta^2} \tag{2}$$

In order to assess consumers' reaction to firms' deviations, we need to consider the case in which only one variety is available in the current period and the decision of a particular consumer that is located in the Hotelling line in the most distant point from the supplier. Such consumer will purchase if and only if the payoff from purchasing, $R - 1 + \mu\delta U^* + (1 - \mu)\,\delta^2 U^*$, is higher or equal than the payoff from abstaining, δU^*. That is, the current surplus, $R - 1$, has to be weakly higher than the option value of waiting, $\delta\,(1 - \delta)\,(1 - \mu)\,U^*$. The reason this option value is positive is the following. By making a purchase in the current period a consumer may delay the next purchase one period with probability $(1 - \mu)$. This means that the discounted continuation value, δU^*, is reduced by a factor $(1 - \delta)\,(1 - \mu)$. Taking into account equation (2), this condition can be written as $R \geq \underline{R} = \frac{4 - \delta - 3\mu\delta - 3(1-\mu)\delta^2}{4(1-\delta)}$. I will maintain this assumption (Assumption

$A2$) throughout the section. That is, I focus on parameter values such that, given the relatively low prices arising in an equilibrium, the current surplus is always higher than the option value of waiting. Notice that $\underline{R}$ increases with δ and decreases with μ. That is, as temporary satiation exacerbates (lower μ) and the future becomes more valuable (higher δ), the minimum value of R required by Assumption $A2$ increases.[12]

Firms' pricing. Under Assumptions $A1$ and $A2$, since firm j charges $p_t^j = 1$ then, for all $t > 0$, x_{t+1} is independent of p_t^i, since all active consumers will purchase in period t, independently of p_t^i. Hence, firm i's pricing problem becomes static. And it is well known that, in the unique Nash equilibrium of the static Hotelling model with linear transportation costs and zero marginal cost, prices are equal to the unit transportation costs: $p_t^A = p_t^B = 1$.

Equilibrium. This discussion can be summarized as follows.

Proposition 1 *Under Assumptions $A1$ and $A2$, there exists a Markov perfect equilibrium in which, for all $t > 0$, both firms introduce a new product every period and charge prices equal to 1. All active consumers purchase one of the new varieties and, as a result, the fraction of active consumers converges to $\frac{1}{2-\mu}$.*

Thus, if the fixed cost is sufficiently low with respect to the rate of demand recovery, and consumers' willingness to pay is sufficiently high, there is an equilibrium in which both firms introduce a new variety every period, and hence they compete head-to-head. Moreover, temporary satiation has no effect on prices, but of course it does affect consumer behavior, and hence the dynamics of aggregate consumption.

First period. In order to complete the analysis, we need to consider the equilibrium outcome at $t = 0$. Clearly, if $x_0 > \mu$ then two varieties are also introduced in period 0. However, if $x_0 < \mu$ there are two possibilities: (i) if x_0 is very small then no firm will find it optimal to introduce a new product, and hence $x_1 = 1$; and (ii) for intermediate values of x_0 one firm will introduce a new variety. Clearly, the specific details are uninteresting and hence omitted.

3.2 Staggered new product introductions

In this subsection I focus on an equilibrium configuration in which, for all $t > 0$, one variety is introduced every period and firms alternate in its provision. In particular, firm

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[12]In a static Hotelling model the market is covered (all consumers purchase) if and only if $R \geq 1$. In our case, consumers demand a strictly positive current surplus and thus $\underline{R} > 1$.

A is a temporary monopolist in even periods and firm B in odd periods. Like before, it will be convenient to restrict attention to those parameter values for which firms find it optimal to sell to all active consumers.

In order to rule out that both firms introduce a new variety in the same period, we assume that the fixed cost is sufficiently high. In particular, I make Assumption $A3$: $\gamma > \frac{1}{2}$. From the analysis of the previous subsection, we know that if both firms choose to introduce a new product in a particular period then they will charge prices equal to one, split the market, and make individual profits equal to $\frac{1}{2}x_t - \gamma$. Hence, Assumption $A3$ excludes the possibility of two varieties introduced in the same period, for any value of x_t.

Consumer choices. Suppose that consumers expect that one new variety will be introduced every period, and such variety will be sold at a (constant) price p^m. Then, analogous to the previous subsection, the continuation value of an active consumer at the beginning of a period, before learning the realization of z_i, can be written as $U^* = R + \frac{1}{2} - p^m + \mu\delta U^* + (1 - \mu)\delta^2 U^*$. Solving for U^*:

$$U^* = \frac{R + \frac{1}{2} - p^m}{1 - \mu\delta - (1 - \mu)\delta^2} \tag{3}$$

The only difference between (2) and (3) is that in the latter the average transportation cost is $\frac{1}{2}$ instead of $\frac{1}{4}$. Once again, we need to pay attention to consumers' reaction to deviations from equilibrium prices. Thus, in any arbitrary period a fraction q of active consumers will purchase the current variety at a price p, provided a consumer located at a distance q of the active firm is indifferent between purchasing and abstaining:

$$R + 1 - q - p + \mu\delta U^* + (1 - \mu)\delta^2 U^* = \delta U^*, \tag{4}$$

where U^* is given by equation (3). Thus, if $q \in [0, 1]$, $q(p)$ is implicitly given by equation (4). Notice that $\frac{dq(p)}{dp} = -1$. Also, if the firm finds it optimal to serve all active consumers, then the equilibrium price, p^m, can be found by setting $q = 1, p = p^m$, in the above equation. If we solve for p^m, using equation (3), we obtain:

$$p^m = R - \frac{\delta(1 - \mu)}{2} \tag{5}$$

In the hypothetical case that consumers were myopic, then the most distant consumer would be willing to purchase the good if the price was below or equal to her willingness to pay, R. With forward-looking consumers, and because of temporary satiation, consumers

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demand a positive current surplus in order to compensate for the option value of waiting. Thus, the maximum price that the most distant consumer is willing to pay, p^m, is lower than R. Notice that the gap between R and p^m is the option value of waiting in equilibrium, and is proportional to the expected reduction in transportation costs, $\frac{1}{2}$, with the factor of proportionality being the product of $1 - \mu$ (the intensity of temporary satiation) and δ (the value of future payoffs).

