Debt Dilution and Debt Overhang

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Debt Dilution and Debt Overhang*

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Abstract

We introduce long-term debt (and a maturity choice) into a standard model of firm financing and investment. This allows us to study two distortions of investment: (1.) *Debt dilution* distorts firms' choice of debt which has an indirect effect on investment; (2.) *Debt overhang* directly distorts investment. In a dynamic model of investment, leverage, and debt maturity, we show that the two frictions interact to reduce investment, increase leverage, and increase the default rate. We provide empirical evidence from U.S. firms that is consistent with the model predictions. Using our model, we isolate and quantify the effect of debt dilution and debt overhang. Debt dilution is more important for firm value than debt overhang. Debt overhang can actually *increase* firm value by reducing debt dilution. The negative effect of debt dilution on investment is about half as strong as that of debt overhang. Eliminating the two distortions leads to an increase in investment equivalent to a reduction in the corporate income tax of 3.5 percentage points.

Keywords: investment, debt dilution, debt overhang.
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1. Introduction

This paper starts out from a simple observation. Empirically, most firm debt is long-term. About 67% of the average U.S. corporation's total stock of debt does not mature within one year. This fact is missing from many economic models. The standard assumption is that all firm debt is short-term, i.e. all debt issued in period t matures in period $t+1$. This paper introduces long-term debt (and a maturity choice) to a standard model of firm financing and investment.[1] This allows us to study two important problems which are absent from standard models: debt dilution and debt overhang.

In a model with long-term debt, firms take decisions in the presence of previously issued outstanding debt. If a firm decides to increase its stock of debt, this raises the risk of default and lowers the price at which the firm can sell new debt. The firm fully internalizes the reduction in the value of newly issued debt. But a higher risk of default also lowers the value of existing debt. This dilution of the value of existing debt is not internalized by the firm (*debt dilution*). The value of existing debt is also affected by the firm's investment decision. If investment increases the value of existing debt, this benefit is not internalized by the firm and reduces the gains which accrue to shareholders (*debt overhang*).

The effect of debt overhang on investment is well known and studied since Myers (1977). The main contribution of our paper is to identify the effect of debt dilution on investment. Debt dilution induces firms to increase leverage and the risk of default. Higher credit spreads imply an increased cost of capital which makes investment less profitable. We describe the effect of debt dilution on investment and firm value, compare it to debt overhang, and study the interaction between the two distortions.

We begin our analysis in a simple two-period economy and derive analytical results on the role of debt dilution and debt overhang. We then proceed to solve a fully dynamic model of investment, leverage, and debt maturity. Investment can be financed through equity and debt. Debt is attractive because of its tax-advantage. The downside is that firms may default because of limited liability. Firms issue both short-term debt and long-term debt. A high share of long-term debt saves roll-over costs, but it also increases the severity of debt dilution and debt overhang in the future.

We calculate the global solution to the problem of a firm which dynamically chooses capital, leverage, and debt maturity. Our quantitative results show that debt dilution and debt overhang reduce investment, increase leverage, and increase the default rate. This leads to high credit spreads and low output. In our model, firms can minimize debt dilution and debt overhang by choosing a low share of long-term debt. However, firms do not internalize all costs of long-term debt. In equilibrium, the share of long-term debt is high and the effects of debt dilution and debt overhang are large.

Using firm-level data from Compustat and Moody's Default & Recovery Database,

[1] In contrast to Gomes, Jermann, and Schmid (2016), we focus on real debt and do not explore the implications of nominal debt. There is a long tradition in corporate finance of modelling long-term debt. In Section 2, we explain how our results relate to this literature. Long-term debt also yields important results in models of sovereign borrowing and default, e.g. Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), or Hatchondo, Martinez, and Sosa-Padilla (2016).

we construct an empirical proxy for the severity of debt dilution and debt overhang. Reduced-form evidence is in line with our model predictions and suggests that the two distortions are economically significant. Just as in the model, our firm-specific proxy for debt dilution and debt overhang is negatively related to a firm's rate of asset growth, and positively related to leverage and default risk. In the model and in the data, these relationships are stronger for firms with a smaller distance to default.

Both debt dilution and debt overhang are the result of a commitment problem. Since firms cannot credibly promise to internalize the payoff to long-term creditors in the future, creditors demand high credit spreads on long-term debt. Our model allows to isolate and quantify the distinct roles of the two distortions. In the last part of the paper, we assume that firms can choose the total stock of debt, capital, or both with full commitment. In that way, we selectively eliminate either debt dilution, debt overhang, or both.

We find that debt dilution is more costly in terms of firm value, whereas debt overhang has a stronger effect on investment. Because of diminishing returns, the marginal unit of capital contributes little to firm value. This is why the large effect of debt overhang on investment does not translate into large effects on firm value. Debt overhang provides an incentive for firms to limit their share of long-term debt which mitigates debt dilution. This positive effect of debt overhang on firm value can actually outweigh the negative effect from reduced investment.

In our model economy, eliminating debt overhang leads to a permanent increase in investment equivalent to a reduction in the corporate income tax of up to 2.75 percentage points. The effect of debt dilution on investment is sizeable as well. Eliminating debt dilution can raise investment by as much as a tax reduction of 1.35 percentage points. If both distortions are eliminated, the resulting increase in investment corresponds to a tax reduction of 3.50 percentage points. These effects are absent from standard models with one-period debt. In this sense, long-term debt amplifies the steady state effect of financial frictions on economic activity.

Our model deliberately abstracts from financial instruments like debt covenants or secured debt. The empirical corporate finance literature finds that less than 25% of investment grade bonds include covenants which address debt dilution, and less than 20% feature restrictions with the potential to limit debt overhang.[2] Costs resulting from reduced flexibility might help explain why firms do not use these covenants more intensively in practice.

Secured debt and seniority structures have opposing effects on the two distortions. Stulz and Johnson (1985) and Hackbarth and Mauer (2011) find that debt overhang is more severe if existing debt is prioritized (or secured). Newly issued debt should be prioritized (or secured) to reduce debt overhang. On the other hand, Chatterjee and Eyigungor (2015) show that debt dilution is reduced if existing debt has priority.

[2]It is very common that covenants limit the issuance of additional secured debt with priority over existing debt. Covenants which limit the issuance of additional unsecured debt with identical (or lower) seniority (e.g. through general leverage limits or minimum interest coverage ratios) are far less common. See Nash, Netter, and Poulsen (2003), Begley and Freedman (2004), Billett, King, and Mauer (2007), and Reisel (2014). We discuss this evidence in more detail in Appendix A.

Granting priority to newly issued debt renders debt dilution more severe. These opposing effects may explain why the use of secured debt is limited. For the median firm in our Compustat sample, the share of secured debt is 19%. We conclude that for the typical U.S. corporation neither debt covenants nor secured debt play a major role in limiting debt dilution or debt overhang.

In Section 2, we survey some related literature. Section 3 provides analytical results on debt dilution and debt overhang in a simple two-period setup. We extend our analysis to a fully dynamic economy in Section 4. We test the model predictions in Section 5 using firm-level evidence from the U.S. corporate sector. In Section 6, we use our model to isolate and quantify the distinct roles of debt dilution and debt overhang. Concluding remarks follow.

2. Related Literature

The paper most closely related to ours is Gomes et al. (2016). Their main result is that shocks to inflation change the real burden of outstanding nominal long-term debt and thereby distort investment. The key difference to our paper is that Gomes et al. (2016) do not discuss and disentangle the joint effect of debt dilution and debt overhang. Furthermore, they focus on cyclical fluctuations while we study the effect of debt dilution and debt overhang on steady state quantities. Their model solution describes deviations from a deterministic steady state whereas we calculate a fully non-linear global solution. Another difference is that they do not allow for short-term debt issuance. This assumption is restrictive since a maturity choice allows firms to respond to and mitigate distortions from debt dilution and debt overhang.

A second related paper is Crouzet (2016). His focus lies on firms' debt maturity choice and he does not discuss the respective roles of debt dilution and debt overhang for investment. Other models of firm investment with long-term debt rule out debt dilution and debt overhang a priori, either by assuming that debt is riskless (e.g. Alfaro, Bloom, and Lin (2016)) or that firms need to retire all outstanding debt before investing and issuing new debt (e.g. Caggese and Perez (2015)). Discrete-time models with one-period debt share this feature by construction (e.g. Bernanke, Gertler, and Gilchrist (1999), Cooley and Quadrini (2001), Hennessy and Whited (2005), Covas and Den Haan (2012), and Katagiri (2014)).

