A Theory of Crowdfunding -A Mechanism Design Approach with Demand Uncertainty and Moral Hazard: Comment

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A theory of crowdfunding — a mechanism design approach with demand uncertainty and moral hazard: Comment*

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Abstract

Strausz (2017) claims that crowdfunding implements the optimal mechanism design outcome in an environment with entrepreneurial moral hazard *and* private cost information. Unfortunately, his analysis, solution and claim depend critically on imposing an untenable condition (29) that he had explicitly discarded from his weak feasibility concept.[1] This condition is essentially equivalent to ex post participation. We explain why it is inconsistent to assume consumers can opt out after learning the entrepreneur's cost structure in a model of fraud. We then study weak feasibility without the corresponding ex post participation constraint. We provide a simple example of a crowdfunding design that raises profit and welfare by tolerating some fraud risk. This shows how cross-subsidizing between cost states relaxes the most restrictive moral hazard constraints and generates better outcomes. We then characterize the optimal mechanism in the case of one consumer and two cost states. In general, this must hide information, including prices, from consumers. So crowdfunding cannot implement these optima.

Keywords: Crowdfunding, mechanism design, moral hazard, private information.
JEL Classifications: C72, D42, D81, D82, D86, L12, L26

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[1] Strausz's corrigendum responding to this comment now adds (29) back, remarkably with neither mention of this vital change nor a justification – see our prelude for details.

Prelude

In response to the previous version of this comment, Strausz now provides a corrigendum[2] in which he changes the definition of weakly feasible mechanisms by adding a condition (29) that had been explicitly disregarded from strict feasibility as if unnecessary for the results. This change fixes a key analytical error, but makes the exercise irrelevant for crowdfunding, because in crowdfunding, bidders have no ex post option of withdrawing their money after it is transferred to the entrepreneur. Our comment provides a thorough explanation of this point and clarifies some misleading claims in Strausz (2017), as well as providing the relevant solution without imposing the ex post outside option. In sum, the main message of our comment is unchanged: The mechanism design approach to modelling moral hazard and private cost information should only impose interim and not ex post individual rationality. The constrained optimum under this weaker constraint produces higher profit and welfare and it cannot be implemented by crowdfunding. This overturns the main message of Strausz (2017). Moreover, we provide a simple mechanism, satisfying interim individual rationality, entirely consistent with Strausz's definition of crowdfunding, that can already generate higher profits and welfare.

We point out the main mistakes in the original article, since the corrigendum does not acknowledge them (not even alerting the reader to the crucial change in the definition of weak feasibility) and Strausz (2017) heavily confuses readers about the role of his key constraint (29) in a number of ways: (i) He calls it plain "individual rationality" even though it conditions on the entrepreneur's privately known cost structure.[3] (ii) Footnote 23 misleads the reader about the meaning of constraint (29) because it argues that crowdfunders may withdraw pledges after they learn the campaign is a success as if that were equivalent to being able to withdraw after the entrepreneur has received their money. The latter would indeed justify (29), but it is obviously impossible for consumers to withdraw after their money is passed to the entrepreneur in the event that she runs off with it.[4] (iii) Strausz (2017) presents two distinct definitions of implementability, but treats them as if equivalent. Allocation functions and output schedules are defined, on p. 1450, to be implementable when they coincide with a perfect Bayesian equilibrium outcome of some game. Strausz (2017) then erroneously states that by the mediated revelation principle of Myerson (1982), this is equivalent to being implementable by means of a strictly feasible mechanism, which explicitly includes (29) in its definition. Note that Myerson (1982)

[2] Available at https://assets.aeaweb.org/assets/production/files/5889.pdf.

[3] It is standard to write individual rationality alone only when referring to the interim stage where private information remains private. We refer to constraint (29) as ex post individual rationality since the entrepreneur's private information on cost structure is assumed known (implementability is not affected by the fact that (29) involves bidder expectations over each others' valuations).

[4] Footnote 23 and the prior text and ensuing notion of the conditional outside option also suggest that (29) is importantly different from ex post individual rationality because there is no conditioning on the realized valuations of other consumers, only on that of the entrepreneur's cost, but this difference is immaterial since consumers have private values and have learned if the project succeeds or fails.

considers only incentive constraints, not participation constraints, because all actions (here including fraud by entrepreneurs and potential withdrawal options by funders) are to be included in the description of the game. Myerson (1982, equation (1)) then requires taking expectations over all other agents' types in the expression of payoffs, so (29) would have to be justified via an explicitly modeled action. Yet explicitly modeling the bidders' ex post outside option reveals that this is tantamount to assuming away all fraud and doing so, of course, undoes all of Strausz's results.[5]

Throughout the remainder of this comment we refer to Strausz (2017) in the **past tense** when our comments apply only to his original version and in the **present tense** when our remarks refer to either version.

1 Introduction

Strausz (2017) investigates whether popular crowdfunding schemes can deal optimally with moral hazard concerns. He suggests that they can. Concretely, he uses mechanism design techniques to solve the problem of maximizing profit in an environment with entrepreneurial moral hazard and private cost information. This is valuable because prior work had abstracted from an explicit treatment of moral hazard. Strausz (2017) derives two main features of his constrained efficient mechanism: it induces deferred payments and limits the entrepreneur's information about their size. He then argues that crowdfunding indirectly implements these two features.[6] Unfortunately, two important errors in the analysis generated a number of invalid lemmas and propositions.[7] As a result, Strausz's analysis of the constrained optimum was incorrect. The two main errors are: (i) the payoff expressions are incorrect since they exclude the possibility of fraud; (ii) the analytic derivation that the optimum satisfies condition (29) effectively assumed[8] the result, and (29) cannot be justified in crowdfunding. In this comment, we relax Strausz's optimization problem by disregarding (29). We provide a simple example of how crowdfunding generates higher profits and welfare than Strausz claimed. Then we characterize the relevant constrained optimum, showing it has the additional interesting feature of cross-subsidization. We also show that this cannot be implemented by crowdfunding.

The most important problem is the unjustifiable assumption of condition (29). In his definition of a constrained efficient mechanism, Strausz (2017) does not only impose budget- and development-feasibility, incentive compatibility and obedience constraints.

[5]The mechanism design literature has considered ex post individual rationality constraints on the distinct ground of seeking dominant strategy implementation but this is not Strausz's approach and dominant strategy implementation cannot deliver his results.

[6]Strausz (2017) does, on the other hand, recognize in footnote 6 that actual crowdfunding cannot *jointly* implement both features unless the constrained optimum is efficient and marginal costs are zero.

[7]Concretely, Lemmas 3, 5 and 6 were incorrect and, because the proofs of Propositions 2 and 3 depend on those lemmas, those Propositions remained unproven.

[8]See prelude on use of past tense for the errors fixed in the corrigendum.