Firms' pricing. The next step is to check that no firm wishes to deviate and set a price different from p^m. I will start by considering the maximization of static profits. Clearly, setting a price $p_t < p^m$ is never profitable since demand is invariant at x_t. However, a firm may benefit from a higher price. In particular, if $p_t = p^m + \varepsilon$ then then the new variety is sold to a fraction $1 - \varepsilon$ of active consumers. Setting $\varepsilon = 0$ is optimal if and only if $p^m \geq 1$, a condition which is implied by Assumption $A2$. Let us now consider the intertemporal effects of current prices. Suppose firm B sells to a fraction q_1 of active consumers in period 1, then $x_2 = 1 - x_1 q_1 (1 - \mu)$. Since for $t > 2$ firms are expected to follow the equilibrium strategy and sell to all active consumers then the law of motion will still be given by equation (1). As a result, $\frac{dx_3}{dq_1} = (1 - \mu)^2 x_1$. That is, raising the price above p^m implies lower q_1 and lower demand in $t = 3$, which is the next point in time the deviating firm is expected to introduce a new product. Hence, if the firm's continuation value does not decrease with x_3, then there exist no future gains of setting a price above p^m. Indeed, firm B's continuation value in period 3 can be written as: $\Pi(x_3) = \Lambda + \frac{p^m x_3}{1 - (1-\mu)^2 \delta^2}$, where Λ is a function of parameter values. Hence, $\frac{d\Pi(x_3)}{dq_1} > 0$. Consequently, under Assumption $A2$ firms find it optimal to set p^m, the price that maximizes short-run profits.

The timing of new product introductions. If firm B introduces a new product in period 1, then current profits are equal to $x_1 p^m - \gamma$, and $x_3 = \mu + (1 - \mu)^2 x_1$. Alternatively if firm B deviates and abstains from introducing a new product, then current profits are zero, and $x_3 = \mu$ (since $x_2 = 1$). Therefore, firm B will be willing to introduce a new product in period 1 provided that $x_1 p^m - \gamma + \delta^2 \Pi \left[\mu + (1 - \mu)^2 x_1 \right] \geq \delta^2 \Pi(\mu)$. If we define by $\overline{x}^m$ the value of x_1 that satisfies the previous condition with equality, then firm B introduces a new product in period 1 only if $x_1 \geq \overline{x}^m$, where

$$\overline{x}^m = \frac{\gamma}{p^m} \left[1 - \delta^2 (1 - \mu)^2 \right]. \tag{6}$$

Independently of x_0, $x_t \geq \mu$ for all $t > 0$. Hence, if $\overline{x}^m \leq \mu$ (Assumption $A4$) there is an equilibrium in which from period $t = 1$ onwards, one new product is introduced

in every period and firms alternate in its provision.[13] Like in the previous subsection, x_t converges to $\frac{1}{2-\mu}$ following an oscillatory trajectory.

Equilibrium. We need to check that Assumptions $A3$ and $A4$ are compatible, since $A3$ requires the fixed cost to be sufficiently high and $A5$ sufficiently low. These two conditions are compatible if and only if $R \geq \widetilde{R} = \frac{1}{2\mu} \left[1 - \delta^2 \left(1 - \mu \right)^2 \right] + \frac{\delta(1-\mu)}{2}$. Notice that if μ is high then this condition is implied by Assumption $A2$. However, if μ is low then this condition is stronger than Assumption $A2$. Hence, I will assume that $R \geq \max \left\{ \widetilde{R}, \underline{R} \right\}$ (Assumption $A5$).

The above discussion can be summarized as follows:

Proposition 2 *Under Assumptions $A3 - A5$, there exists a subgame perfect equilibrium in which, for $t > 0$, firms alternate in the introduction of new varieties, and charge the short-run profit maximizing price, p^m. All active consumers purchase the new variety, and as a result the fraction of active consumers converges to $\frac{1}{2-\mu}$.*

Thus, if the fixed cost is not too low then firms have incentives to avoid synchronization of new product introductions and avoid head-to-head competition. In fact, firms are temporary monopolists. However, their ability to extract surplus from consumers is limited by both consumers' and firms' intertemporal substitution. First, notice that if consumers were myopic the optimal price would be R, and firms would still sell to all active consumers. In contrast, when firms face forward-looking consumers the optimal price is $p^m < R$ because consumers require a strictly positive current surplus in order to compensate for the option value of waiting. Second, given that current prices can affect future demand, and since alternating firms do not internalize the effect of current prices on their rivals' demand, prices tend to be lower than in the case all varieties are provided by the same firm. Hence, in a duopoly market with staggered provision of new varieties, firms have less incentives to engage in intertemporal price discrimination. In other words, even though firms do not compete head-to-head there exists some kind of "intertemporal" price competition.

In order to illustrate the latter claim, it will be useful to compare the above equilibrium price with the optimal price of a single firm that introduces a new product every period. In particular, we can examine the conditions under which, a single monopolist optimally sets a price equal to p^m (given by equation (5)) and sells new varieties to all active consumers.

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[13]If $x_0 \geq \overline{x}^m$ then firm A also introduces a new product in period 0, otherwise, there is no product introduction and $x_1 = 1$.

In this case, the continuation value of the firm at the beginning of period 2 can be written as: $\Pi(x_2) = \Phi + \frac{p^m x_2}{1-(1-\mu)\delta}$, where Φ is a function of parameters. Also, $\frac{dx_2}{dq_1} = -(1-\mu)x_1$. Hence, in this case the effect of a higher price on future demand is positive.[14] Thus, the optimal price in period 1, p, is the solution to maximize $\{x_1 p q(p) - \gamma + \delta\Pi(x_2)\}$. The solution is $p = p^m$ and $q = 1$ provided:

$$1 - p^m + \frac{p^m \delta (1-\mu)}{1 - \delta(1-\mu)} \leq 0. \tag{7}$$

We need to consider two different cases. First, if $\delta(1-\mu) < \frac{1}{2}$, then condition (7) is equivalent to $p^m \geq \frac{1-\delta(1-\mu)}{1-2\delta(1-\mu)} > 1$. Thus, if R is such that $1 \leq p^m < \frac{1-\delta(1-\mu)}{1-2\delta(1-\mu)}$, then duopolists find it optimal to cover the market but a monopolist would rather set a price above p^m and sell the variety only to a fraction of active consumers. Second, if $\delta(1-\mu) > \frac{1}{2}$ then for all $p^m \geq 1$ condition (7) fails and hence the monopolists finds it optimal to set a price above the duopoly equilibrium.

Summarizing, when firms alternate in their introduction of new products prices tend to be lower than in the case varieties are provided by the same firm. The reason is that alternating firms disregard the positive consequences of higher prices on next period demand. Hence, the presence of two firms, even though they are temporary monopolist, moderates prices.

3.3 Discussion: Other patterns of new product introductions

Assumptions $A2$ and $A5$ guarantee that firms have incentives to sell to all active consumers. These assumptions drastically enhance the tractability of the model. If we were to relax these assumptions, and allow for equilibria in which only a fraction of active consumers purchase the good, then both consumers and firms' optimization problems would become quite cumbersome. In particular, small changes in prices would affect the future path of x_t. Explicit consideration of such intertemporal margin in a discrete time model would imply to renounce to analytical solutions. By assuming $A2$ and $A5$ we shut down this margin, which highly simplifies the analysis. I believe that the cost in terms of economic insight is moderate.

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[14]Such intertemporal effect on monopoly prices was studied in detail in Caminal (2016). In particular, in a set up with a smooth demand function, it is shown that a monopoly firm finds it optimal to set prices above the short-run profit maximizing level. A small increase with respect to the short-run profit maximizing level generates second order losses. However, consumers excluded by the higher current price are more likely to be willing to pay a higher price for future varieties, which implies a first order gain.