Debt dilution has previously been identified as a mechanism which generates excessive leverage and default risk (e.g. Admati, DeMarzo, Hellwig, and Pfleiderer (2013)).[3] The effect of debt dilution on investment has not been systematically studied. Most closely related to our work is the model of debt dilution by DeMarzo and He (2016) which includes an extension with endogenous investment. The authors do not solve for the optimal firm policy under full commitment and therefore do not identify the separate effects of debt dilution and debt overhang on investment. Brunnermeier and Oehmke (2013) show that debt dilution influences the maturity choice even if a firm's debt level is fixed. In their setup, creditors learn about a firm's default risk over time. In our

[3] See also Bizer and DeMarzo (1992), Kahn and Mookherjee (1998), and Parlour and Rajan (2001).

setup, the "rat race" mechanism is absent because all creditors can exactly predict a firm's current and future default risk.

Debt overhang is a key concept in corporate finance since the seminal contribution by Myers (1977). Subsequent studies of debt overhang include Hennessy (2004), Moyen (2007), Titman and Tsyplakov (2007), Diamond and He (2014), and Occhino and Pescatori (2015). Debt dilution is not a concern in this literature, either because debt is exogenous, chosen with full commitment, or fully retired before the issuance of new debt. Our result that debt overhang can mitigate debt dilution is therefore absent from these contributions.

Degryse, Ioannidou, and von Schedvin (2016) provide empirical evidence on debt dilution. Empirical studies of debt overhang are Lang, Ofek, and Stulz (1996), Hennessy (2004), and Hennessy, Levy, and Whited (2007).

The literature on sovereign default has found that debt dilution helps to generate realistic levels of sovereign debt and credit spreads. A non-exhaustive list includes Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2013), Chatterjee and Eyigungor (2015), Hatchondo et al. (2016), and Aguiar, Amador, Hopenhayn, and Werning (2016). Since these are models of endowment economies, there is no effect of debt dilution on investment and there is no debt overhang.

3. Two-period Model

We begin our analysis by studying a two-period model of a firm which finances its capital stock through equity and debt. The optimal capital structure solves a trade-off between the tax advantage of debt and the expected costs of default. The firm chooses investment and leverage in the presence of previously issued long-term debt. This variable is exogenous in the two-period setup. It will be endogenized in the fully dynamic economy of Section 4.

In the presence of previously issued long-term debt, two investment distortions arise: (1.) *Debt dilution* affects the firm's incentive to borrow because not all costs from additional debt are internalized by the firm. This has an indirect effect on investment. (2.) *Debt overhang* directly affects investment because the firm does not internalize all associated benefits. We use the simple two-period setup to derive analytical results on the effect of debt dilution and debt overhang on investment.

3.1. Setup

There are two periods: $t = 1, 2$. In period 2, a firm uses capital k to produce output y using a technology with diminishing returns:

$$y = f(k)\,. \tag{1}$$

The production function $f(k)$ is increasing and concave. Capital depreciates at rate δ.[4] Firm earnings are uncertain because of an earnings shock ε. Earnings before interest and taxes are given as:

$$f(k) - \delta k + \varepsilon k\,. \tag{2}$$

At t=1, ε is a random variable with probability density $\varphi(\varepsilon)$. There are two ways to finance the capital stock k in the initial period: equity and debt.

Definition: Debt. A debt security is a promise to pay one unit of the numéraire good together with a fixed coupon payment c at the end of period 2.

In this two-period model, the firm issues new debt only once. Because we are interested in the question of how previously issued long-term debt affects the firm's behavior, we introduce an exogenous variable b which denotes the quantity of bonds outstanding at the beginning of the initial period. These bonds mature at time 2 just like the one-period bonds which the firm can issue in period 1. One may think of b as long-term debt which has been issued before period 1.

Let p be the market price of a one-period bond sold by the firm in period 1. If the firm sells an amount Δ of new bonds, it raises an amount $p\Delta$ on the bond market. This brings its stock of debt to $b + \Delta = \tilde{b}$. Let e be the stock of equity. The capital stock in period 1 is given as:

$$k \;=\; e + p\Delta \;=\; e + p\,(\tilde{b} - b)\,. \tag{3}$$

Firm earnings are taxed at rate τ. The firm's stock of equity after production in period 2 is:

$$q = k - \tilde{b} + (1-\tau)[f(k) - \delta k + \varepsilon k - c\tilde{b}]\,. \tag{4}$$

The fact that coupon payments are tax-deductible lowers the total tax payment by the amount $\tau c\tilde{b}$. This is the benefit of debt. The downside is that the firm cannot commit to repaying its debt after production in period 2.

Definition: Limited Liability. Shareholders are protected by limited liability. They are free to default and hand over the firm's assets to creditors for liquidation. Default is costly. A fixed fraction ξ of the firm's assets is lost in this case.

The timing can be summarized as follows.

t=1 Given an existing stock of debt b, the firm chooses capital k. Capital is financed using equity e and through the revenue $p\,(\tilde{b} - b)$ from the sale of additional bonds.

t=2 The firm's stock of debt is $\tilde{b}$. Earnings are: $f(k) - \delta k + \varepsilon k$. The firm decides whether to default.

[4] Capital is the only factor of production. This is not restrictive if one assumes that variable production factors, e.g. labor, are optimally chosen conditional on capital. In this case, the parameter δ also captures the cost of variable production factors, e.g. wages. See Cooley and Quadrini (2001).

3.2. Firm Problem

The firm maximizes shareholder value. Since shareholders are risk neutral, the firm's objective is the expected present value of net cash flows from the firm to shareholders.

We can solve the firm's problem using backward induction, beginning with the default decision after the realization of firm earnings in period 2. Limited liability protects shareholders from large negative realizations of the earnings shock ε. Given a firm's stock of capital k and debt $\tilde{b}$, there is a unique threshold realization $\bar{\varepsilon}$ which sets the firm's equity stock after production q equal to zero:

$$\bar{\varepsilon}: \quad q = 0 \quad \Leftrightarrow \quad k - \tilde{b} + (1-\tau)[f(k) - \delta k + \bar{\varepsilon} k - c\tilde{b}] = 0\,. \tag{5}$$

If ε is smaller than $\bar{\varepsilon}$, full repayment would result in negative equity q while default provides an outside option of zero. In this case, the firm optimally decides to stop paying its liabilities and defaults.

In period 1, the firm decides on its scale of production k, and its preferred financing mix of equity and debt. The firm anticipates that shareholders receive q whenever $\varepsilon \geq \bar{\varepsilon}$ and zero otherwise:

$$\max_{k,e,\tilde{b},\bar{\varepsilon}} \quad -e + \frac{1}{1+r} \int_{\bar{\varepsilon}}^{\infty} [k - \tilde{b} + (1-\tau)[f(k) - \delta k + \varepsilon k - c\tilde{b}]\; \varphi(\varepsilon)\, d\varepsilon \tag{6}$$

$$\text{subject to:} \quad 0 = k - \tilde{b} + (1-\tau)[f(k) - \delta k + \bar{\varepsilon} k - c\tilde{b}]$$

$$k = e + p\,(\tilde{b} - b)\,,$$

where r is the risk-free interest rate. The optimal firm policy crucially depends on the bond price p. A high bond price implies a low credit spread which reduces the firm's cost of capital and makes it attractive to finance investment through debt instead of equity. We derive the firm-specific bond price from the creditors' optimization problem.

3.3. Creditors' Problem

Creditors are risk-neutral and discount the future at the same rate $1/(1+r)$ as shareholders. They buy the firm's debt in period 1. If the firm does not default in period 2, they receive full repayment. In case of default, they receive the firm's liquidation value $(1-\xi)\,\tilde{q}$, where:

$$\tilde{q} \equiv k + (1-\tau)[f(k) - \delta k + \varepsilon k]\,. \tag{7}$$

Competitive creditors break even on expectation. The break-even price p of firm debt in period 1 depends on the probability $\Phi(\bar{\varepsilon})$ that the firm defaults in period 2:

$$p = \frac{1}{1+r}\left[[1 - \Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}} \int_{-\infty}^{\bar{\varepsilon}} \tilde{q}\,\varphi(\varepsilon)\, d\varepsilon\right]. \tag{8}$$

If creditors expect a positive risk of default, they will charge a credit spread over the riskless rate.[5]

3.4. Equilibrium

We solve for the partial equilibrium allocation given the risk-free rate r. In equilibrium, the firm maximizes shareholder value (6) subject to creditors' break-even condition (8). We can simplify this problem by re-writing it in terms of only two endogenous variables: the scale of production k, and the default threshold $\bar{\varepsilon}$.