He additionally imposes inequality constraint (29), which says that each consumer receives at least "his outside option *conditional* on his own type *and the project's cost structure*" (p.1449, our italics).[9] This ex post constraint is unjustified and inappropriate in the context of crowdfunding with private cost information. First, by the very definition of private information, consumers do not observe the entrepreneur's cost when they decide whether to participate by bidding. Moreover, they do not update their beliefs about the entrepreneur's cost when all types of entrepreneur pool on the same crowdfunding price and threshold. Second, consumers may eventually learn about the entrepreneur's cost after a successful campaign, as when low cost entrepreneurs produce while high cost entrepreneurs run with the money. But they have no option to withdraw their pledge at that stage: it is then simply too late to withdraw and avoid the loss.[10]

In this way, Strausz (2017) seems merely interested in characterizing optimal *loss-free* mechanisms. However, all statements and analysis in Lemmas 2 through 7 referred to weakly feasible mechanisms that were defined by *explicitly disregarding* (29). This suggested that all results were derived without imposing (29). In particular, Lemma 5 implied (29), so that (29) seemed to be a strong result rather than a strong assumption. However, as we show in this comment, the proof of Lemma 5 assumed (29), in fact, it even assumed something slightly stronger. In private communication with us, the author claimed that it was a lapse to (explicitly) disregard (29) from his definition of *weakly feasible* mechanisms. An online corrigendum now includes (29) as a defining feature of *weakly feasible* mechanisms, weakens the statement of Lemma 5 and corrects its proof. Unfortunately, the corrigendum does not inform the reader about this change in definition, instead mentioning only the weakening of Lemma 5 and the correction of its proof. To emphasize and clarify the strength of condition (29), this comment explains how key results of Strausz (2017) were incorrect when (29) was not included in the definition of weakly feasible mechanisms, and that the corrigendum, while technically fixing those errors by imposing (29), does not speak to the crowdfunding context.

The first error in the paper remains present in the corrigendum. It lies in the payoff expressions which neglect the possibility of fraud, representing an excessively reduced-form analysis. Hence, Strausz (2017) tacitly restricts attention to mechanisms that prevent all fraud. However, crowdfunding that tolerates some fraud already (even before moving to more general mechanisms) generates better outcomes than Strausz's solution achieves.

[9]In the case of perfect information about the project's cost structure, this is just interim individual rationality, but it goes far beyond that when there is truly imperfect information about costs. Indeed, for the case of just one consumer (which we consider later on in detail), this assumption is identical to ex post individual rationality.

[10](29) might be justified if crowdfunding platforms guaranteed to prosecute fraudulent entrepreneurs and fully reimburse funders, but: (1) this is clearly not the case; e.g., Kickstarter explicitly warns pledgers that "*... a project may not work out the way everyone hopes. Kickstarter creators have a remarkable track record, but nothing's guaranteed. Keep this in mind when you back a project.*" 2) Strausz's model would still not fit; even if such guarantees existed and were perfect, the whole moral hazard problem disappears as no entrepreneur would ever try to run with the money.

The second important error lay in the proof of Lemma 5: in that proof, the derivation that the optimum satisfies condition (29) effectively assumed the result. It is only in the absence of cost uncertainty (or via the untenable addition of (29) to weak rationality, as in the corrigendum) that these errors turn out to be inconsequential.

As Strausz (2017) notes, it is important to take private information about cost seriously. To do so, we analyze the relevant case: we solve for the optimum without imposing (29). We show our constrained optimum generates more trade and welfare than Strausz's solution. This uncovers a third feature (on top of transfer deferral and hiding information from entrepreneurs) of the constrained optimal mechanism: it involves restricting consumer's information about the entrepreneur's cost. The intuition is straightforward. Hiding this information enables subsidization by consumers across cost states so as to increase the amount of deferred payments in those states where moral hazard concerns are most severe. Note that popular crowdfunding cannot perfectly implement this third feature, because thresholds and prices chosen by the entrepreneur signal her cost to consumers. This forces us to conclude that crowdfunding cannot implement the optimum.

Nevertheless, we can readily construct an intuitive example of how crowdfunding, by hiding cost information and tolerating some fraud, can already generate better outcomes than Strausz (2017) claims. So we start with our simplest example. Strausz (2017) states that crowdfunding is completely useless if there is any probability, no matter how small, of an entrepreneur with costs high enough to make production inefficient even when all consumers value the product highly. Now, when such an entrepreneur manages to raise funds from consumers who pay her at most the high valuation, she would not produce, instead committing fraud by absconding with the funds. Because cost information is private, crowdfunding can only fully prevent such fraud by not funding any project. So this zero production solution is optimal when (29) is imposed. However, intuition and evidence strongly suggest that crowdfunders should tolerate some fraud risk if sufficiently small – as in the classic refrain, "*nothing ventured, nothing gained.*" Example 1 in Section 2 vindicates this intuition: a large probability of productive crowdfunding outweighs a small risk of fraud and generates a positive net surplus. This example shows crowdfunding can do strictly better than Strausz (2017) claims. Moreover, it is consistent with the evidence of small amounts of fraud (see Mollick, 2014).

This example neither proves nor disproves the claim that crowdfunding can implement the constrained efficient outcome. To shed light on that issue, we need to characterize correctly our constrained efficient outcome without imposing (29). Unfortunately, we cannot build too much on Strausz's prior analysis. The prior example demonstrates that Lemmas 3 and 5 in Strausz (2017) were false. This in turn casted doubt on the validity of Lemma 6 and Propositions 2 and 3 which build on Lemma 5. Indeed, Example 2 in Section 3 presents an efficient output schedule that can be implemented despite not being "affluent" (as defined by Strausz, 2017, condition (44), p. 1454).

In Section 4, we provide a complete characterization of optimal mechanisms in the simplest case of one consumer and two types of entrepreneur. The optimal solution involves cross-subsidization to relax the obedience constraints in those cost states where moral hazard concerns are more severe. It hides entrepreneur's cost information from consumers, just as Strausz's solution hid some consumer demand information from the entrepreneur. Hiding cost information is necessary because consumers effectively subsidize the entrepreneur by paying a higher price and taking a utility loss in high cost states. By contrast, popular reward-based crowdfunding platforms directly let consumers know the price they will pay when they bid on a project that gets successfully funded. It follows that popular crowdfunding platforms, in general, *cannot* implement the constrained optimum in an environment with entrepreneurial moral hazard and private cost information.

2 Notation and errors

To facilitate comparison, we use the same notation and setup as in Strausz (2017). In particular, the set of consumers is $\mathcal{N} = \{1, \ldots, n\}$. The vector of valuations is $v = (v_1, \ldots, v_n) \in \{0,1\}^n$ and $n(v) \equiv \sum_{i\in\mathcal{N}} v_i$ denotes the number of high value consumers. An allocation $a = (t, x) = (t^a_1, \ldots, t^a_n, t^p_1, \ldots, t^p_n, x_0, x_1, \ldots, x_n)$ consists of ex ante and ex post transfers t^a_i and t^p_i from consumer i, the probability that investment takes place x_0 and the probability that consumer i consumes one unit of the good, x_i. Ex post transfers are conditioned on the behavior of the entrepreneur. They are not paid when the entrepreneur acts fraudulently and makes a run with the funds raised. In this fraud event, the entrepreneur can only enjoy a fraction α of the ex ante transfers. The remaining fraction is lost. $\alpha > 0$ captures the degree of moral hazard. A project is characterized by $(I, c) \in \mathcal{K}$, where I denotes fixed cost and c marginal cost. The entrepreneur observes her cost structure (I, c) privately; this defines her type. A direct mechanism $(\mathbf{t}, \mathbf{x})$ assigns an allocation $(\mathbf{t}(I, c, v), \mathbf{x}(I, c, v))$ to each demand and cost state. Since none of our points require the marginal cost feature or the existence of more than two cost states, we set $c = 0$ and $\mathcal{K} = \{(I_1, 0), (I_2, 0)\}$. So the type k entrepreneur has cost I_k and we denote the probability by $\rho_k > 0$.