Assumptions $A1, A3$, and $A4$ are sufficient conditions on the value of the fixed cost to generate stationary equilibrium paths; that is, from $t = 1$ onwards, consumers always enjoy either one variety or two. We can now relax these assumptions and consider intermediate values of the fixed cost: $\gamma \in \left[\frac{\mu}{2}, \frac{1}{2}\right]$. We need to separately consider two alternative cases. First, if $\gamma \in \left(\frac{\mu}{2}, \frac{1}{2(2-\mu)}\right)$ the fraction of active consumers that involves zero duopoly profits, $\overline{x}^d = 2\gamma$, belongs to the interval $\left(\mu, \frac{1}{2-\mu}\right)$. In this case, if $x_0 \in \left[\overline{x}^d, 1 - (1-\mu)\overline{x}^d\right]$ then both firms introduce new varieties in all periods, including $t = 0$ (See Figure 2a). Alternatively, if $x_0 \notin \left[\overline{x}^d, 1 - (1-\mu)\overline{x}^d\right]$ other patterns can arise. In particular, one and two varieties per period can alternate during the initial periods, but in the medium and long-run the equilibrium converges to a pattern where two varieties are introduced every period (See Figure 2b). That is, initial conditions can delay the permanent provision of two varieties per period, but eventually the equilibrium looks like in Proposition 1. Second if $\gamma \in \left[\frac{1}{2(2-\mu)}, \frac{1}{2}\right]$, then $\overline{x}^d \in \left[\frac{1}{2-\mu}, 1\right]$ and $\overline{x}^m$ can still be higher or lower than μ. In this case, if the equilibrium converges to a constant pattern, then in the long-run only one variety per period is provided and hence the equilibrium looks like in Proposition 2. For instance, Figure 3a illustrates the possibility that after an initial period with two new varieties, only one variety is provided for all $t > 0$. Of course, depending on parameter values, the transition could be longer than one period. Finally, the equilibrium may not converge to a constant pattern of new product introductions. For instance, Figure 3b suggests that along the equilibrium path periods with two varieties may alternate with periods with no product introduction.[15]

Summarizing, if the fixed cost takes an intermediate value, and if the equilibrium converges to a constant pattern of new product introductions converges, then in the medium and long-run the equilibrium looks like either Proposition 1 or 2. However, the system may not converge and exhibit a permanent cyclical pattern.

4 Short time periods

In this section I examine the case of arbitrarily short time periods. In this case, the "discrete-time" friction is negligible and hence the timing of new product introductions can be studied more transparently. In this context, new product introductions are relatively infrequent in the sense that after the consumption of a variety, it takes many periods

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[15]Such an equilibrium configuration arises if γ is lower, but sufficiently close, to $\frac{1}{2}$, and Assumption $A4$ does not hold.

for demand to recover and reach a level that renders the introduction of a new variety profitable. In order to enhance tractability I specialize the model and assume that $\mu_1 = \mu_2 = \mu > 0$. The assumption of short time periods implies that μ (the rate of demand recovery per period) is arbitrarily small and δ is arbitrarily close to 1. However, no restriction is placed on the actual time distance between two consecutive varieties. As discussed below, some of the welfare results depend on the balance between discounting and demand recovery: i.e., on the ratio $\frac{\mu}{1-\delta}$.

If a new variety is introduced in period t, and provided that all active consumers purchase such variety, then $x_{t+1} = \mu$, independently of x_t. As long as no new variety is introduced in $t+1$ then $x_{t+2} = \mu + (1 - \mu)\mu$. By iteration, the fraction of active consumers T periods after the last product introduction is

$$\alpha_T = 1 - (1 - \mu)^T. \tag{8}$$

Note that α_T increases with T and converges to 1 as T goes to infinity. Indeed, α_T can also be interpreted as the probability that a consumer who purchased the last product introduction is active again T periods later.

I focus on equilibria in which one firm introduces a new product as soon as demand has grown sufficiently to generate enough revenue to cover costs. Thus, active firms make an amount of profits that is small but (generically) strictly positive. Since current prices can affect future demand, such intertemporal effect, together with the endogenous timing of new product introductions, generate complex strategic incentives. In order to keep the presentation simple I focus on equilibria in which along the equilibrium path all active consumers purchase the new variety. First, I present an equilibrium in which only one firm is active along the equilibrium path. The characterization of such type of equilibrium is relatively straightforward. Next, I discuss a symmetric equilibrium in which both firms alternate in the introduction of new products, which is probably a more natural representation of competition. Both types of equilibrium look essentially identical, but in the latter case firms' incentives to engage in intertemporal price discrimination are enhanced and the conditions for the existence of such equilibrium are more restrictive.

4.1 One active firm

Along the stationary equilibrium path a new variety is introduced every T periods, and all varieties are provided by the same firm (call it firm A), and sold at a constant price, p^m. Moreover, varieties are purchased by all active consumers.

Consumer choices. In a period in which a new variety is introduced, an active consumer will find it optimal to purchase the good depending on the price charged, p, and the realization of the preference shock, z. If we let U^* be the continuation value of a consumer at the beginning of a period in which a new product is introduced (before learning the preference shock z), then if the consumer chooses not to purchase her discounted continuation value is $\delta^T U^*$. Alternatively, if the consumer is located at a distance q from the seller and purchases one unit of the good at price p, then she gets $R + 1 - q - p + \alpha_T \delta^T U^* + (1 - \alpha_T) \alpha_T \delta^{2T} U^* + (1 - \alpha_T)^2 \alpha_T \delta^{3T} U^* + \ldots$Hence a consumer located at q will be indifferent between purchasing and not purchasing if and only if

$$R + 1 - q - p = \frac{(1 - \mu)^T (1 - \delta^T)}{1 - (1 - \mu)^T \delta^T} \delta^T U^* \tag{9}$$

The right hand side of (9) represents the option value of waiting. Hence, a consumer located at q will be willing to purchase the new variety at the price p only if she obtains a sufficiently high current surplus that compensates for the option value of waiting.

In such a stationary scenario, the continuation value of an active consumer is $U^* = R + \frac{1}{2} - p^m + \alpha_T \delta^T U^* + (1 - \alpha_T) \alpha_T \delta^{2T} U^* + (1 - \alpha_T)^2 \alpha_T \delta^{3T} U^* \ldots$That is,

$$U^* = \frac{1 - (1 - \mu)^T \delta^T}{(1 - \delta^T)} \left(R + \frac{1}{2} - p^m \right) \tag{10}$$

Clearly, U^* decreases with p^m and T and increases with μ (through α^T) and δ.[16] Thus, if the rate of demand recovery, μ, increases, then the probability of being active when future varieties are introduced increases, and hence U^* also increases.