3.4.1. Consolidated Problem

We begin by expressing the stock of debt $\tilde{b}$ in terms of k and $\bar{\varepsilon}$. From the definition of $\bar{\varepsilon}$ it follows:

$$\tilde{b}\,[1+(1-\tau)c] = k + (1-\tau)[f(k) - \delta k + \bar{\varepsilon}k] \;\Leftrightarrow\; \tilde{b} = \frac{k + (1-\tau)[f(k) - \delta k + \bar{\varepsilon}k]}{1+(1-\tau)c}\,. \tag{9}$$

Consider first the left hand side of the first equation in (9). Creditors are entitled to a fixed payment of $\tilde{b}[1+c]$ in period 2. But effectively the firm only pays $\tilde{b}[1+(1-\tau)c]$ because it can deduct $\tilde{b}\tau c$ from its tax bill. The right hand side of the first equation states that this payment consists of two parts: the safe part of firm assets after production, $k+(1-\tau)[f(k)-\delta k]$, and a fixed promised amount of the risky part of earnings, $(1-\tau)\bar{\varepsilon}k$.

Similarly, we can express equity e in terms of k, $\tilde{b}$, and p:

$$e = k - p\,(\tilde{b} - b)\,. \tag{10}$$

Using these two expressions, the firm's problem can be re-written as:

$$\max_{k,\tilde{b},\bar{\varepsilon},p} \quad -k \,+\, p\,(\tilde{b} - b) \,+\, \frac{1-\tau}{1+r}\,k \int_{\bar{\varepsilon}}^{\infty} [\varepsilon - \bar{\varepsilon}]\;\varphi(\varepsilon)\,d\varepsilon\,. \tag{11}$$

From (8), we know that p depends on k, $\bar{\varepsilon}$, and $\tilde{b}$. Since $\tilde{b}$ itself is a function of k and $\bar{\varepsilon}$, (11) characterizes the equilibrium allocation in terms of k and $\bar{\varepsilon}$ only. The firm maximizes (11) subject to (8) and (9).

The firm's objective is to maximize shareholder value. But in (11), the firm maximizes the total return to capital k including the value of newly issued debt $p(\tilde{b} - b)$. Shareholders benefit from a high value of newly issued debt as less equity e is required for a

[5] Sometimes more than one bond price satisfies creditors' break-even condition. In this case, different bond prices also imply different default probabilities. See Calvo (1988). The conditions which introduce multiplicity are described in Nicolini, Teles, Ayres, and Navarro (2015). By allowing the firm in (6) to directly select the default probability through $\bar{\varepsilon}$, we implicitly assume that the firm sells its bonds to creditors by making a take-it-or-leave-it offer specifying both a price p and a quantity $\tilde{b}$ of bonds. This implies that the firm is always able to select the preferred default probability and there is a unique equilibrium. See also Crouzet (2016).

given level of capital k (see equation (10)). Because creditors break even on expectation, shareholders appropriate the entire surplus created by the investment of k.

3.4.2. Debt Dilution

First, consider the special case of $\xi = 1$. This means that the liquidation value of the firm is zero in case of default and the bond price in (8) only depends on $\bar{\varepsilon}$:

$$p = \frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\,. \tag{12}$$

The derivative of (11) with respect to k yields a first order condition for the optimal scale of production:

$$\underbrace{-1}_{\substack{Marginal\\cost\,of\\capital}} + \underbrace{\frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\frac{1+(1-\tau)[f'(k)-\delta+\bar{\varepsilon}]}{1+(1-\tau)c}}_{\substack{Marginal\,increase\\in\,value\,of\,newly\,issued\,debt}} + \underbrace{\frac{1-\tau}{1+r}\int_{\bar{\varepsilon}}^{\infty}[\varepsilon-\bar{\varepsilon}]\;\varphi(\varepsilon)\,d\varepsilon}_{\substack{Marginal\,increase\\in\,expected\,dividend}} \;=\; 0 \tag{13}$$

A marginal increase in k has an opportunity cost of one. The benefit consists of an increase in the value of newly issued debt and equity. Because of diminishing returns to production, the marginal increase in the value of newly issued debt is falling in k. Note that the number of previously issued bonds b does not appear in the first order condition for k. Conditional on the firm's choice of $\bar{\varepsilon}$, the existing stock of debt b does not influence investment. In other words, with $\xi = 1$ there is no debt overhang.

A first order condition for an optimal choice of $\bar{\varepsilon}$ is:

$$\underbrace{[1-\Phi(\bar{\varepsilon})](1-\tau)k\frac{\tau c}{1+(1-\tau)c}}_{Marginal\,tax\,benefit\,of\,\bar{\varepsilon}} \quad - \quad \underbrace{\varphi(\bar{\varepsilon})(1+c)(\tilde{b}-b)}_{\substack{Marginal\,increase\,in\\expected\,costs\,of\,default\\internalized\,by\,the\,firm}} \;=\; 0 \tag{14}$$

The first term is the marginal benefit of an increase in $\bar{\varepsilon}$. It is weighted with the repayment probability $[1-\Phi(\bar{\varepsilon})]$. If default is avoided, a higher value of $\bar{\varepsilon}$ increases the fixed amount promised to creditors by $(1+c)\partial\tilde{b}/\partial\bar{\varepsilon}$, and reduces the dividend by $(1-\tau)k$.[6] Since coupon payments are tax-deductible, it costs shareholders only $1+(1-\tau)c$ to increase the promised payment to creditors by $1+c$. Because competitive creditors break even, the entire tax benefit generated from substituting equity with debt is captured by shareholders.

The second term in (14) is the marginal cost of an increase in $\bar{\varepsilon}$. The probability of default increases by $\varphi(\bar{\varepsilon})$ and creditors lose the entire amount of $(1+c)\tilde{b}$ in this case.

[6] The marginal tax benefit of $\bar{\varepsilon}$ can be written as:

$$[1-\Phi(\bar{\varepsilon})](1-\tau)k\frac{\tau c}{1+(1-\tau)c} = [1-\Phi(\bar{\varepsilon})]\left[(1+c)\frac{\partial\tilde{b}}{\partial\bar{\varepsilon}} - (1-\tau)k\right]\,.$$

While the firm internalizes the tax benefit of the entire stock of debt $\tilde{b}$, it does not internalize all associated costs. The firm takes into account that an increase of $\bar{\varepsilon}$ lowers the value of newly issued bonds $p(\tilde{b} - b)$. But it disregards the fact that this also lowers the value of previously issued debt pb. This *dilution* of the value of previously issued debt through the sale of additional debt is not internalized by the firm.

The optimal value of $\bar{\varepsilon}$ is pinned down by the trade-off between the tax advantage of debt and the internalized part of the expected costs of default. Proposition 3.1 describes the effect of debt dilution on the firm's behavior at an interior solution where the two first order conditions hold.

Proposition 3.1. ***Debt Dilution:*** *Assume* $\xi = 1$.

1. *The default rate* $\Phi(\bar{\varepsilon})$ *is increasing in the stock of existing debt* b.
2. *For* $b < \underline{b}$, *capital* k *is increasing in* b. *For* $b > \underline{b}$, *capital is falling in* b. *The threshold value* $\underline{b}$ *is:*
$$\underline{b} \equiv \frac{(1-\tau)k\left[\frac{f(k)}{k} - f'(k)\right]}{1 + (1-\tau)c}.$$
3. *If* $b > \underline{b}$, *leverage* $\tilde{b}/k$ *is increasing in* b.

A proof can be found in Appendix B. The first part of Proposition 3.1 is an immediate consequence of debt dilution. If $b = 0$, the entire stock of debt $\tilde{b}$ is issued in period 1 and the firm fully internalizes all expected default costs through the break-even price of debt. But with positive b, a part of the expected costs of default is not borne by the firm but by the holders of previously issued debt. This allows the firm to enjoy a given amount of the tax benefits of debt at a lower cost. As a result, the firm optimally decides to utilize the tax benefit of debt more intensively by raising $\tilde{b}$ and $\bar{\varepsilon}$. This effect of debt dilution on borrowing and default rates is well understood in corporate finance (e.g. Bizer and DeMarzo (1992)).[7]

The increase in $\bar{\varepsilon}$ has an ambiguous effect on investment as described by the second part of Proposition 3.1. A higher value of $\bar{\varepsilon}$ reduces the effective tax rate as a larger part of firm earnings is paid out in the form of tax-deductible debt coupons. This encourages investment. The downside is that the bond price p in (12) falls in $\bar{\varepsilon}$ which raises the cost of capital and discourages investment. Once b rises above $\underline{b}$, the second effect dominates. This effect of debt dilution on firm investment is different from debt overhang. If the firm did not respond to the increase in b by choosing a higher value of $\bar{\varepsilon}$, there would be no effect on k. It is the endogenous response of borrowing which has implications

[7] A remark on terminology: In corporate finance, sometimes the term 'debt dilution' is only used for the specific situation that an increased number of creditors needs to share a given liquidation value of a bankrupt firm. We use the term in a more general sense as the same mechanism is at work even if the liquidation value is zero ($\xi = 1$) or if existing debt is fully prioritized (as in Bizer and DeMarzo (1992)). In our usage of the term 'debt dilution', we therefore follow the literature on sovereign debt (e.g. Hatchondo et al. (2016)).

for investment. This effect of debt dilution on investment has not been systematically studied by the existing literature.