We begin with the simple example of how crowdfunding, of exactly the kind depicted by Strausz, can generate higher profits and welfare than Strausz has claimed are possible. The example also illustrates the inappropriateness of (29).

Example 1 (Fraud-tolerant crowdfunding)**.** *There is one consumer ($n = 1$), with valuation 0 or 1, and two types of entrepreneur. Type $k \in \{1, 2\}$ has fixed cost I_k where $0 < I_1 < \frac{1}{1+\alpha} < 1 < I_2$. Let $R = \rho_1/\rho_2$ denote the relative likelihood of type 1. If the consumer with valuation 1 pays $t^a = I_1$ as an ex ante transfer and commits to pay $t^p = p - I_1$ upon receiving the good, he runs the risk of losing I_1 when dealing with the* *high type entrepreneur. So his expected utility is* $\rho_1(1-p) - \rho_2 I_1 = \rho_2\big[R(1-p) - I_1\big]$ *and he is willing to pay* $p \le 1 - I_1/R$*. Moreover, the type 1 entrepreneur will produce instead of running with the money if and only if* $p - I_1 \ge \alpha I_1$*, equivalent to* $p \ge (\alpha+1)I_1$*. Now* $1 - I_1/R \ge (\alpha+1)I_1$ *if and only if* $R \ge I_1/(1-(1+\alpha)I_1)$*, that is, if the risk* ρ_2 *is sufficiently low. For any such* R*, type 1 produces for sure while type 2 commits fraud, generating strictly positive expected surplus and profits. Any price* $p \in [(\alpha+1)I_1, 1-I_1/R]$ *will do. The entrepreneur prefers the highest one:* $p = 1 - I_1/R$*.*

It is easy to verify that Example 1 already demonstrates that Lemmas 3 and 5 in Strausz (2017) were incorrect. Formally, the direct mechanism $(\mathbf{t}, \mathbf{x})$ defined by

$$\begin{aligned} (\mathbf{t}(I_k,0), \mathbf{x}(I_k,0)) &= (0,0,0,0) \text{ for } k = 1,2 \\ (\mathbf{t}(I_1,1), \mathbf{x}(I_1,1)) &= (I_1, p - I_1, 1, 1) \\ (\mathbf{t}(I_2,1), \mathbf{x}(I_2,1)) &= (I_1, p - I_1, 0, 0) \end{aligned}$$

is clearly weakly feasible (original definition) for any of the prices p indicated in Example 1.[11] Yet $\alpha t^a(I_2,1) = \alpha I_1 > 0 = x_0(I_2,1)I_2$, contradicting Lemma 3. Moreover, conditional on dealing with type 2, the high valuation consumer incurs a loss of I_1, contradicting Lemma 5.

The origin of these mistakes lies in three errors in Strausz's (2017) analysis. First, the payoff expressions neglect the possibility of fraud. Strausz's (2017) equations (6) and (7) state the consumer utility and profit as

$$\begin{aligned} U_i(a|v_i) &= v_i x_i - t_i^a - t_i^p \\ \Pi(a|I,c) &= \sum_{i\in\mathcal{N}} [t_i^a + t_i^p - x_i c] - x_0 I \end{aligned}$$

However, the entrepreneur can act fraudulently by setting $x_0 = 0$ despite receiving ex ante transfers $t_i^a > 0$, so the correct expressions are

$$\begin{aligned} U_i(a|v_i) &= x_0[v_i x_i - t_i^a - t_i^p] + (1-x_0)[-t_i^a] \\ \Pi(a|I,c) &= x_0\left[\sum_{i\in\mathcal{N}} [t_i^a + t_i^p - x_i c] - I\right] + (1-x_0)\left[\alpha \sum_{i\in\mathcal{N}} t_i^a\right] \end{aligned}$$

The combination of positive ex ante transfers ($\sum_{i\in\mathcal{N}} t_i^a > 0$) without production ($x_0 = 0$) represents the fraud event where the entrepreneur can only enjoy fraction α of the funds. These expressions also make the conditionality of the ex post transfers t_i^p explicit: they are only paid when the entrepreneur produces obediently; the ex ante transfers t_i^a are always paid, though the entrepreneur only fully enjoys them conditional on producing (see

[11] Strausz (2017, p. 1451) defined weak feasibility via constraints (21)-(26), (28) and (30). Note that a range of other ex post transfers, including $t_2^p = 0$, are equally effective since the entrepreneur absconds and does not receive t^p for the indicated value.

Strausz, 2017, p. 1445). In addition, Strausz's definition of a crowdfunding mechanism, via his equations (10)–(13) excludes the possibility of fraud.[12]

The second and equally fundamental error lay in the analytic derivation of the claim that the weakly feasible optimum satisfies ex post individual rationality constraint (29) – the derivation essentially assumes the result.[13] Strausz (2017, p. 1449) writes "*Yet, rather than imposing ex post participation by assumption, we will [...] show the extent to which ex post individual rationality of the optimal mechanism is a result rather than an assumption.*" Formally, Strausz (2017, Sect. III.B) defines a constrained efficient mechanism as one that maximizes welfare subject to constraints (21)-(29). He then claimed to solve the optimal weakly feasible mechanism by setting up the relaxed problem that explicitly dropped (29) and then showing in his Lemma 5 that it satisfied (29) with equality. However, the proof of Lemma 5 was incorrect, because it assumed that the consumer's utility is non-negative for all cost structures in any weakly feasible mechanism. Assuming non-negative utility in all cost states is essentially equivalent to assuming ex post individual rationality. So this completely invalidated the proof.[14]

The only way to restore all of Strausz's analytical results would be to assume ex post rationality (29) throughout (which is done in his corrigendum), but this is inconsistent with the assumption that entrepreneurs' costs are private information and the well-motivated objective of solving for the general optimum with private information:

> "*It seems, however, natural that entrepreneurs are better informed about their cost structure than consumers or the crowdfunding platform. Also in practice, consumers and crowdfunding platforms reportedly worry that crowdfunding will attract fraudulent entrepreneurs [...] Taking this informational asymmetry seriously, an implementation of our crowdfunding scheme would then require the entrepreneur to first truthfully report her cost structure.*" (Strausz, 2017, p. 1441)[15]

[12]Note that Strausz's online corrigendum does not address the reported errors in the payoff functions.

[13]Strausz (2017) refers to constraint (29) as "individual rationality," but since it conditions on the entrepreneur's cost, the adjective "ex post" is more appropriate. Constraint (29) is precisely ex post individual rationality when $n = 1$. By contrast, ex ante and interim individual rationality would require taking expectations over all possible costs.

[14]To be concrete, Strausz (2017, p.1468) constructed transfer $\hat{t}^p_{li}(I, c, 1, v_{-i})$ from $\tilde{t}^p_{li}(I, c, 1, v_{-i})$ by adding $U_i^{\tilde{\gamma}_i}(1|I, c, 1)$ and claimed that this weakly raises the original transfer, but this is only true if $U_i^{\tilde{\gamma}_i}(1|I, c, 1) \geq 0$, which is even stronger than (29). His corrigendum fixes this mild gap by weakening the statement of Lemma 5 so that then imposing (29) is necessary and sufficient for the proof.