If $q \in [0, 1]$, then using (10) to rewrite (9) the fraction of active consumers willing to purchase the good is given by:

$$q = R + 1 - (1 - \mu)^T \delta^T \left(R + \frac{1}{2} - p^m \right) - p \tag{11}$$

Hence, $\frac{dq}{dp} = -1$.

Optimal pricing in a covered market. Suppose that firm A always sets the maximum price that induces all active consumers to purchase the new variety. Such a price can be found by evaluating equation (11) at $q = 1$ and $p = p^m$, and solving for p^m:

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[16]Equations (9) and (10) are similar to (4) and (3) in section 3.2, respectively, but do not coincide for $T = 1$. The reason is that μ_2 was assumed to be equal to 1 in Section 3.2., and equal to μ_1, which is lower than 1, in this section.

$$p^m(T) = R - \frac{\delta^T(1 - \alpha_T)}{2\left[1 - (1 - \alpha_T)\delta^T\right]} \tag{12}$$

Note that $p^m(T)$ increases with T, and converges to R as T goes to infinity. I will argue below that, under certain conditions, $p^m(T)$ is indeed the equilibrium price.

The timing of new product introductions. If firm A charges a price $p^m(T)$ for every new variety, then we can denote the fraction of active consumers that involves zero profits by $\overline{x}^m(T)$, which is given by

$$\overline{x}^m(T) = \frac{\gamma}{p^m(T)}. \tag{13}$$

In order to specify the Markov strategies, it will be useful to introduce the following sequence: $y_{T+n} = \mu + (1 - \mu)y_{T+n-1}, n > 0, y_T = \overline{x}^m(T)$. That is, if firm A fails to introduce a new variety when the fraction of active consumers is equal to $\overline{x}^m(T)$ then, in the next period, such state variable will be equal to $y_{T+1} = \mu + (1 - \mu)\overline{x}^m(T)$. Other values of y_{T+n} are constructed following the same logic. Using this notation I can now define the following intervals: $I_{T+n} = [y_{T+n}, y_{T+n+1})$. Note that any value of x in the interval $[\overline{x}(T), 1)$ belongs to one and only one $I_{T+n}, n \geq 0$. Moreover, since μ is arbitrarily small, the length of I_{T+n} is also arbitrarily small.

In equilibrium firm A introduces a new product as soon as instantaneous profits turn non-negative. Thus, the equilibrium value of T is the number of periods such that $\alpha_T \in I_T$. Notice that α_T increases with T and takes values between μ and 1. Also, $\overline{x}^m(T)$ decreases with T and take values between $\frac{\gamma}{p^m(1)}$ and $\frac{\gamma}{R}$. Hence, since μ is arbitrarily small, if $\gamma < R$ (Assumption A6) there exists a unique value of T such that $\alpha_T \in I_T$.

Thus, instantaneous profits, $\overline{\pi}$, are non-negative and given by $\overline{\pi} = p^m\alpha_T - \gamma = p^m[\alpha_T - \overline{x}^m(T)]$ and lie in the interval $[0, \mu p^m(1 - \overline{x}^m(T)))$. Since μ is very low then profits are also very low.

Out of the equilibrium path, if x_t is sufficiently high then there may be room for two varieties to be introduced in the same period. In other words, like in the previous section, $\overline{x}^d = 2\gamma$ is the threshold value of the fraction of active consumers such that both firms charging prices equal to 1 would make zero instantaneous profits. Thus, any $x_t \geq \overline{x}^d$ can sustain two varieties in the same period. If $\gamma > \frac{1}{2}$ then $\overline{x}^d \geq 1$ and hence for all $x \in [\overline{x}^m(T), 1]$ only one firm can introduce a new product and make non-negative profits. However, if $\gamma < \frac{1}{2}$ then there is a value of $n > 0$, call it $\overline{n}$, such that $y_{T+\overline{n}-1} < \overline{x}^d \leq y_{T+\overline{n}}$.

I restrict attention to Markov strategies. Thus, production and pricing decisions only depend on x_t. In particular, firm A introduces a new variety if and only if the fraction of active consumers either lies in I_{T+n}, where n is an odd number lower than $\overline{n}$, or any number higher or equal than $\overline{n}$. Also, firm B introduces a new variety only if the fraction of active consumers either lies in I_{T+n}, where n is an even number lower than $\overline{n}$, or is higher or equal than $\overline{n}$. Thus, as soon as I_T is reached firm A is expected to introduce a new variety. If it fails to do so, then interval I_{T+1} is reached, and firm B is expected to introduce a new product, and if it fails to do so, then it is firm A's turn again, and so on until we reach $I_{T+n}, n \geq \overline{n}$, at which point both firms are expected to introduce a new variety.

We must check that these strategies conform a subgame perfect Nash equilibrium. For now, I still assume that new varieties are sold to all active consumers at the price p^m. Suppose firm A deviates and introduces a new variety when the fraction of active consumers is $x < \overline{x}(T)$, τ periods before expected. Then the payoff is $xp^m - \gamma + \frac{\delta^T}{1-\delta^T}\overline{\pi}$, which is lower than waiting for τ periods, in which case the firm will obtain $\frac{\delta^\tau}{1-\delta^T}\overline{\pi}$. Hence, such a deviation is not profitable. Also, firm B has no incentive to introduce a new variety when the fraction of active consumers is $x < \overline{x}(T)$ since it would also involve negative profits.

Suppose now that firm A deviates and does not introduce a new variety when the fraction of active consumers belongs to the interval I_{t+n}, n being an odd number, $0 \leq n < \overline{n}$. Then, it will be firm B who introduces a new variety next period and hence firm A's payoff will be $\frac{\delta^{T+1}}{1-\delta^T}\overline{\pi}$, which is lower than in case it does introduce a new variety right away: $p^m x - \gamma + \frac{\delta^T}{1-\delta^T}\overline{\pi} > \frac{\delta^{T+1}}{1-\delta^T}\overline{\pi}$. Similarly, if firm B fails to introduce a new variety when the fraction of active consumers belongs to I_{T+n}, n being an even number, $0 < n < \overline{n}$, then it misses a unique opportunity to make a profit.

Finally, if $x \geq \overline{x}^d$ then no firm has incentives to deviate and give away current profits with no consequences on future payoffs.

Optimal pricing. Finally, we need to provide conditions under which firms have no incentives to set a price different from p^m. Obviously, firms will never consider setting a price below p^m, since it has no effect on current demand or future profits, and reduces current profits. However, they may consider the possibility of setting a price above p^m. Provided p^m maximizes short-run profits, firm B will never consider a price different from p^m, since future profits will be zero in any case. However, firm A, may be willing to set

a price above p^m if the current losses are lower than future gains. A price above p^m will discourage those consumers who are located further away from the firm. As a result, the fraction of active consumers in the following periods will be higher. This may imply that firm A may enjoys higher demand and/or bring forward future product introductions. The next lemma (proved in the Appendix) shows that if consumers' willingness to pay is sufficiently high then firms will choose p^m in any node of the game. In particular, consider Assumption $A7$:

$$R \geq \overline{R} = \frac{1 + \frac{1}{2}\delta^T (1 - \mu)^T}{1 - \delta^T (1 - \mu)^T}.$$

If Assumption $A7$ holds, then $p^m > 1$, which implies that p^m is the short-run profit maximizing price.