The third part of Proposition 3.1 characterizes the effect on leverage. As can be seen from (9), leverage $\tilde{b}/k$ is increasing in $\bar{\varepsilon}$ and in the average return on capital $f(k)/k$. If both $\bar{\varepsilon}$ and k increase, the joint effect on leverage is ambiguous because the average return on capital is falling in k. But once $b > \underline{b}$ and k begins to fall in b, this theoretical ambiguity disappears and leverage necessarily increases in b.

3.4.3. Debt Overhang

In the previous subsection, we deliberately ruled out any role for debt overhang by setting $\xi = 1$. Now we do the opposite. We neutralize debt dilution by assuming that the firm's stock of debt $\tilde{b}$ in period 2 is exogenous. The firm cannot dilute the value of existing debt by choosing the number of additional bonds. Any remaining effect from the existing stock of debt b on firm investment must be due to debt overhang.

Formally, we study the firm problem (11) subject to the two constraints (8), (9), and some exogenous value $\tilde{b}$. Because $\tilde{b}$ is fixed, (9) imposes a unique functional relationship between k and $\bar{\varepsilon}$.

$$\frac{d\bar{\varepsilon}}{dk} = -\frac{1 + (1-\tau)[f'(k) - \delta + \bar{\varepsilon}]}{(1-\tau)k}. \tag{15}$$

Constraining the firm's choice of $\tilde{b}$ in this way leaves only one choice variable and only one first order condition. We obtain it from the derivative of (11) with respect to k:

$$\underbrace{-1}_{\substack{\text{Marginal}\\\text{cost of}\\\text{capital}}} + \underbrace{\frac{dp}{dk}(\tilde{b} - b)}_{\substack{\text{Marginal increase}\\\text{in value of}\\\text{newly issued debt}}} + \underbrace{\frac{1-\tau}{1+r}\left[\int_{\bar{\varepsilon}}^{\infty}[\varepsilon - \bar{\varepsilon}]\varphi(\varepsilon)d\varepsilon - \frac{d\bar{\varepsilon}}{dk}[1 - \Phi(\bar{\varepsilon})]k\right]}_{\substack{\text{Marginal increase}\\\text{in expected dividend}}} = 0 \tag{16}$$

With $\tilde{b}$ fixed, the firm's choice of k simultaneously controls $\bar{\varepsilon}$ and therefore the risk of default $\Phi(\bar{\varepsilon})$. The key difference to the first order condition in (13) is that now the firm's choice of k affects the bond price p. The firm takes into account that an increase in k affects the value of newly issued bonds $p(\tilde{b} - b)$. But it does not internalize the effect on the value of existing debt pb.

Proposition 3.2 describes the consequences for firm behavior at an interior solution.

Proposition 3.2. ***Debt Overhang:*** *Assume that the stock of debt $\tilde{b}$ in period 2 is fixed.*

1. *Capital k is falling in b if and only if the bond price p is increasing in k.*
2. *Leverage $\tilde{b}/k$ is increasing in b if and only if k is falling in b.*
3. *The default rate $\Phi(\bar{\varepsilon})$ is falling in k if and only if: $1 + (1-\tau)[f'(k) - \delta + \bar{\varepsilon}] > 0$.*

The proof is deferred to Appendix B. The first part of Proposition 3.2 is an application of the classic debt overhang result from Myers (1977), p. 164-165. Because $\tilde{b}$ is fixed, the marginal unit of capital comes from an increase in equity. Shareholders internalize that an increase in capital raises the value of both equity and newly issued debt. But they do not benefit if an increase in capital also raises the value of existing debt. In this case, a part of the benefit from investing constitutes a transfer from shareholders to the holders of existing debt. The size of this transfer is increasing in the stock of existing debt b. The larger this transfer, the lower the incentive for shareholders to increase capital.

The opposite is true if the bond price p is falling in k. This can be the case if the increase in k raises the riskiness of the firm and makes default more likely. In this case, investment transfers value from the holders of existing debt to shareholders which increases their incentive to invest.

With $\tilde{b}$ fixed, the effect of b on leverage $\tilde{b}/k$ directly follows from the behavior of capital k. The effect of an increase in k on the default rate $\Phi(\bar{\varepsilon})$ is ambiguous. An increase in k lowers leverage which reduces the risk of default. At the same time, it also raises the variance of earnings. If the latter effect dominates, a higher value of k may imply a higher default rate.

3.4.4. Summary of Analytical Results

Propositions 3.1 and 3.2 describe two different channels through which investment is affected by the stock of existing debt b. In Section 3.4.2, the stock of debt $\tilde{b}$ is endogenous and we assume $\xi = 1$. In this special case, an increase in capital has no effect on the value of existing debt. Either default is avoided and the value of debt is $b(1+c)$, or default occurs and the value of debt is zero, independently of the amount of capital. For this reason, there is no debt overhang in Section 3.4.2. For a given value of $\bar{\varepsilon}$, an increase in b has no effect on k. It is only through the endogenous response of $\bar{\varepsilon}$ and $\tilde{b}$ induced by debt dilution that b affects k. One of the main contributions of this paper is to identify this effect of debt dilution on investment.

In Section 3.4.3, the stock of debt $\tilde{b}$ is exogenous. This is similar to Myers (1977) or other studies of debt overhang. In this special case, debt dilution does not play any role for k since the firm is unable to dilute existing debt by choosing a high value of $\tilde{b}$. For a given value of k, an increase in existing debt b has no effect on $\bar{\varepsilon}$. But k responds to the increase in b because of the direct externality of k on the value of existing debt pb.

In practice, the liquidation value of a firm is generally positive ($\xi < 1$) and firms may not be able to commit to a fixed value of debt $\tilde{b}$. This implies that both debt dilution and debt overhang distort firms' investment decisions. We are interested in studying the two distortions together in order to understand their respective roles. Should firms or policy makers primarily try to address one of the two distortions? Which of the two is more severe? Do they amplify or dampen one another? To answer these questions, we extend our analysis to a fully dynamic model in which debt dilution and debt overhang simultaneously affect firm investment.

4. Dynamic Model

In the two-period model studied above, the stock of existing debt b is an exogenous variable. Propositions 3.1 and 3.2 show that this variable determines the severity of debt dilution and debt overhang. For this reason, it is important to endogenize firms' choice of b in a fully dynamic model.

The main additional feature of the dynamic model with respect to the two-period setup is that firms have a maturity choice. They can sell short-term bonds and long-term bonds. The issuance of bonds is costly. This makes long-term debt attractive because it allows to maintain a given level of leverage at lower levels of debt issuance. The downside of long-term debt is that it gives rise to debt dilution and debt overhang in the future.

The dynamic setup is otherwise kept as close as possible to the two-period model from above. This ensures that our analytical results continue to be useful to interpret the quantitative results from the dynamic model. It also means that we abstract from several model elements which are likely to matter for firm behavior, in particular in the short-run (e.g. adjustment costs to capital, equity issuance costs). Our results capture the steady state effects of debt dilution and debt overhang.

4.1. Setup

There is a unit mass of firms. As in the two-period economy, a firm i uses capital k_{it} to produce output y_{it} using a technology with diminishing returns:

$$y_{it} = k_{it}^{\alpha}, \quad \alpha \in (0,1). \tag{17}$$

Earnings before interest and taxes are given as:

$$k_{it}^{\alpha} - \delta k_{it} + \varepsilon_{it} k_{it}. \tag{18}$$

The firm-specific earnings shock ε_{it} is i.i.d. and follows a probability distribution $\varphi(\varepsilon)$.

In contrast to the two-period economy, the firm can now choose between short-term debt and long-term debt.

Definition: Short-term Debt. A short-term bond issued at the end of period $t-1$ is a promise to pay one unit of the numéraire good together with a fixed coupon payment c in period t. The quantity of these short-term bonds sold by firm i is $\tilde{b}_{it}^{S}$.

Definition: Long-term Debt. A long-term bond issued at the end of period $t-1$ is a promise to pay a fixed coupon payment c in period t. In addition, the firm repays a fraction $\gamma \in (0,1)$ of the principal in period t. In period $t+1$, a fraction $1-\gamma$ of the bond remains outstanding. The firm pays a coupon payment $(1-\gamma)c$ and repays the fraction γ of the remaining principal: $(1-\gamma)\gamma$. In this manner, payments geometrically decay over time. The maturity parameter γ controls the speed of decay. The quantity of long-term bonds chosen by the firm at the end of period $t-1$ is $\tilde{b}_{it}^{L}$.