[15]At the same time, in footnote 23, Strausz (2017) does briefly attempt to justify ex post individual rationality by claiming that consumers can withdraw pledges after a successful campaign. This attempt fails on two grounds. First, the only given evidence, a personal experience at Kickstarter of dropped pledges, misconstrues how Kickstarter works: Kickstarter *does* require consumers to honor any pledges active at the deadline of a successful campaign; see Section 5 of Kickstarter's terms of use at `https://www.kickstarter.com/terms-of-use` (last accessed August 14, 2017). Dropped pledges can occur when a consumer's credit card expires but represent a consumer default. Second, and more fundamentally, even if consumers did have the option to withdraw their pledges after a campaign is successfully funded,

It should go without saying that truthful revelation to the intermediary does not imply revelation of cost to the consumers. Strausz already showed that the optimal mechanism typically must hide demand information from entrepreneurs, but his solution failed to take private cost information seriously: our characterization in Section 4 will show that the optimal mechanism typically must also hide cost information from consumers. In fact, Example 1, while not optimal in the class of general mechanisms, already illustrates how hiding cost information can raise profits and welfare relative to Strausz's solution. To see this, note that the direct mechanism $(\mathbf{t}, \mathbf{x})$ implementation of the solution in Example 1 features transfer contracts, (t^a, t^p), that are identical for both entrepreneur types or costs, I_1 and I_2. It immediately follows that this solution can be implemented via an indirect crowdfunding mechanism in which entrepreneurs with differing costs pool on the same crowdfunding proposal. In other words, that improved solution involves hiding cost information from consumers.

A third, and less serious, error is that the mechanism design analysis considers transfer instruments, t^a, t^p, while neglecting the much simpler possibility of unconditional transfers when the mechanism recommends non-production. We let p' denote the transfer from high valuation consumers to the entrepreneur when the mechanism recommends non-production, so that p' is the mirror image of t^p. Paying for nothing may seem pointless but can avoid wasting the factor $1-\alpha$ in Example 1: a transfer $p'_2 = \alpha I_1$ to type 2 can just keep her from pretending to be type 1 and running off with the higher transfer $t^a = I_1$. Paying "money for nothing" is not in the spirit of actual crowdfunding, but is a natural device in general mechanism design. In Section 4, we characterize optimal mechanism design. Allowing for such transfers simplifies the analysis.[16]

3 Efficient outcomes

Example 1 showed that Strausz (2017) did not find the constrained optimal mechanism, and that Lemmas 3 and 5 were incorrect. This casted doubt on the validity of Propositions 2 and 3 because they build on Lemma 5. Proposition 2 claims that the efficient output schedule is implementable if and only if it is "affluent," as defined by condition (44) in Strausz (2017, p. 1454). Proposition 3 claims that if the efficient output schedule is not affluent, then the optimal solution is only constrained efficient but can still be implemented by a crowdfunding mechanism, as defined by equations (10–13). Example 2 below shows that Propositions 2 and 3 were invalid, because the efficient output schedule can be implemented, despite the fact that it is not affluent. On the other hand, that

they would *not* be able to retrieve all their money after the entrepreneur runs with the funds transferred ex ante. Such a full retrieval option would justify ex post individual rationality, but would clearly contradict the basis of the moral hazard problem.

[16]We show that one can actually do without them, although it sometimes requires approaching the optimum as the limit of stochastic mechanisms using only the instruments t^a and t^p.

schedule cannot be implemented by a crowdfunding mechanism or any other mechanism that satisfies (29).

Example 2. *Let* $n = 2$, $I_1 = \frac{1}{2}$, $I_2 = \frac{3}{2}$, $\rho_1 = \rho_2 = \frac{1}{2}$, $\alpha = \frac{7}{32}$. *Let* $q = \frac{1}{10}$ *denote the probability that a consumer has valuation 1. The efficient output schedule has type 1 produce when at least one consumer has valuation 1, i.e.* $v \neq (0,0)$, *and type 2 produce when both consumers have the high valuation, i.e.* $v = (1,1)$. *Note that*

$$q^2(2 - I_2) < (1 - (1-q)^2)\alpha I_1$$

so that the efficient output schedule is not affluent. According to Strausz (2017, Prop. 2), this implies it is not implementable, but we show it is implementable. Let $p_1 = \frac{9}{10}$ *and* $p_2 = 2$ *and define direct mechanism* $(\boldsymbol{t}, \boldsymbol{x})$ *as follows. For* $k = 1, 2$, $(\boldsymbol{t}(I_k, v), \boldsymbol{x}(I_k, v)) = (0,0,0,0,0,0,0)$ *if* $v = (0,0)$, *and for* $v \neq (0,0)$ *define*

$$x_0(I_k, v) = \begin{cases} 0 \text{ if } n(v) < k \\ 1 \text{ if } n(v) \geq k \end{cases}$$
$$x_i(I_k, v) = v_i x_0(I_k, v)$$
$$t_i^a(I_k, v) = x_i(I_k, v) I_k / n(v)$$
$$t_i^p(I_k, v) = x_i(I_k, v)(p_k - I_k/n(v))$$

This mechanism clearly implements the efficient output schedule. We now show it is a weakly feasible mechanism by verifying that the individual rationality, incentive compatibility and obedience constraints hold. A high valuation consumer has expected utility

$$\rho_1(1 - p_1) + \rho_2 q(1 - p_2) = 0,$$

so his individual rationality constraint is satisfied. A low valuation consumer never pays or consumes anything, so his individual rationality constraint is satisfied. Consumers have also no incentive to misrepresent their type. The entrepreneur of type 1 is obedient and has no incentive to run off with the money, even in the least favorable demand state where $n(v) = 1$, *because* $p_1 - I_1 > \alpha I_1$. *Type 2 is obedient because* $2p_2 - I_2 > \alpha I_2$. *The entrepreneur of type 1 has no incentive to misrepresent her type because her expected profit is higher than she can obtain from misrepresentation:*

$$2q(1-q)(p_1 - I_1) + q^2(2p_1 - I_1) \geq q^2(2p_2 - I_1)$$

Observe that in case of misrepresentation, it is optimal for her to produce obediently because $2p_2 - I_1 > 2p_2 - I_2 > \alpha I_2$. *The entrepreneur of type 2 has no incentive to* *misrepresent her type (and then run with the money)*

$$q^2(2p_2 - I_2) \geq (1-(1-q)^2)\alpha I_1$$

Note that p_k equals the total price (the sum of ex ante and ex post transfers) paid by a high valuation consumer when the entrepreneur is of type k and produces. When the entrepreneur of type k is recommended to produce, she only learns that $n(v) \geq k$ and she receives ex ante an amount equal to I_k. So payments are deferred and information is restricted. These are the two tools for controlling moral hazard identified by Strausz (2017). The optimal weakly feasible mechanism in Example 2 uses a third tool: cross-subsidization over cost states. The intuition for how this helps is that the moral hazard concern is more severe in the only high cost state with production ($2 - I_2 < \alpha I_2$) than in states with $n(v) = 1$ and a low cost ($1 - I_1 > \alpha I_1$). Using the slack in the latter moral hazard constraint, one can relax the former by having consumers pay a higher total price when production occurs with high cost: $p_2 > 1 > p_1$. This implies that the optimal weakly feasible mechanism does not satisfy ex post individual rationality (29). Conditional on the high cost state, a high valuation consumer expects a loss of 1 (paying $p_2 = 2$ for a good worth only 1) with probability $q = 0.1$. But since cost information is private and because the consumer is compensated by paying only $p_1 = 0.9$ in the low cost state, $k = 1$, he is willing to participate ex ante.[17]

The cross-subsidization implies that the efficient output schedule cannot be implemented by a crowdfunding mechanism as defined by Strausz (2017) since that requires consumers to pay their valuation when production occurs. Actual reward-based crowdfunding, defined by a pair (p, T), also has the feature that consumers know what they pay in case of production, and is thus also unable to implement the efficient output schedule in Example 2. In the next section, we show that cross-subsidization is generically valuable in the constrained optimal outcome in the simplest setting of one consumer and two cost states.