Lemma 3 *Under Assumption $A7$, no firm has incentives to deviate from p^m.*

Like in the previous section, an equilibrium with the market covered (all active consumers purchasing the new variety) is only possible if consumers' willingness to pay is sufficiently high. Moreover, the lower bound of consumers willingness to pay increases with the value of temporary satiation. In particular, a higher value of $\delta^T (1 - \mu)^T$ implies a higher value of $\overline{R}$. Notice that $(1 - \mu)^T$ is the probability that a consumer has not recovered from the consumption of the previous variety, and δ^T is the discount factor applied to payoffs obtained T periods later.

All this discussion can be summarized as follows.

Proposition 4 *If μ and $(1 - \delta)$ are sufficiently close to zero, and Assumptions A6 and A7 hold, then there exists a Markov perfect equilibrium in which a new variety is introduced every T periods, all active consumers purchase the new variety at the price that maximizes short-run profits, p^m, and profits per variety are nearly zero.*

Thus, along the equilibrium path firm A is a temporary monopolist that introduces new products as soon demand becomes compatible with non-negative profits, and charges the short-run profit maximizing price. Such a price is limited by the extent of consumers' temporary satiation; that is, their willingness to pay is reduced by the option value of waiting. Hence, firms compete essentially through the timing of new product introductions, but prices are monopolistic.

The characteristics of this equilibrium are reminiscent of contestability. Only one firm is active, but its behavior is tightly constrained by the presence of the rival firm, who can

credibly enter the market in case the incumbent deviates. In the next subsection I study a more natural equilibrium in which firms alternate in the provision of new varieties. The characteristics of such equilibrium are basically identical but, unfortunately, conditions for existence are more demanding as strategic pricing incentives become more complex.

4.2 Two active firms

Consider a stationary equilibrium in which a new variety is introduced every T periods, and firms alternate. Moreover, every time a new variety is introduced all active consumers make a purchase. Firms' strategies are essentially the same than in the previous subsection. The only difference is that in this subsection, because we want firms to alternate in the introduction of new products, they switch roles after any product introduction. In particular, if B was the last firm who introduced a new product, then firms A and B play the same strategies as in the previous subsection, but if it was firm A then firms' strategies are reversed. The main difference with the previous analysis is that in this case firms' incentives to deviate and set a price higher than p^m (and not sell to all active consumers) are reinforced if instantaneous profits are not sufficiently close to zero. In this case we need a lower bound on R (Assumption $A8$: $R \geq \frac{1 - \frac{1}{2}\delta^T(1-\mu)^T}{1 - \delta^T(1-\mu)^T}$) and also an upper bound on $\Delta = \alpha^T - \overline{x}^m(T)$. Notice that instantaneous profits are equal to $p^m(T)\Delta$ and Δ only depends on exogenous parameters. In the Appendix I specify the upper bound on $\Delta, \overline{\Delta}$, that defines Assumption $A9 : \Delta \in [0, \overline{\Delta}]$. There I also complete the proof of the following proposition.

Proposition 5 *If μ and $(1 - \delta)$ are sufficiently close to zero, and Assumptions $A8$ and $A9$ hold, there exists a subgame perfect equilibrium with alternating firms such that a new variety is introduced every T periods, all active consumers purchase the new variety at the price that maximizes short-run profits, p^m, and profits per variety are nearly zero.*

As in Section 3.2, despite of the fact that firms are temporary monopolists, their ability to extract surplus from consumers is limited by the same two factors. First, consumers' willingness to pay is reduced by their option value of waiting. That is, $p^m(T)$ falls with $\delta^T(1-\mu)^T$. Second, because of competition, firms are less likely to engage in intertemporal price discrimination and stick to the short-run profit maximizing price. In order to illustrate this second point, notice that a monopolist would prefer to introduce products less frequently, and sell each one at a higher price: $p^m(T)$ increases with T

and, moreover, a monopolist could still find it profitable raise the price above the short-run profit maximizing level. This argument combines pricing incentives with the optimal frequency of new product introductions. Alternatively, we may want to separate pricing incentives from the frequency of new product introductions. More specifically, suppose that the frequency of new product introductions is exogenous: a new variety must be introduced every T periods. Then, in an equilibrium with alternating firms no one wishes to deviate from the short-run profit maximizing price, because any increase in future demand will be exclusively enjoyed by the rival firm. However, a monopolist may be willing to raise its price above the short run profit maximizing level in order to enjoy higher future demand (intertemporal price discrimination). More precisely, if a monopolist sets a price equal to $p^m + \varepsilon$ then current profits per active consumer, decrease by $\varepsilon (p^m - 1 + \varepsilon)$, but future profits increase by $\delta^T (1 - \mu)^T \varepsilon$. Hence, a monopolist will set $p^m (T)$ only if $p^m (T) \geq \frac{1}{1 - \delta^T (1-\mu)^T} > 1$, whereas alternating firms will set a price equal to $p^m (T)$, provided $p^m (T) \geq 1$.

4.3   Welfare analysis

Consider the following second best benchmark: the social planner chooses the number of periods between two product introductions, τ, and the price, p, in order to maximize total surplus under the constraint that firms cannot make negative profits (and consumers maximize their utility). Clearly, the social planner cannot choose a frequency below the equilibrium level, $\tau < T$, since no price allows firms to cover costs. In contrast, if $\tau > T$, since demand has more time to recover, there exists prices below $p^m (\tau)$ that induces all active consumers to purchase the new variety and allows firms to cover costs. In this case, the present value of total surplus, $W (\tau)$, can be written as $W (\tau) = \frac{\alpha_\tau \left(R + \frac{1}{2} \right) - \gamma}{1 - \delta^\tau}$. Thus, by increasing τ, $W (\tau)$ raises through α_τ but decreases through δ^τ. In other words, raising the distance between two new product introductions raises welfare by raising the fraction of consumers that enjoy each new variety, but lowers welfare by decreasing frequency. If we denote by $\Delta W = W (\tau + 1) - W (\tau)$, its sign is given by:

$$sign (\Delta W) = sign \left\{ -\delta^\tau (1 - \delta) \left[\left(R + \frac{1}{2} \right) \alpha_\tau - \gamma \right] + \left(R + \frac{1}{2} \right) \mu (1 - \alpha_\tau) (1 - \delta^\tau) \right\}$$

Since $R\alpha_\tau > \gamma$, then the first term is negative and represents the losses caused by less frequent new product introductions, and its value is proportional to $1 - \delta$. The second term is positive and represents the gains derived from higher consumption of each variety.