This computationally tractable specification of long-term debt goes back to Leland (1994). Short-term debt and long-term debt are of equal seniority.

Definition: Floatation cost. The firm pays an amount η for each bond sold (or re-purchased). The total floatation cost $H(\tilde{b}_{it}^S, \tilde{b}_{it}^L, b_{it})$ is therefore:

$$H(\tilde{b}_{it}^S, \tilde{b}_{it}^L, b_{it}) = \eta(\, \tilde{b}_{it}^S + |\tilde{b}_{it}^L - b_{it}|\,)\,, \tag{19}$$

where b_{it} is the stock of previously issued long-term bonds outstanding before the firm decides on its investment and financing policy at the end of period $t-1$.

The firm finances its capital stock by injecting equity and selling new short- and long-term bonds:

$$k_{it} = e_{it} + p_{it}^S \tilde{b}_{it}^S + p_{it}^L(\tilde{b}_{it}^L - b_{it}) - H(\tilde{b}_{it}^S, \tilde{b}_{it}^L, b_{it})\,. \tag{20}$$

The firm's equity stock after production in period t is:

$$q_{it} = k_{it} - \tilde{b}_{it}^S - \gamma \tilde{b}_{it}^L + (1-\tau)[k_{it}^\alpha - \delta k_{it} + \varepsilon_{it} k_{it} - c(\tilde{b}_{it}^S + \tilde{b}_{it}^L)]\,. \tag{21}$$

Definition: Limited Liability. Shareholders are protected by limited liability. They are free to default and hand over the firm's assets to creditors for liquidation. A fixed fraction ξ of the firm's assets is lost in this case. Shareholders' outside option in case of default is V_D.

We assume that a defaulting firm is replaced by a new firm with zero debt and equity.

Timing

End of period $t-1$: Firm i has an amount b_{it} of long-term debt outstanding. Given b_{it}, the firm chooses next period's book value of equity e_{it}. It also decides on how to adjust its level of long-term debt $\tilde{b}_{it}^L$ and how many short-term bonds $\tilde{b}_{it}^S$ to sell. This determines next period's stock of capital k_{it}.

Beginning of period t: The firm draws the realization ε_{it}. This determines firm earnings. The firm decides whether to default. If it decides not to default, it pays corporate income tax on its earnings net of depreciation and coupon payments. This leaves the firm with a stock of equity after production of q_{it}. Next period's amount of long-term debt is $b_{it+1} = (1-\gamma)\tilde{b}_{it}^L$.

4.2. Firm Problem

As in the two-period economy, the firm maximizes expected shareholder value. Because of limited liability, there is a unique threshold realization $\bar{\varepsilon}_{it}$ which determines whether the firm prefers to default after earnings are realized:

$$\bar{\varepsilon}_{it}: \quad k_{it} - \tilde{b}_{it}^S - \gamma \tilde{b}_{it}^L + (1-\tau)[k_{it}^\alpha - \delta k_{it} + \bar{\varepsilon}_{it} k_{it} - c(\tilde{b}_{it}^S + \tilde{b}_{it}^L)] + V_t\Big((1-\gamma)\tilde{b}_{it}^L\Big) = V_D,$$

where $V_t((1-\gamma)\tilde{b}^L_{it})$ denotes the end-of-period-t stock market value of a firm with outstanding long-term debt $(1-\gamma)\tilde{b}^L_{it}$. If ε_{it} is smaller than $\bar{\varepsilon}_{it}$, the firm optimally decides to stop paying its debt liabilities and defaults. Shareholders receive an outside option V_D in this case.

We assume that the firm has no ability to commit to future actions. This lack of commitment not only affects the firm's default choice, but also its decision of how much to produce and how to finance capital. The firm must therefore take its own future behavior as given. The only way in which it can influence future shareholder value $V_t((1-\gamma)\tilde{b}^L_{it})$ is through today's choice of long-term debt $\tilde{b}^L_{it}$. At the end of period $t-1$, the firm solves:

$$\begin{aligned}
\max_{k_{it},e_{it},\tilde{b}^S_{it},\tilde{b}^L_{it},\bar{\varepsilon}_{it}} \quad & -e_{it} + \frac{1}{1+r}\left[\int_{\bar{\varepsilon}_{it}}^{\infty}\Big[q_{it} + V_t((1-\gamma)\tilde{b}^L_{it})\Big]\varphi(\varepsilon)d\varepsilon + \Phi(\bar{\varepsilon}_{it})\,V_D\right] \qquad (22)\\
\text{subject to:} \quad & q_{it} = k_{it} - \tilde{b}^S_{it} - \gamma\tilde{b}^L_{it} + (1-\tau)[k^\alpha_{it} - \delta k_{it} + \varepsilon_{it}k_{it} - c(\tilde{b}^S_{it} + \tilde{b}^L_{it})]\\
& \bar{\varepsilon}_{it}: \quad q_{it} + V_t((1-\gamma)\tilde{b}^L_{it}) = V_D\\
& k_{it} = e_{it} + p^S_{it}\tilde{b}^S_{it} + p^L_{it}(\tilde{b}^L_{it} - b_{it}) - H(\tilde{b}^S_{it},\tilde{b}^L_{it},b_{it})\,.
\end{aligned}$$

4.3. Creditors' Problem

As in the two-period setup, the optimal firm policy crucially depends on the two bond prices p^S_{it} and p^L_{it}. Risk-neutral and competitive creditors break even on expectation. In case the firm stops paying its debt liabilities and defaults in period t, the value of the firm's assets is:[8]

$$\tilde{q}(k_{it},\bar{\varepsilon}_{it}) \equiv k_{it} + (1-\tau)[k^\alpha_{it} - \delta k_{it} + \bar{\varepsilon}_{it}k_{it}]\,. \qquad (23)$$

At this point, creditors liquidate the firm's assets and receive $(1-\xi)\tilde{q}(k_{it},\bar{\varepsilon}_{it})$. Since short-term debt and long-term debt have equal seniority, the price of short-term debt is:

$$p^S_{it}(k_{it},\tilde{b}^S_{it},\tilde{b}^L_{it},\bar{\varepsilon}_{it}) = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon}_{it})]\,(1+c) + \Phi(\bar{\varepsilon}_{it})\,\frac{(1-\xi)\tilde{q}(k_{it},\bar{\varepsilon}_{it})}{\tilde{b}^S_{it}+\tilde{b}^L_{it}}\right]. \qquad (24)$$

The price of long-term debt p^L_{it} not only depends on the firm's choices today, but also on the future value of long-term debt p^L_{it+1}:

$$\begin{aligned}
p^L_{it}(k_{it},\tilde{b}^S_{it},\tilde{b}^L_{it},\bar{\varepsilon}_{it}) = \frac{1}{1+r}\Bigg[&[1-\Phi(\bar{\varepsilon}_{it})]\,\Big(\gamma + c + (1-\gamma)\,p^L_{it+1}((1-\gamma)\tilde{b}^L_{it})\Big)\\
&+ \Phi(\bar{\varepsilon}_{it})\,\frac{(1-\xi)\tilde{q}(k_{it},\bar{\varepsilon}_{it})}{\tilde{b}^S_{it}+\tilde{b}^L_{it}}\Bigg]. \qquad (25)
\end{aligned}$$

[8]This specification of $\tilde{q}(k_{it},\bar{\varepsilon}_{it})$ differs slightly from the one used in the two-period setup. It facilitates numerical computations as it makes sure that the firm's liquidation value is always positive.

The future price of long-term debt p^L_{it+1} depends on the firm's future behavior which today's firm must take as given. The only way in which it can influence the future price is through today's choice of long-term debt $\tilde{b}^L_{it}$.

4.4. Markov Perfect Equilibrium

As in the two-period economy, we solve for the partial equilibrium allocation given the risk-free rate r. In equilibrium, a firm maximizes shareholder value (22) subject to creditors' two break-even conditions (24) and (25). Because we assume that the firm has no ability to commit to future actions, it plays a game against its future selves. We restrict attention to the Markov Perfect equilibrium, i.e. we consider strategies which are functions of the current state of the firm.