4 Optimal mechanism design

In this section, we characterize the constrained optimum of the general mechanism in the case where $n = 1$ and the entrepreneur has two possible costs. The interesting case is that in which $I_1 < 1/(1+\alpha) < I_2$. It is straightforward to see that if $I_1 < I_2 < 1/(1+\alpha)$, the first-best can be implemented, because it can be done even when it is known that the cost state is the least favorable one, I_2, while if $1/(1+\alpha) < I_1 < I_2$, no production can

[17] Example 2 also demonstrates that Lemma 6 was false. Lemma 6 claims that, for any optimal weakly feasible mechanism, social surplus equals profit in each cost state. However, social surplus is strictly higher than profit in cost state I_1, while social surplus is strictly lower than profit in cost state I_2.

be implemented because it cannot be done even when the cost state is known to be the most favorable one, I_1. There are two subcases of interest: (a) $I_2 > 1$ and (b) $I_2 < 1$. Notice that, conditional on the valuation of the consumer being 1, it is efficient for both types of entrepreneur to produce in case (b) and for only type 1 to produce in case (a).

We maximize expected social surplus subject to IR and IC constraints. When the single consumer has valuation 0, it is efficient to not produce. One can show that in any feasible allocation schedule, there must be no transfers when the consumer has no value for the good. So we can, and do, simplify by considering the case where the consumer has valuation 1 for the good. This event has probability $\pi(1)$.

A direct mechanism must decide for each entrepreneurial type, $k = 1, 2$, the probability γ_k with which she is instructed to produce and the total price p_k she then receives, and the probability $\gamma'_k = 1 - \gamma_k$ with which she is instructed to not produce and the reward p'_k she then receives. To relax the moral hazard constraint, an optimal mechanism only pays $t^a_k = I_k$ as an ex ante transfer, when the entrepreneur is recommended to produce, and defers $t^p_k = p_k - I_k$, which is paid after verifying that investment has been made. When instead the entrepreneur is told not to produce, no ex ante transfer is made and the reward p'_k is also deferred.

The general optimization problem is thus

$$\max_{\gamma_1,\gamma'_1,\gamma_2,\gamma'_2,p_1,p'_1,p_2,p'_2} \pi(1)\Big[\gamma_1\rho_1(1-I_1) + \gamma_2\rho_2(1-I_2)\Big]$$

subject to

$$\gamma_1\rho_1(1-p_1) + \gamma'_1\rho_1(-p'_1) + \gamma_2\rho_2(1-p_2) + \gamma'_2\rho_2(-p'_2) \geq 0 \qquad (IR_C)$$

$$\gamma_1 = 0 \text{ or } p_1 - I_1 \geq \alpha I_1 \qquad (MH_1)$$

$$\gamma_2 = 0 \text{ or } p_2 - I_2 \geq \alpha I_2 \qquad (MH_2)$$

$$\gamma_1(p_1 - I_1) + \gamma'_1 p'_1 \geq \gamma_2 \max\{p_2 - I_1, \alpha I_2\} + \gamma'_2 p'_2 \qquad (IC_{12})$$

$$\gamma_2(p_2 - I_2) + \gamma'_2 p'_2 \geq \gamma_1 \alpha I_1 + \gamma'_1 p'_1 \qquad (IC_{21})$$

$$0 \leq \gamma_1, 0 \leq \gamma'_1 = 1 - \gamma_1 \qquad (1)$$

$$0 \leq \gamma_2, 0 \leq \gamma'_2 = 1 - \gamma_2 \qquad (2)$$

$$p'_1, p'_2 \geq 0 \qquad (3)$$

Proposition 1. *Let $n = 1$, $R = \rho_1/\rho_2 > 0$, $0 < I_1(1+\alpha) < 1 < I_2(1+\alpha)$. Define*

$$R_1 \equiv \frac{\alpha I_1}{1-(1+\alpha)I_1} \text{ and } R_2 \equiv \frac{I_1\big[(1+\alpha)I_2 - 1\big]}{I_1^2 + I_2 - (2+\alpha)I_1 I_2}$$

(a) When $I_2 > 1$, the first-best has $(\gamma_1, \gamma_2) = (1, 0)$ and:
If $R \geq R_1$, this can be implemented by setting $p'_2 = R(1-p_1) \geq \alpha I_1$ and any $p_1 \in [\frac{R+I_1}{R+1}, 1 - \frac{\alpha I_1}{R}]$. Raising p_1 transfers rent from high to low cost entrepreneurs. The first-

best can also be implemented without rewarding non-production, by using the limit of a sequence of mechanisms with $p_2' = 0$, by letting $p_2 = \alpha I_1/\gamma_2$ and $\gamma_2 \to 0$.
If $R < R_1$, no production can be implemented.

(b) When $I_2 < 1$, the first-best has $(\gamma_1, \gamma_2) = (1, 1)$; it cannot be implemented. Instead:
If $R \geq R_2$, the constrained optimum is uniquely determined by $(\gamma_1, \gamma_2) = (1, \gamma_2)$ where $\gamma_2 \in (0, 1)$ is defined by $p_2' = 0$ and binding constraints (IR_C), (IC_{12}) and (MH_2).
If $R_1 \leq R < R_2$, the constrained optimum is uniquely determined by $(\gamma_1, \gamma_2) = (1, \gamma_2)$ where $\gamma_2 \in (0, 1)$ is defined by $p_2' = 0$ and binding constraints (IR_C), (IC_{12}) and (IC_{21}); the range $(R_1 \leq R < R_2)$ is non-empty if and only if $\alpha > (1 - I_2)/I_1$.
If $R < \min\{R_1, R_2\}$, no production can be implemented.

Observe that Example 1's parameters lie in case (a). That example showed how inducing fraud was a feasible Pareto improvement over no production when $R \geq I_1/(1 - (1+\alpha)I_1)$. The proposition now reveals that the *optimal* solution does not involve fraud, but rather a "legitimated" reward (p_2') to type 2 for revealing her type without incurring the social waste $(1-\alpha)I_1$. This saving explains the less stringent condition on R. If such rewards were not allowed, the mechanism could still approximate the solution by instead having type 2 produce with a very small probability γ_2 in return for a very high price $p_2 = \alpha I_1/\gamma_2$, as indicated in part (a) of the proposition.

In case (b), it is efficient to have type 2 produce with probability 1, but the constrained optimum always has $\gamma_2 < 1$. We illustrate this possibility and an interesting aspect of the solution in the following numerical example.