This second term is proportional to μ. Hence, for a fixed value of μ, arbitrarily small, as δ approaches 1 the first term vanishes and hence $\Delta W > 0$. In this case, the equilibrium is second best inefficient: new products are introduced too frequently. Instead, if $1 - \delta$ is not too small, the negative term dominates and $\Delta W < 0$. Hence, the equilibrium is second best efficient.

In the absence of price controls, the reduction in the frequency of new product introductions has non-trivial distributional consequences; in particular, it can raise profits but is likely to reduce consumer surplus. More specifically, if firms jointly determine the timing of new product introductions, increasing the distance between two consecutive varieties, then they will be able to raise prices ($p^m(\tau)$ increases with τ, as consumers' option value of waiting shrinks), and sell each variety to more consumers, which implies that the profits associated to each new product introduction will increase. Hence, since equilibrium profits are nearly zero then, starting at $\tau = T$, a small increase in τ is bound to raise the present value of profits. However, consumers will enjoy less frequent varieties, sold at higher prices, although each consumer will have a higher chance of being active when new products are introduced. Clearly, if total surplus decreases as a result of a higher τ, then consumer surplus must decrease even further (as profits increase). If total surplus increases moderately, the increase in profits will still imply a reduction in consumer surplus. Only if total efficiency gains are high enough consumers have a chance of capturing some of this extra rents. In other words, in the absence of price controls, the distributional consequences of a reduction in the frequency of new product introductions are biased against consumers.

Summarizing, authorities concerned about total welfare should pay close attention to firms' attempts at coordinating the timing of releases. If authorities put a higher weight on consumer surplus then they should scrutinize even more closely such coordination efforts.[17]

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[17]In some countries, including the UK, competition authorities have been concerned about film distributors coordinating the timing of their releases. As far as I know some of these attempts have been taken to courts only in Spain. In 2006 five major distributors were fined.

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6   Appendix

6.1   Proof of Lemma 3

For the moment, I consider price deviations in those periods for which the equilibrium prescribes the introduction of a new variety. Below, I consider joint deviations of pricing and introductory policies.

As an intermediate step, we can provide conditions under which, setting a price equal to p^m maximizes current profits, $\alpha_T q(p) p - \gamma$, where $q(p)$ is given by equation (11). The optimal static price is p^m if and only if $p^m \geq 1$, i.e., $R \geq 1 + \frac{\delta^T [1-\alpha_T]}{2\{1-[1-\alpha_T]\delta^T\}}$, which is implied by Assumption $A7$. However, a forward-looking firm must also consider the effect of the current price on future profits. In particular, suppose that in period 0 firm A is supposed to introduce a new product. The fraction of active consumers is $x_0 \in [\overline{x}(T), \mu + (1-\mu)\overline{x}(T))$. If firm A sets a price, $p = p^m + \varepsilon$, then current profits decrease by $\varepsilon (p^m - 1 + \varepsilon) x_0$. There is a value of ε, that I denote by $\overline{\varepsilon}_1$, such that $T - 1$ periods later the fraction of active consumers will be $\overline{x}(T)$. $\overline{\varepsilon}_1$ is given by the law of motion: $\overline{x}(T) = \alpha_{T-1} + (1-\mu)^{T-1} x_0 \overline{\varepsilon}_1$. It is convenient to denote by $\Delta = \alpha_T - \overline{x}(T)$, Δ can take values in the interval $\left[0, \mu(1-\mu)^{T-1}\right)$. Hence,

$$x_0 \overline{\varepsilon}_1 = \mu - \frac{\Delta}{(1-\mu)^{T-1}}$$

Thus, if $\varepsilon < \overline{\varepsilon}_1$ such a price deviation will not change the timing of the new product introduction, but will increase future profits because of a higher fraction of active consumers. In particular, the net losses from such a price deviation can be written as:

$$\Gamma(\varepsilon) = \varepsilon (p^m - 1 + \varepsilon) x_0 - \delta^T p^m (1-\mu)^T \varepsilon x_0$$

Assumption $A7$ implies that $p^m \geq \frac{1}{1-\delta^T(1-\mu)^T}$, which in turn implies that $\frac{d\Gamma(\varepsilon)}{d\varepsilon} > 0$ for all $\varepsilon > 0$. Hence, firm A has no incentives to deviate from p^m.

Next, let us define by $\bar{\varepsilon}_2$ the price deviation such that $T-2$ periods later the fraction of active consumers will be $\bar{x}(T) : x_0\bar{\varepsilon}_2 = 1 - \frac{(1-\mu)^T + \Delta}{(1-\mu)^{T-2}}$. Thus, if $\varepsilon \in [\bar{\varepsilon}_1, \bar{\varepsilon}_2)$ then the net gains from such a deviation are:

$$\Gamma(\varepsilon) = \varepsilon\left(p^m - 1 + \varepsilon\right)x_0 - \delta^{T-1}p^m\left[\frac{\Delta(1-\delta)}{1-\delta^T} - (1-\mu)^{T-1}\left(\mu - \varepsilon x_0\right)\right]$$

Notice that in this case all future product introductions are brought forward by one period. If there exists a value of $\varepsilon \in [\bar{\varepsilon}_1, \bar{\varepsilon}_2)$ such that $\Gamma(\varepsilon) < 0$ then, it must be the case that:

$$x_0\varepsilon\left[p^m - 1 - p^m\delta^{T-1}(1-\mu)^{T-1}\right] + \delta^{T-1}p^m\left[-\frac{(1-\delta)\Delta}{1-\delta^T} + (1-\mu)^{T-1}\mu\right] < 0$$

Taking into account that $p^m \geq \frac{1}{1-\delta^T(1-\mu)^T}$ and that $x_0\varepsilon$ is lower than $x_0\bar{\varepsilon}_2$ then

$$1 - \frac{(1-\mu)^T + \Delta}{(1-\mu)^{T-2}} > \frac{(1-\mu)^{T-1}\mu - \frac{(1-\delta)\Delta}{1-\delta^T}}{(1-\mu)^{T-1}\left[1 - \delta(1-\mu)\right]} \tag{14}$$

Since δ is arbitrarily close to 1 then the right hand side is also arbitrarily close to 1. However, the left hand side is strictly lower than 1. We have reached a contradiction. Therefore there is no profitable deviation in this interval either.