In the absence of adjustment costs to capital or equity, the stock of existing debt b_{it} is the only state variable. The equilibrium can be defined recursively. In each period, the firm chooses a policy vector $\phi(b) = \{k, e, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}\}$ which solves:

$$V(b) = \max_{\phi(b)=\{k,e,\tilde{b}^S,\tilde{b}^L,\bar{\varepsilon}\}} \quad -e + \frac{1}{1+r}\left[\int_{\bar{\varepsilon}}^{\infty}\left[q + V((1-\gamma)\tilde{b}^L)\right]\varphi(\varepsilon)d\varepsilon + \Phi(\bar{\varepsilon})\,V_D\right] \tag{26}$$

$$\begin{aligned}
\text{subject to:} \quad & q \;=\; k - \tilde{b}^S - \gamma\tilde{b}^L + (1-\tau)[k^\alpha - \delta k + \varepsilon k - c(\tilde{b}^S + \tilde{b}^L)] \\
& \bar{\varepsilon}: \quad q + V((1-\gamma)\tilde{b}^L) = V_D \\
& k \;=\; e + p^S(b)\,\tilde{b}^S + p^L(b)\,(\tilde{b}^L - b) - H(\tilde{b}^S, \tilde{b}^L, b) \\
& p^S(b) = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})]\,(1+c) \;+\; \Phi(\bar{\varepsilon})\,\frac{(1-\xi)\tilde{q}(k,\bar{\varepsilon})}{\tilde{b}^S + \tilde{b}^L}\right] \\
& p^L(b) = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})]\left(\gamma + c + (1-\gamma)\,p^L((1-\gamma)\tilde{b}^L)\right)\right. \\
& \qquad\qquad \left. +\; \Phi(\bar{\varepsilon})\,\frac{(1-\xi)\tilde{q}(k,\bar{\varepsilon})}{\tilde{b}^S + \tilde{b}^L}\right].
\end{aligned}$$

Since a firm's policy $\phi(b) = \{k, e, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}\}$ only depends on its state b and since a firm's future state only depends on its current policy, the equilibrium bond prices $p^S(b)$ and $p^L(b)$ likewise only depend on b.

4.5. Quantitative Analysis

The Markov Perfect equilibrium in (26) can only be computed using numerical methods. Before choosing parameter values, we briefly describe our solution method.

4.5.1. Solution Method

We solve the model using value function iteration and interpolation. Following the literature on sovereign default with long-term debt (e.g. Hatchondo and Martinez (2009)), we compute the equilibrium allocation of a finite-horizon economy. Starting from the final date, we iterate backward in time until the firm's value function and the two bond prices have converged. We then use the first-period equilibrium functions as the infinite-horizon-economy equilibrium.

Common practice in the literature on risky debt is to compute the complete bond price schedules for all possible actions: $p^S(k, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon})$ and $p^L(k, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon})$. These price schedules from the 'outer loop' are then used to compute the optimal policy in an 'inner loop'. We find this 'inner-loop-outer-loop' procedure to be costly in terms of computing time. The 'outer loop' for the bond price schedules needs to be highly precise in order to get meaningful results from the 'inner loop' which computes the optimal firm policy.

For this reason, we resort to an alternative solution method. Similar to the approach used in the consolidated problem of Section 3.4.1, we express equilibrium bond prices as a function of today's choice variables. Given the firm's future policy, both bond prices are pinned down by the firm's choices today. This allows us to compute equilibrium bond prices and today's firm policy in a single step. This reduces the number of necessary computations and allows for a faster and more precise solution.

4.5.2. Parametrization

We choose a model period of one year. The annual rate of return on a riskless asset is set to $r = 3.09\%$. We also specify $c = r$, which implies that the price of a riskless short-term bond and a riskless long-term bond are both equal to one. Shareholders' outside option in case of default is specified to be equal to the equilibrium value of a firm with zero equity and zero debt: $V_D = V(0)$.

Because we do not explicitly model labor, the parameter value α controls the degree of diminishing returns. Empirical estimates by Blundell and Bond (2000) suggest a value close to one. Accordingly, we choose $\alpha = 0.9$. Based on Hennessy and Whited (2005), we set the tax rate τ to 0.3. For the floatation cost η, we are also able to use micro evidence. Altınkılıç and Hansen (2000) report this value to be 1.09%.[9]

The firm-specific earnings shock is Normal with zero mean and standard deviation σ_ε. This leaves us with four unspecified parameters: γ, δ, σ_ε, and ξ. We choose these values to replicate four key statistics from the U.S. corporate sector given in Table 1: the capital-output ratio, corporate leverage, the long-term debt share, and the average credit spread.[10] Model counterparts of empirical moments are derived in Appendix C.

In our model, firms differ with respect to the stock of existing debt b. Given the stationary equilibrium distribution of firms over b, aggregate variables are constructed as weighted averages of firm policies. Table 2 reports our choice for the full set of parameter values.[11]

[9] Altınkılıç and Hansen (2000) find that floatation costs for bond offerings consist of a (small) fixed cost and a (large and convex) variable part. Modeling convex instead of linear floatation costs does not affect our quantitative results.

[10] In our model without labor, δ not only accounts for depreciation but also captures the costs of variable production factors, e.g. wages. This is why in the choice of δ we do not target the empirical rate of depreciation but the capital-output ratio. See Cooley and Quadrini (2001).

[11] The specified standard deviation of the earnings shock σ_ε is high. Given the stylized nature of our

Table 1: Empirical Moments

	Data	Model
1984-2015:		
Capital-output ratio	2.07	2.07
Leverage: Debt / Assets	27.2%	27.1%
Long-term Debt Share	67.4%	67.4%
1998-2010:		
Credit spread	2.30%	2.33%

Note: The capital-output ratio is from the Flow of Funds. It is calculated as non-financial assets (marked-to-market) of non-financial corporate businesses divided by revenue from sales of goods and services. We prefer the Flow of Funds for data on assets because Compustat measures assets at historical costs. Leverage and the long-term debt share are from Compustat (excluding financial firms and utilities). Leverage is the average (across all firm-year observations) of the ratio of the book value of total debt to the book value of total assets. The long-term debt share is the average ratio of debt due more than one year from today to total debt. The credit spread is computed based on Adrian, Colla, and Shin (2013). It is the amount-weighted average of the credit spread for loan and bond issuance. The model counterpart is the amount-weighted average of the credit spread for short-term debt issuance and long-term debt issuance.

Table 2: Parametrization

Variable	Description	Value	Target/Source
r	riskless rate	0.0309	
c	debt coupon	r	
V_D	outside option	$V(0)$	
α	technology parameter	0.9	Blundell and Bond (2000)
τ	corporate income tax rate	0.3	Hennessy and Whited (2005)
η	marginal floatation cost	0.0109	Altınkılıç and Hansen (2000)
γ	repayment rate long-term debt	0.1283	Long-term debt share 67.4%
δ	depreciation	0.391	Capital-output ratio 2.07
σ_ε	standard deviation firm earnings	0.6275	Leverage 27.2%
ξ	default cost parameter	0.62	Credit spread 2.30%

To assess our parametrization, we use untargeted empirical moments. Gilchrist and Zakrajsek (2012) calculate an average Macaulay duration of 6.47 years for a sample of U.S. corporate bonds with remaining term to maturity above one year. Our model counterpart (for long-term debt) is very similar: 6.48 years. Bris, Welch, and Zhu (2006) document a mean recovery rate of 27% for Chapter 7 liquidations. Our parametrized model generates an average equilibrium recovery rate $(1-\xi)\tilde{q}[\tilde{b}^S + \tilde{b}^L]^{-1}$ of 36%.

The annual default rate generated by the model is 3.26%. This is high compared to an empirical value of around 1.05% reported by Duan, Sun, and Wang (2012). It is well known that empirical credit spreads are not fully explained by realized default risk (e.g. Elton, Gruber, Agrawal, and Mann (2001)). In our model, credit spreads are driven exclusively by default risk. This means that we have to decide whether we want

model with iid shocks and no costs to equity issuance, large shocks are necessary to generate a positive default rate for realistic levels of leverage.

our model to match the default rate and generate unrealistically low credit spreads, or if want to match credit spreads at the cost of generating unrealistically high default rates. In our model, the bond price schedule is key to understanding firm behavior. We therefore choose to match the average credit spread rather than the default rate.

4.6. Quantitative Results

In this section, we describe the numerical solution to the Markov Perfect equilibrium of the fully dynamic model described above. The results from the two-period model of Section 3 continue to be useful to understand the role of debt dilution and debt overhang. In contrast to the two-period setup, the state variable b is no longer exogenous. A firm's choice of $\tilde{b}^L$ today determines how much long-term debt tomorrow's firm will inherit from the past.

In the current setup, debt dilution and debt overhang simultaneously distort the firm's equilibrium policy. We will isolate and quantify the respective roles of debt dilution and debt overhang in Section 6.

Figures 1 and 2 show firms' equilibrium policies as functions of the existing stock of debt b. Debt is normalized by the optimal capital stock k^* of a frictionless economy without taxation and default or floatation costs.