Example 3. *Let $\alpha = 1/2$, $I_1 = 1/2$, $I_2 = 7/10$ and $R = 1$, so both types of entrepreneur are equally likely. In this case, $R > 1/3 = R_2$. Strausz (2017) claims nothing can be done but that is wrong. Let $p_1 = \frac{23}{24}$, $p_2 = \frac{21}{20}$, $\gamma_1 = 1$ and $\gamma_2 = \frac{5}{6}$. It is easily verified that all constraints are satisfied, with (IR_C), (MH_2) and (IC_{12}) binding.*

Note that if there was perfect information about the entrepreneur's cost, there would be no production for type 2 because $\alpha I_2 > 1 - I_2$. Hence, the example shows that private information may in fact alleviate incentive problems, contrary to Strausz's claim that private information always intensifies them (Strausz, 2017, p. 1431, 1442).

Returning to the general picture, notice that hiding cost information is generically valuable for increasing profits and welfare over Strausz's solution. To see this, note that consumers would never agree to $p_2 > 1$ if able to learn the cost level and equally obviously, they would never agree to pay $p_2' > 0$ if they knew this cost state had arisen.

5 Concluding remarks

We have identified a number of important errors and incorrect results (Lemmas 3, 5 and

6 and Propositions 2 and 3). In addition, we reject Strausz's claim that popular crowdfunding platforms implement the crucial features of the constrained optimal mechanism in the presence of moral hazard and private cost information. First, his analysis of the constrained optimum was incorrect. Second, the optimum cannot be implemented by existing crowdfunding platforms. We characterize the constrained optimum in the setting with one consumer and two types of entrepreneur. Example 1 shows how crowdfunding is more powerful than Strausz claims. Our Proposition 1 shows how optimal mechanism design can do even better. The optimum typically must hide cost information from consumers to reduce the impact of moral hazard constraints. Sometimes, this permits implementation of the first-best as shown in Example 2. In addition, we find that cost uncertainty can sometimes alleviate the moral hazard problem, as Example 3 illustrates.

The most fundamental analytical error was that Strausz (2017) tacitly assumed condition (29), the ex post individual rationality constraint of high types, when trying to prove his Lemma 5 which claimed that the optimal weakly feasible outcome satisfies (29). The only possible way to restore his results is to assume (29) throughout as Strausz now does consistently in his online corrigendum. However, as we argued, this is inconsistent with the assumption that cost information is private. Consumers only learn about a fraud event when the entrepreneur has run with the money, by which time it is simply too late to withdraw the pledge.

If there is no private cost information, so that there is only one type of entrepreneur, (29) is simply the interim individual rationality constraint which must be satisfied to guarantee consumer participation. In this case, Strausz (2017) does find the constrained optimal solution. His insight that deferred payments then help deal with moral hazard concerns nicely complements the results of Ellman and Hurkens (2015) who abstracted from moral hazard and showed that reward-based crowdfunding implements the general optimal mechanism under a more general formulation of demand uncertainty, generated by any two-point distribution of the individual valuation. Their results hold whether or not the entrepreneur has private information about her cost. Our analysis above shows that Strausz (2017)'s insight does not similarly extend to private cost information. This suggests that, to benefit from the attractive simplicity of crowdfunding, there may be a role for platforms to actively monitor entrepreneurs to compensate for their inability to implement cross-subsidization. It also presents an interesting challenge for future research on crowdfunding.

6 Appendix

Proof of Proposition 1.

We begin with a number of observations that help simplify the optimization problem. First, note that $I_1 < I_2$ implies that, if $\gamma_2 > 0$, $p_2 - I_1 > p_2 - I_2 \geq \alpha I_2$, where the weak inequality is just (MH_2). So $\gamma_2 \max\{p_2 - I_1, \alpha I_2\} = \gamma_2(p_2 - I_1)$.

Second, if $\gamma_1 = 0$, then $\gamma_2 = 0$. Suppose instead that $\gamma_2 > 0$. Then (IC_{12}) and (IC_{21}) imply that $p'_1 > p'_1$, a clear contradiction. Setting all prices equal to zero as well as $\gamma_1 = \gamma_2 = 0$, implements the no production solution with no waste. This has zero surplus, so we henceforth restrict attention to $\gamma_1 > 0$. From (MH_1), it then follows that $p_1 > 0$.

Third, without loss of generality, we can set $p'_1 = 0$. This is clear when $\gamma_1 = 1$. If $(1 - \gamma_1)p'_1 > 0$, then one can reduce p'_1 and increase p_1 while keeping γ_1 and $\gamma_1 p_1 + (1 - \gamma_1)p'_1$ constant. This relaxes constraints (MH_1) and (IC_{12}) and affects neither the other constraints nor the objective function.

Fourth, $\gamma_2 < 1$. This obviously holds for $\gamma_2 = 0$, while if $\gamma_2 > 0$, $p_2 \geq (\alpha+1)I_2 > 1$ by (MH_2). (IR_C) then implies that $p_1 < 1 < p_2$. This in turn implies that $\gamma_2 < 1$ because of (IC_{12}).

Fifth, (IR_C) must bind: an increase in p_1 relaxes (IC_{12}), tightens (IR_C) and does not affect objective function or other constraints.

Finally, note that (MH_1) is implied by (IC_{12}) and (IC_{21}). Hence, we can ignore this constraint in the optimization problem, which can therefore be rewritten as follows:

$$\max_{\gamma_1, \gamma_2, p_1, p_2, p'_2} \gamma_1 \rho_1 (1 - I_1) + \gamma_2 \rho_2 (1 - I_2)$$

subject to

$$-\left[\gamma_1 \rho_1 (1 - p_1) + \gamma_2 \rho_2 (1 - p_2) + (1 - \gamma_2)\rho_2(-p'_2)\right] = 0 \tag{4}$$
$$\gamma_2 \big[(\alpha + 1)I_2 - p_2\big] \leq 0 \tag{5}$$
$$\gamma_2(p_2 - I_1) + (1 - \gamma_2)p'_2 - \gamma_1(p_1 - I_1) \leq 0 \tag{6}$$
$$\gamma_1 \alpha I_1 - \gamma_2(p_2 - I_2) - (1 - \gamma_2)p'_2 \leq 0 \tag{7}$$
$$0 < \gamma_1 \leq 1 \tag{8}$$
$$0 \leq \gamma_2 < 1 \tag{9}$$
$$p'_2 \geq 0 \tag{10}$$
$$p_1 > 0 \tag{11}$$

The optimal solution $(\gamma_1, \gamma_2, p_1, p_2, p'_2)$ has $\gamma_1 = 1$, because otherwise $(\tilde{\gamma}_1, \tilde{\gamma}_2, p_1, p_2, \tilde{p}'_2)$ would be strictly better for $\tilde{\gamma}_k = \gamma_k(1 + \varepsilon)$ and $(1 - \tilde{\gamma}_2)\tilde{p}'_2 = (1 + \varepsilon)(1 - \gamma_2)p'_2$ for some small $\varepsilon > 0$.