We can generalize the last step to price deviations that involve anticipating n periods the next product introduction. In particular, for $n > 0, x_0\bar{\varepsilon}_n = 1 - (1-\mu)^n - \frac{\Delta}{(1-\mu)^{T-n}}$. If $\varepsilon \in [\bar{\varepsilon}_n, \bar{\varepsilon}_{n+1})$ the net gains from such a deviation are:

$$\Gamma(\varepsilon) = \varepsilon\left(p^m - 1 + \varepsilon\right)x_0 - \delta^{T-n}p^m\left[\frac{\Delta(1-\delta^n)}{1-\delta^T} - (1-\mu)^{T-n}\left\{\left[1 - (1-\mu)^n\right] - \varepsilon x_0\right\}\right]$$

If there exists a value of $\varepsilon \in [\bar{\varepsilon}_1, \bar{\varepsilon}_2)$ such that $\Gamma(\varepsilon) < 0$ then, the condition analogous to (14) is :

$$1 - (1-\mu)^{n+1} - \frac{\Delta}{(1-\mu)^{T-n-1}} > \frac{1 - (1-\mu)^n}{1 - \delta^n(1-\mu)^n} - \frac{1-\delta^n}{\left[1 - \delta^n(1-\mu)^n\right]\left(1-\delta^T\right)}\frac{\Delta}{(1-\mu)^{T-n-1}} \tag{15}$$

Notice that the RHS decreases with Δ faster than the LHS. Hence, if we evaluate (15) at the maximum value of Δ, $\mu(1-\mu)^{T-1}$, it has to be the case that

$$-\delta^n(1-\mu)^n\left[1 - (1-\mu)^{n+1}\right] + \mu(1-\mu)^n\frac{1 - \delta^n + \left(1-\delta^T\right)\delta^n(1-\mu)^n}{1-\delta^T} > 0$$

Since δ and $(1-\mu)$ are arbitrarily close to 1, we have reached a contradiction.

Finally, let us consider price deviations in periods in which along the equilibrium path there is no product introduction. Suppose that firm B considers to set a price different

from p^m. Since future profits are equal to zero independently of its actions, if firm B introduces a new product, then it will set the price that maximizes current profits, p^m. Firm A's incentives to set a price higher than p^m when $x \notin I_T$ are analogous to those faced in the interval I_T. In fact, I showed above that the losses from deviations are bigger than when we disregard the quadratic component. Moreover, when we only consider the linear terms, then net losses do not depend on the fraction of active consumers but on $x\varepsilon$.

6.2 Proof of Proposition 4

Let us consider the incentives of the firm who is supposed to introduce a new product when the game reaches the interval I_T to set $p = p^m + \varepsilon$. In this case the intertemporal effect of prices are more complex than in the equilibrium with only one active firm. In particular, if the firm sets $\varepsilon < \bar{\varepsilon}_1$ then this does not affect the timing of the next product introductions, and only the fraction of active consumers that the rival will enjoy. Therefore, the firm has no incentives to deviate within this interval. Suppose now that $\varepsilon \in [\bar{\varepsilon}_1, \bar{\varepsilon}_2)$. Such a deviation will bring forward the next product introduction by one period. Since all future prices are expected to be equal to p^m, then such a deviation will not affect instantaneous profits but it will bring forward by one period all future product introductions. Since this effect is independent on ε, as long as it belongs to the interval $[\bar{\varepsilon}_1, \bar{\varepsilon}_2)$, then the optimal deviation would be the one that minimizes the current loss in profits: $\varepsilon = \bar{\varepsilon}_1$, which implies that the rival will make zero profits in the next product introduction. Thus the net losses from such a deviation will be

$$\Gamma(\bar{\varepsilon}_1) = \bar{\varepsilon}_1 (p^m - 1 + \bar{\varepsilon}_1) x_0 - (1 - \delta) p^m \Delta \frac{\delta^{2T-1}}{1 - \delta^{2T}}$$

where $x_0 \bar{\varepsilon}_1 = \mu - \frac{\Delta}{(1-\mu)^{T-1}}$. Under assumption $A8$, that $p^m \geq 1$. If $\Delta = 0$, future expected profits are zero and hence potential gains from deviation are zero. As a result $\Gamma(\bar{\varepsilon}_1) > 0$. In the other extreme, if Δ is arbitrarily close to $\mu(1-\mu)^{T-1}$ and hence $\bar{\varepsilon}_1$ is arbitrarily close to zero, and $\Gamma(\bar{\varepsilon}_1) < 0$. Moreover, $\Gamma(\bar{\varepsilon}_1)$ monotonically decreases with Δ. Hence, for all $x_0 \in [\bar{x}(T), \mu + (1 - \mu)\bar{x}(T))$ there exists $\overline{\Delta}_1(x_0) \in \left(0, \mu(1-\mu)^{T-1}\right)$ such that if $\Delta \leq \overline{\Delta}_1(x_0)$ then $\Gamma(\bar{\varepsilon}_1) \geq 0$. Notice that $\overline{\Delta}(x_0)$ increases with R, through p^m, and decreases with x_0.

Finally, we need to check that there are no incentives to deviate for any value of $\bar{\varepsilon}_n$. In this case the net losses are

$$\Gamma(\bar{\varepsilon}_n) = \bar{\varepsilon}_n (p^m - 1 + \bar{\varepsilon}_n) x_0 - (1 - \delta^n) p^m \Delta \frac{\delta^{2T-n}}{1 - \delta^{2T}}$$

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where $x_0 \bar{\bar{\varepsilon}}_n = \bar{\bar{\varepsilon}}_n = 1 - (1 - \mu)^n - \frac{\Delta}{(1-\mu)^{T-n}}$. Once again, if $\Delta = 0$ then $\Gamma(\bar{\bar{\varepsilon}}_n) > 0$. However, in this case if Δ is arbitrarily close to $\mu(1-\mu)^{T-1}$, and $n > 1$, then the sign of $\Gamma(\bar{\bar{\varepsilon}}_n)$ is ambiguous since $\bar{\bar{\varepsilon}}_n$ is arbitrarily close to $1 - (1-\mu)^{n-1} > 0$. In either case, $\Gamma(\bar{\bar{\varepsilon}}_n)$ monotonically decreases with Δ. Hence, we can still define by $\overline{\Delta}_n(x_0)$ as the value of Δ in the interval $\left(0, \mu(1-\mu)^{T-1}\right)$ such that $\Gamma(\bar{\bar{\varepsilon}}_n) = 0$, if such a value exists. If it does not and $\Gamma(\bar{\bar{\varepsilon}}_n) > 0$ for all $\Delta \in \left(0, \mu(1-\mu)^{T-1}\right)$ then we let $\overline{\Delta}_n(x_0) = \mu(1-\mu)^{T-1}$. Once again, $\overline{\Delta}_n(x_0)$ is a weakly decreasing function of x_0.

Finally, we define $\overline{\Delta}$ and the lowest value of $\overline{\Delta}_n(x_0)$ evaluated at $x_0 = \mu + (1-\mu)\overline{x}(T)$, for all $n, 0 < n < T$. Clearly, $\overline{\Delta} \in \left(0, \mu(1-\mu)^{T-1}\right)$ and if $\Delta \leq \overline{\Delta}$ then firms have no incentives to deviate from p^m.