Capital. The top left panel of Figure 1 shows the firm's choice of capital (relative to k^*). Capital is monotonically falling in b. This is a quantitative result. On the one hand, bond prices are increasing in k in our parametrization. According to Proposition 3.2, debt overhang therefore reduces a firm's incentive to invest as b rises. On the other hand, Proposition 3.1 states that debt dilution alone would induce an initial rise and subsequent fall in capital.[12]

Leverage. Given that k is monotonically falling in b, it follows both from Proposition 3.1 and 3.2 that leverage increases with b. This is confirmed by Figure 1.

Default Rate. The firm-specific default risk is shown in the right panel of the second row of Figure 1. Proposition 3.1 implies that the default rate is increasing in b. The same is true for Proposition 3.2 in our parametrization.[13] Both debt dilution and debt overhang induce the firm to accept a higher risk of default as b rises.

Note that the effect of b on capital, leverage, and the default rate becomes stronger as b and the risk of default increase. We will empirically test this model prediction in Section 5.

Maturity Choice. One important difference with respect to the two-period model of Section 3 is that the state variable b is endogenous. Firms choose the mix between short-term debt and long-term debt. By issuing primarily short-term debt, a firm can reduce the future stock of outstanding debt and thereby minimize future debt dilution and debt overhang.

[12]Our parameter choice of $\alpha = 0.9$ implies that firms' returns to scale are only mildly decreasing. The average product of capital is close to its marginal product. This implies that the threshold value $\underline{b}$ from Proposition 3.1 is close to zero.

[13]The condition $1 + (1-\tau)[f'(k) - \delta + \bar{\varepsilon}] > 0$ is satisfied in equilibrium. Since k is falling in b, according to Proposition 3.2 this implies that the default rate increases in b.

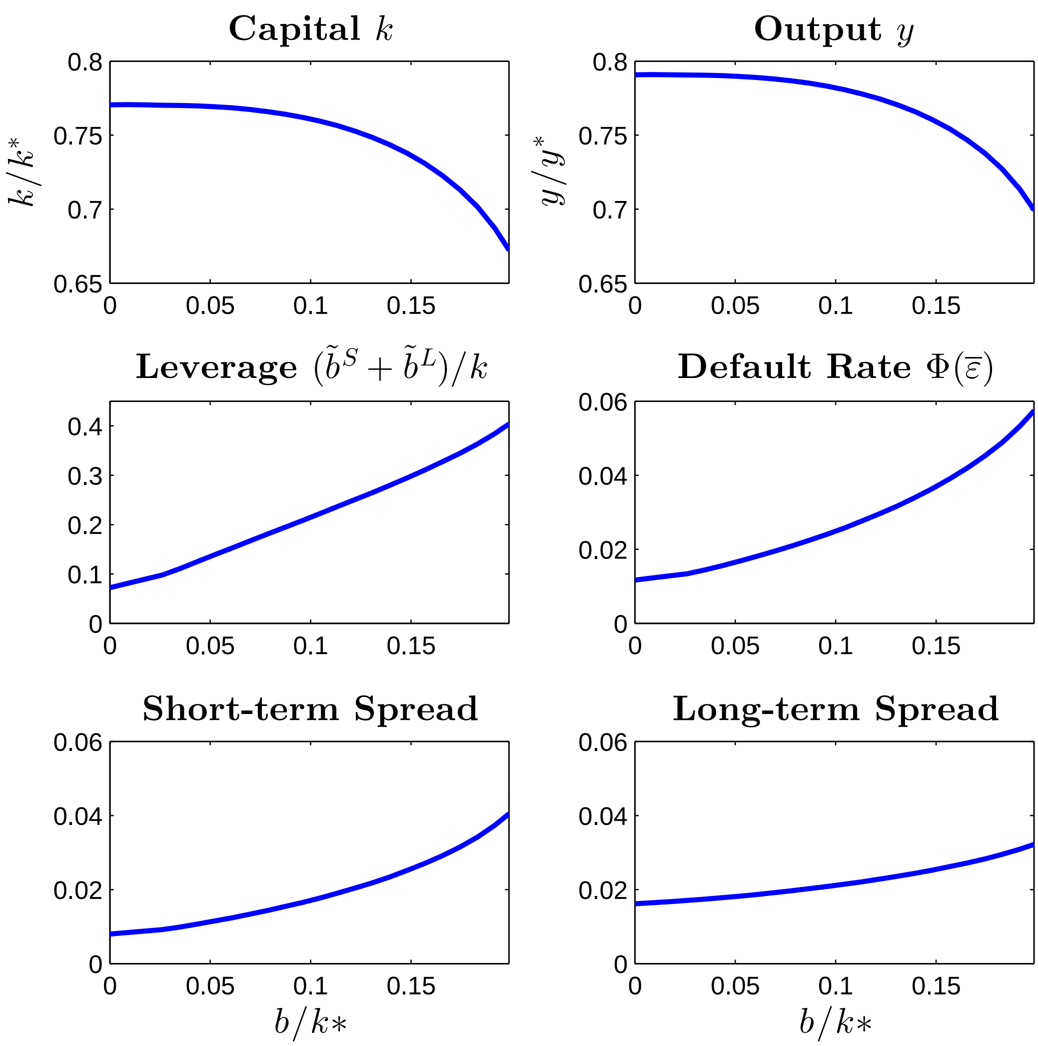

Figure 1: Policy Functions Part I

Capital and output are normalized by their respective frictionless values k^* and y^*. The x-axes show the existing stock of debt b as a fraction of k^*.

The firm's maturity choice is shown in the top four panels of Figure 2. Consider the share of long-term debt, i.e. the ratio of 'debt due more than one year from today' to total debt $\tilde{b}^S + \tilde{b}^L$. For very low values of b, firms choose a corner solution and issue only long-term debt. Given our parameterization of $\gamma = 0.1283$, this implies a share of long-term debt of 84.55%.[14] As b rises further, eventually firms begin to issue short-term debt and the share of long-term debt begins to fall. The negative effect of additional outstanding debt on the long-term debt share is reversed as b becomes very large.

[14]The corner solution for the share of long-term debt is smaller than 1 because a fraction γ of long-term debt is due within one year. See Appendix C for a derivation of the share of long-term debt.