Using Lagrange multipliers λ for the binding (IR_C) constraint and $\mu_1, \mu_2, \mu_3 \geq 0$ for the respective inequality constraints, we write the Lagrangian $\mathcal{L}$ as follows:

$$
\begin{aligned}
\mathcal{L} &= \rho_1(1-I_1) + \gamma_2\rho_2(1-I_2) + \lambda\big[\rho_1(1-p_1) + \gamma_2\rho_2(1-p_2) - (1-\gamma_2)\rho_2 p_2'\big] \\
&\quad -\mu_1\gamma_2\big[(\alpha+1)I_2 - p_2\big] - \mu_2\big[\gamma_2(p_2 - I_1) + (1-\gamma_2)p_2' - (p_1 - I_1)\big] \\
&\quad -\mu_3\big[\alpha I_1 - \gamma_2(p_2 - I_2) - (1-\gamma_2)p_2'\big]
\end{aligned}
$$

Necessary complementary slackness conditions for an optimum are

$$\gamma_2 \geq 0 \ \& \ \frac{\partial \mathcal{L}}{\partial \gamma_2} \leq 0 \ \& \ \gamma_2 \frac{\partial \mathcal{L}}{\partial \gamma_2} = 0 \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial p_1} = 0 \tag{13}$$

$$p_2 \geq 0 \ \& \ \frac{\partial \mathcal{L}}{\partial p_2} \leq 0 \ \& \ p_2 \frac{\partial \mathcal{L}}{\partial p_2} = 0 \tag{14}$$

$$p_2' \geq 0 \ \& \ \frac{\partial \mathcal{L}}{\partial p_2'} \leq 0 \ \& \ p_2' \frac{\partial \mathcal{L}}{\partial p_2'} = 0 \tag{15}$$

where

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \gamma_2} &= \rho_2(1-I_2) + \lambda\rho_2\big[1 - p_2 + p_2'\big] - \mu_1\big[(\alpha+1)I_2 - p_2\big] \\
&\quad -\mu_2\big[p_2 - I_1 - p_2'\big]) + \mu_3\big[p_2 - I_2 - p_2'\big] \qquad (16) \\
\frac{\partial \mathcal{L}}{\partial p_1} &= -\lambda\rho_1 + \mu_2 \qquad (17) \\
\frac{\partial \mathcal{L}}{\partial p_2} &= \gamma_2\big[-\lambda\rho_2 + \mu_1 - \mu_2 + \mu_3\big] \qquad (18) \\
\frac{\partial \mathcal{L}}{\partial p_2'} &= (1-\gamma_2)\big[-\lambda\rho_2 - \mu_2 + \mu_3\big] \qquad (19)
\end{aligned}
$$

CASE 1: We first consider the candidate solutions with $\gamma_2 = 0$. This can be implemented if and only if there exist $p_1, p_2' \geq 0$ such that

$$
\begin{aligned}
&\rho_1(1-p_1) - \rho_2 p_2' = 0 \\
&p_2' - (p_1 - I_1) \leq 0 \\
&\alpha I_1 - p_2' \leq 0
\end{aligned}
$$

Hence, writing $R = \rho_1/\rho_2$, this requires

$$\alpha I_1 \leq p_2' = R(1-p_1) \leq p_1 - I_1,$$

or, equivalently,

$$\frac{R+I_1}{R+1} \leq p_1 \leq \frac{R-\alpha I_1}{R}$$

which has a solution if and only if

$$R \geq \frac{\alpha I_1}{1-(\alpha+1)I_1} = R_1$$

Of course, when the latter condition is satisfied and $I_2 \geq 1$, this is in fact the optimal solution.

CASE 2: We now consider solutions with $\gamma_2 > 0$. Hence, (12) can be replaced by $\partial\mathcal{L}/\partial\gamma_2 = 0$. Also note that if $p_2' > 0$, then raising p_2 and lowering p_2' while keeping $\gamma_2 p_2 + (1-\gamma_2)p_2'$ constant, relaxes (MH_2) while not affecting the objective function or the other constraints. So, w.l.o.g. we may set $p_2' = 0$ in this case.

Note that if $\mu_2 = 0$, then also $\lambda = 0$ from (17). But then also $\mu_1 = \mu_3 = 0$ from (14) and (18), because $\mu_1 \geq 0$ and $\mu_3 \geq 0$. This contradicts $\partial\mathcal{L}/\partial\gamma_2 = 0$ when $I_2 \neq 1$. Hence, $\mu_2 > 0$ and the corresponding constraint (IC_{12}) must bind:

$$\gamma_2(p_2 - I_1) = p_1 - I_1 \tag{20}$$

Writing as before $R = \rho_1/\rho_2 \in (0,\infty)$ for the relative probability of a low cost type 1, we can rewrite the binding (IR_C) constraint as

$$\gamma_2 = \frac{R(1-p_1)}{p_2 - 1} \tag{21}$$

This is well-defined because by (MH_2), $p_2 \geq (\alpha+1)I_2$ and the latter strictly exceeds 1. From (20) and (21) one can express p_2 and γ_2 as functions of p_1:

$$p_2(p_1) = \frac{p_1 - I_1 - R(1-p_1)I_1}{p_1 - I_1 - R(1-p_1)} \tag{22}$$

$$\gamma_2(p_1) = \frac{p_1 - I_1 + R(p_1 - 1)}{1 - I_1} \tag{23}$$

Note that $\gamma_2(p_1)$ is a linear, strictly increasing function and that $0 < \gamma_2(p_1) < 1$ if and only if $p_1 \in (p_{min}, 1)$, where $p_{min} = \frac{I_1+R}{1+R}$. Note that $p_2(p_1)$ is decreasing on this domain, with $\lim_{p_1 \uparrow 1} p_2(p_1) = 1$ and $p_2(p_1) \to \infty$ as $p_1 \downarrow p_{min}$.

The question is now whether p_1 can be chosen within this domain so that constraints (MH_2) and (IC_{21}) are also satisfied. (MH_2) requires that $p_2(p_1) \geq \hat{p}_2 = (\alpha+1)I_2$. Hence, $p_1 \leq p_2^{-1}(\hat{p}_2) \equiv \bar{p}_1$. It is straightforward to show that

$$\bar{p}_1 = \frac{(I_1+R)(\alpha+1)I_2 - I_1(1+R)}{(1+R)(\alpha+1)I_2 - (1+RI_1)}$$

(IC_{21}) requires that $Z(p_1) \equiv \gamma_2(p_1)(p_2(p_1) - I_2) \geq \alpha I_1$. It can be verified that $Z(p_1)$ is a linear function of p_1:

$$Z(p_1) = \frac{I_1(I_2 - (R+1)) + I_2 R + p_1(1 + I_1 R - I_2(R+1))}{1 - I_1}$$

This function is increasing if and only if

$$R \leq \frac{1 - I_2}{I_2 - I_1}$$

Straightforward calculations show that

$$Z(\bar{p}_1) = \frac{(1 - I_1)I_2 R\alpha}{I_2(1+R)(1+\alpha) - (1 + I_1 R)} \text{ and } \lim_{p_1 \downarrow p_{min}} Z(p_1) = \frac{(1 - I_1)R}{1 + R}$$

If $I_2 > 1$, $Z(p_1)$ is strictly decreasing. Note that the objective function is now decreasing in γ_2 so that lower p_1 is better. There are feasible allocations if and only if $\lim_{p_1 \downarrow p_{min}} Z(p_1) > \alpha I_1$, that is, when

$$R > \frac{\alpha I_1}{1 - (\alpha + 1)I_1} \equiv R_1(\alpha)$$

But in this case there exists the solution from case 1 with $\gamma_2 = 0$, which implements the first-best. If the condition is not met, there are no feasible allocation schedules with positive production.

If $I_2 < 1$, the objective function is increasing in γ_2. Hence, one seeks the maximal p_1. We consider two cases.