Finally, just like in Lemma 3, firms have no incentives to deviate either in intervals different from I_T, since the sign of the linear component of the net losses is independent of x_0.
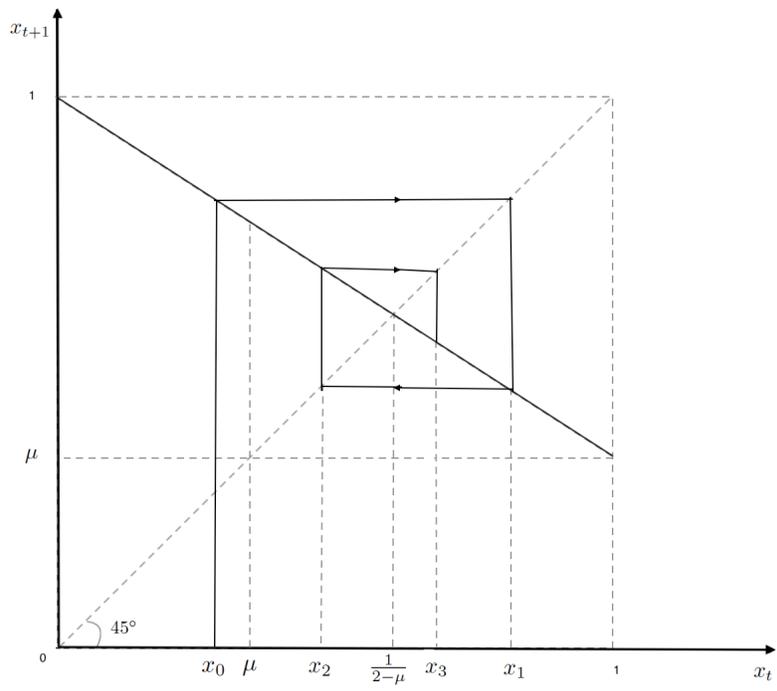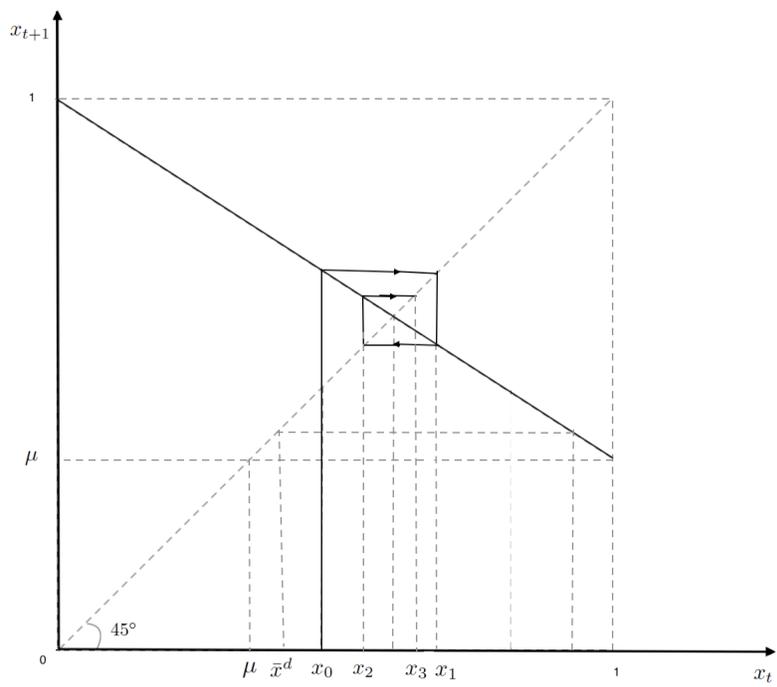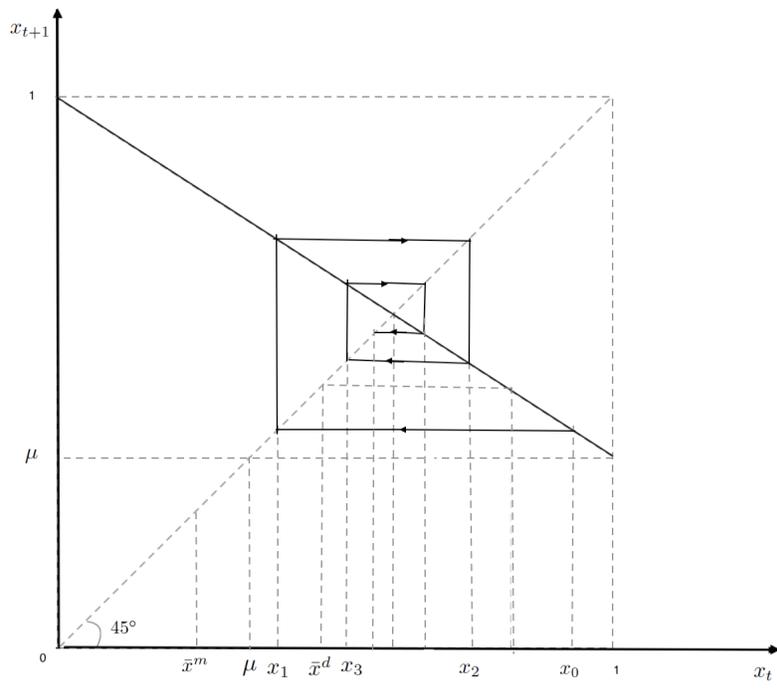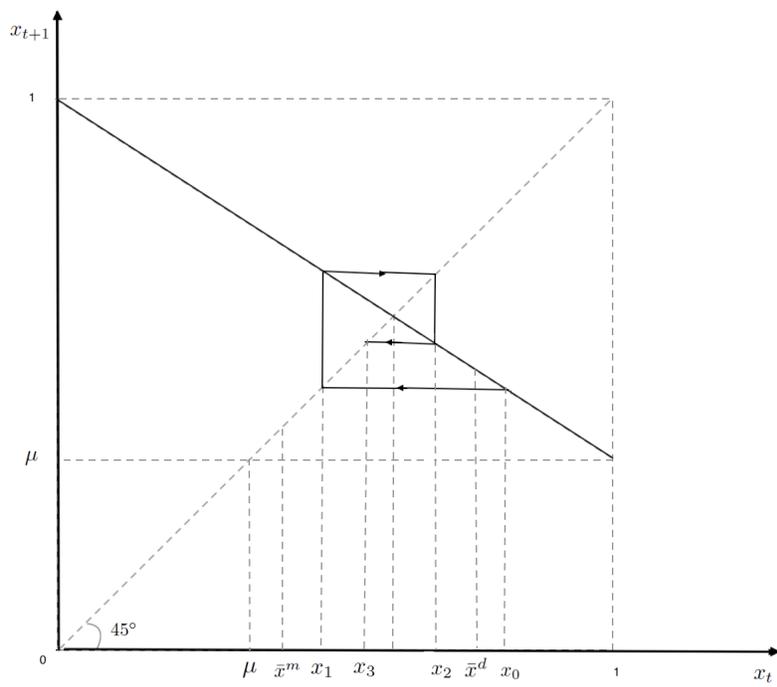
Figure 1

Figure 2a

Figure 2b

Figure 3a

Figure 3b