First, suppose $R \leq (1 - I_2)/(I_2 - I_1)$. Now $Z(p_1)$ is increasing. There is a solution with production if and only if $Z(\bar{p}_1) \geq \alpha I_1$. This is satisfied if and only if

$$R \geq \frac{I_1((1+\alpha)I_2 - 1)}{I_1^2 + I_2 - (2+\alpha)I_1 I_2} \equiv R_2(\alpha)$$

The solution is determined by $\bar{p}_1$. The constraints (IR_C), (MH_2) and (IC_{12}) are binding.

Second, consider the case with $R > (1 - I_2)/(I_2 - I_1)$. Then $Z(p_1)$ is decreasing. If $\lim_{p_1 \downarrow p_{min}} Z(p_1) = \frac{(1-I_1)R}{1+R} > \alpha I_1$, there exists a unique $\tilde{p}_1 > p_{min}$ such that $Z(\tilde{p}_1) = \alpha I_1$. The optimal solution involves production by type 2 and has optimal price $p_1 = \min\{\tilde{p}_1, \bar{p}_1\}$. In particular, when $\tilde{p}_1 < \bar{p}_1$, constraint (MH_2) is slack while (IC_{21}) then binds. If $\lim_{p_1 \downarrow p_{min}} Z(p_1) \leq \alpha I_1$, there exists no implementable allocation in which type 2 produces. Note that $\lim_{p_1 \downarrow p_{min}} Z(p_1) \leq \alpha I_1$ if and only if

$$R \leq \frac{\alpha I_1}{1 - (1+\alpha)I_1} = R_1(\alpha)$$

Figure 1 illustrates.

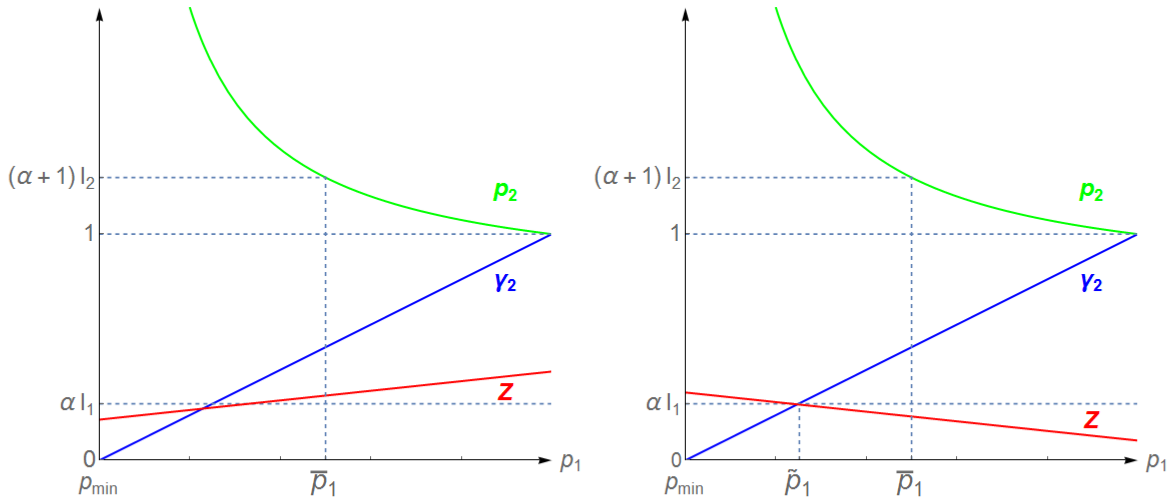

Figure 1: Production by type 2 is feasible if and only if $Z(p_1) \geq \alpha I_1$ for some $p_{min} < p_1 \leq \bar{p}_1$

The conclusions from Proposition 1 now follow from the following observations. Note that R_1 and R_2 are strictly increasing in α. Given that $I_1 > 0$, straightforward calculations show that $R_1(\alpha) = R_2(\alpha)$ if and only if $\alpha = \hat{\alpha} \equiv (1 - I_2)/I_1$, and that $R_1(\hat{\alpha}) = R_2(\hat{\alpha}) = (1 - I_2)/(I_2 - I_1) \equiv \bar{R}$. Moreover, $R_2 - R_1$ is strictly increasing at $\alpha = \hat{\alpha}$:

$$R'_2(\hat{\alpha}) - R'_2(\hat{\alpha}) = \frac{I_1 I_2(1 - I_1)}{(I_2 - I_1)^3} - \frac{I_1(1 - I_1)}{(I_2 - I_1)^2} = \frac{I_1^2(1 - I_1)}{(I_2 - I_1)^3} > 0$$

Figure 2 illustrates the three different subcases when $I_2 < 1$. Note that the feasible region of α is restricted to $(\underline{\alpha}, \bar{\alpha})$ where $\underline{\alpha} = 1/I_2 - 1$ and $\bar{\alpha} = 1/I_1 - 1$.

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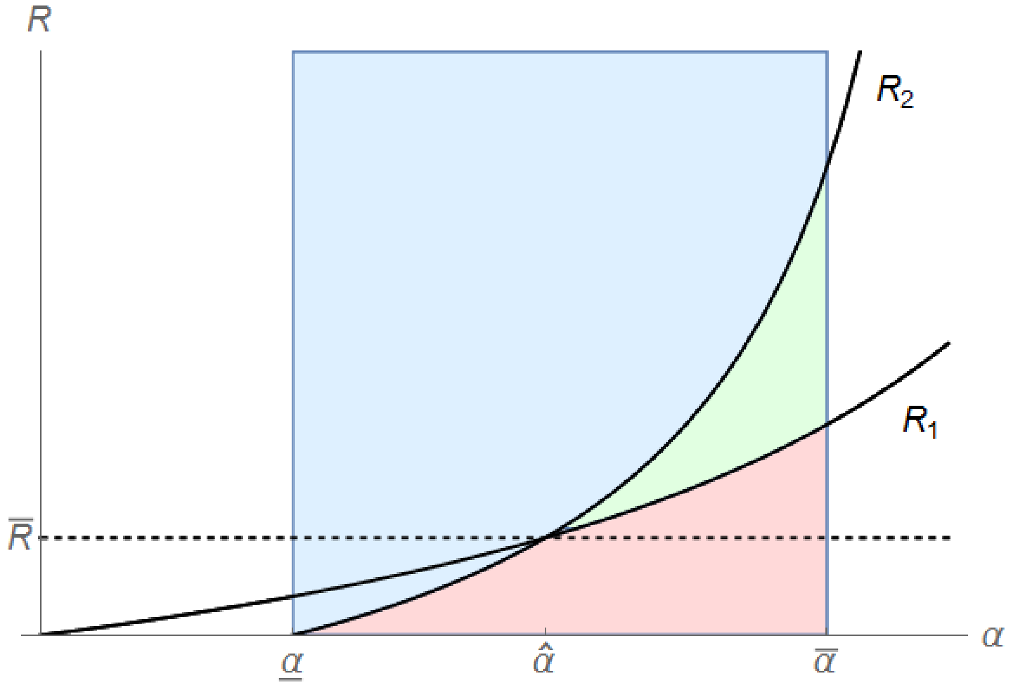

Figure 2: Constrained optima when $I_2 < 1$. In the lower (red) region, there is no production for any type. In the upper green and blue regions, type 1 produces with probability 1 and type 2 with some probability $\gamma_2 \in (0, 1)$. The constrained optima are characterized by the binding constraints (IR_C), (IC_{21}), and (MH_2) in the large blue region and by (IR_C), (IC_{21}), and (IC_{12}) in the smaller green region.