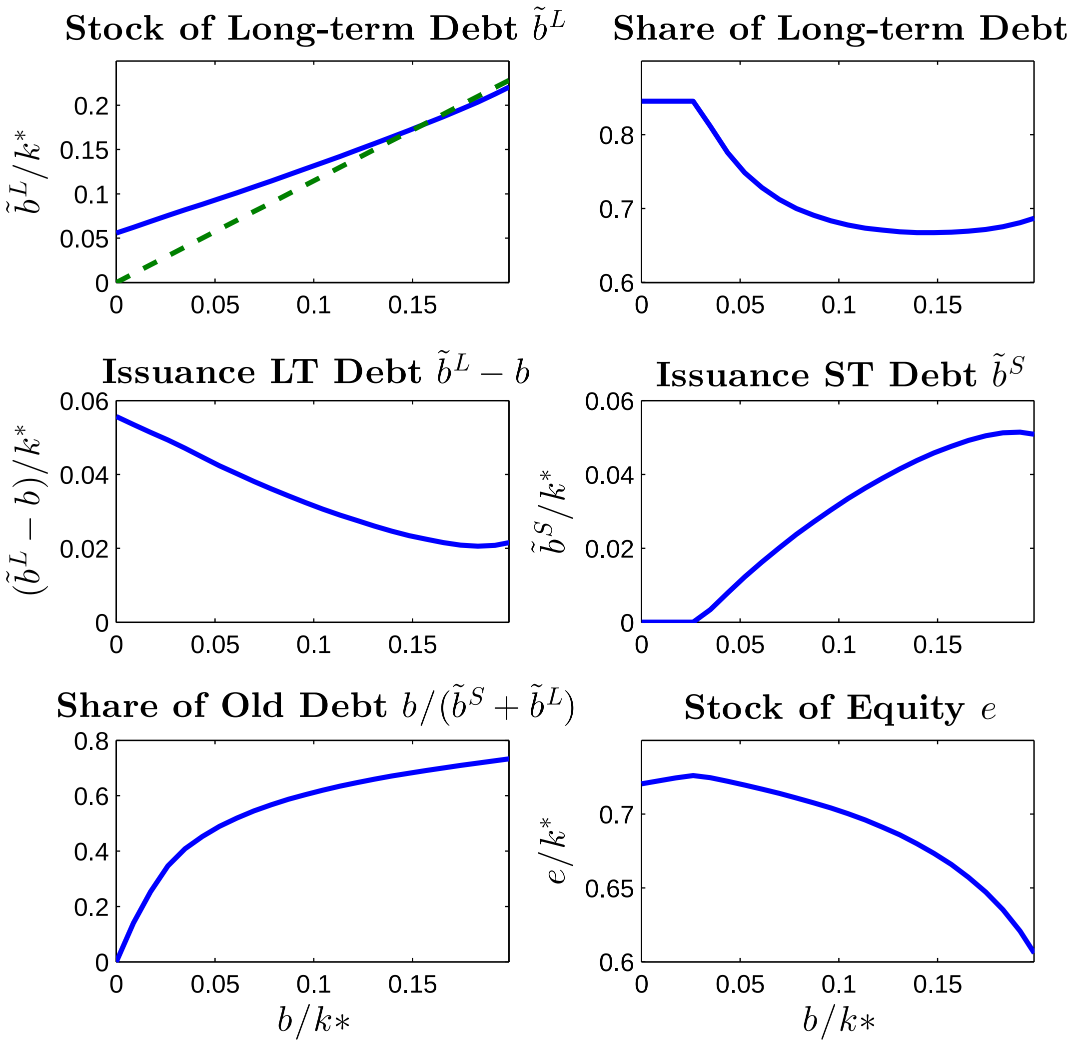

Figure 2: Policy Functions Part II

Debt and Equity are normalized by the frictionless capital stock k^*. The x-axes show the existing stock of debt b as a fraction of k^*.

This non-monotonicity in the long-term debt share can be explained as follows. Firms pay a floatation cost of 1.09% for every bond sold. This floatation cost makes long-term debt attractive as it allows to maintain a given level of leverage for a lower number of new bonds issued each period. The downside of long-term debt is that it gives rise to debt dilution and debt overhang in the future. The fact that creditors will suffer the consequences of debt dilution and debt overhang affects the firm through the price of long-term bonds sold today. Since the effect of b on the default rate and capital becomes stronger as b rises, the marginal cost of long-term debt is increasing in b. This explains why the long-term debt share initially decreases in b.

Future debt dilution and debt overhang lower the value of long-term bonds today. A part of this loss is borne by the owners of the existing stock of long-term debt. This part is not internalized by the firm. Furthermore, this part is increasing in b. The higher is b, the lower is the fraction of the total cost of long-term debt which is internalized by the firm. This explains why the long-term debt share starts to increase for very high values of b.

As shown in the top left panel of Figure 2, the stock of long-term debt $\tilde{b}^L$ increases in b. In the words of Gomes et al. (2016), long-term debt is "sticky". The dashed line indicates stable values of $\tilde{b}^L$ with $b = (1-\gamma)\tilde{b}^L$. In our parametrization, the stable choice of long-term debt $\tilde{b}^L$ for non-defaulting firms lies around 17.5% of k^* which implies a corresponding value for the stock of existing debt $b = (1-\gamma)\tilde{b}^L$ of around 15.3%. If b is higher, the firm will reduce its amount of outstanding long-term debt over time.

Credit Spreads. The behavior of credit spreads at the bottom of Figure 1 reflects current and future risk of default. The short-term spread is more sensitive to changes in b than the long-term spread. For low values of b, creditors correctly anticipate that the firm will increase the amount of long-term debt in the future. Because of debt dilution and debt overhang, this will increase the risk of default. Since this affects the value of long-term debt more than the value of short-term debt, the long-term spread is higher than the short-term spread. If b is very high, creditors anticipate that the firm will lower the amount of long-term debt in the future. Since this will lower default risk in the future, the long-term spread is lower than the short-term spread for high values of b.

Note that for high values of b the firm issues more short-term debt than long-term debt even though the long-term spread is lower and increasing more slowly in b than the short-term spread. This reflects the fact that credit spreads indicate the average cost of debt and not its true marginal cost (see Aguiar et al. (2016)).

Share of Old Debt. In the bottom left panel of Figure 2, we plot the share of old debt, i.e. the existing stock of debt b relative to total debt $\tilde{b}^S + \tilde{b}^L$. We will use this measure as an indicator for debt dilution and debt overhang in the empirical analysis below. If b is close to its median in the firm distribution, a 1 percentage point increase in the share of old debt is associated to an increase in default risk of 0.24 percentage points, an increase in leverage of 1.43 percentage points, and a 0.9% drop in capital k. This suggests that the joint effect of debt dilution and debt overhang is quantitatively important.

Firm Distribution. Figure 3 shows the stationary firm distribution. At each point in time, firms differ with respect to the existing stock of debt b. The majority of firms has not defaulted for a long time. These firms eventually find themselves near the stable value for outstanding long-term debt of around 15.3% of k^*. But firms always choose a positive risk of default. In our model, a defaulting firm is replaced by a new firm with $b = 0$. This firm initially chooses low values of $\tilde{b}^L$. But with a positive amount of long-term debt outstanding, debt dilution induces the firm to take on more and more long-term debt over time until the firm reaches the stable value.

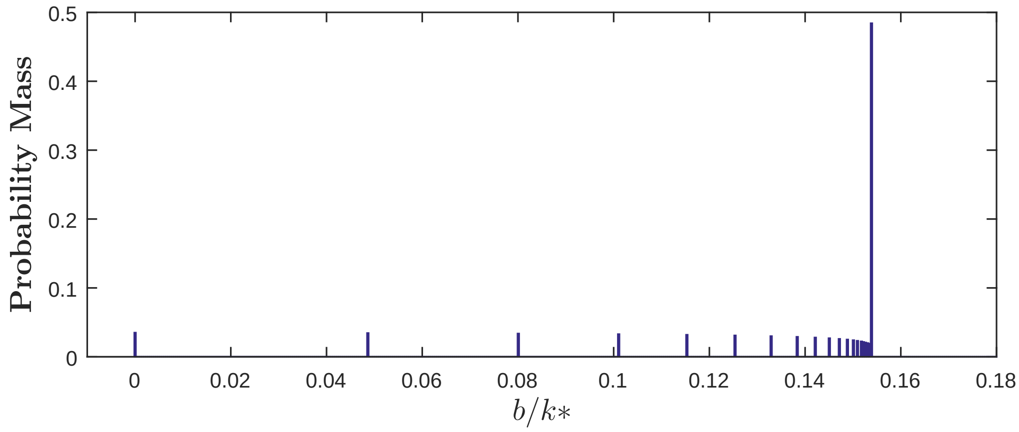

Figure 3: Firm Distribution

5. Empirical Evidence

In the previous section, we studied the joint effect of debt dilution and debt overhang on firm behavior. Our model produces novel predictions which are empirically testable using firm-level data.

To construct our sample, we merge Compustat data for publicly traded U.S. firms from 1984-2015 with default events recorded in Moody's Default & Recovery Database.[15] We exclude financial firms and utilities. In each given year, our sample accounts for about one fourth of total U.S. employment and more than one third of total assets of non-financial firms. Additional details can be found in Appendix D.

We define the `OLD-Share` as:

$$\texttt{OLD-Share} = \frac{b}{\tilde{b}^S + \tilde{b}^L} \ ,$$

where b is debt which has been issued in year t-1 or before and is outstanding at the end of year t, and $\tilde{b}^S + \tilde{b}^L$ is total firm debt at the end of year t. In the theoretical model, this variable is an indicator of debt dilution and debt overhang as it co-moves positively with default risk and leverage, and negatively with capital (see Figures 1 and 2). We use the `OLD-Share` as an empirical proxy for the severity of debt dilution and debt overhang and estimate its relationship with leverage, default risk, and asset growth.

To be clear, the goal of this exercise is not to establish causality. The variables `OLD-Share`, default risk, leverage, and asset growth are all choice variables. Correlations between these variables do not readily admit conclusions about causality (see e.g. Roberts and Whited (2013)). Nevertheless, these correlations provide reduced-form evidence which allows us to test if the data displays patterns which are consistent with the

[15] Jermann and Quadrini (2012) suggest 1984 as a starting date because of the beginning of the Great Moderation and regulatory changes in U.S. financial markets at that time. Moody's default data is available from 1988 onwards.

model of debt dilution and debt overhang developed in the previous sections.

We focus on the cross-section of firms, i.e. the `OLD-Share` of firm j in our empirical analysis is the median of firm j's `OLD-Share` for all years in which firm j appears in our sample. With all other variables we proceed the same way. We choose to ignore the time dimension because our stylized model deliberately abstracts from many factors which are likely to be important in explaining short-run variations in the data (e.g. adjustment costs to capital, equity issuance costs). Our model is designed to capture slow-moving or time-invariant patterns in firm behavior.[16] All of our regressions include industry-fixed effects and robust standard errors.

5.1. Leverage

Table 3 shows regression results with `Leverage` as the dependent variable. `Leverage` is the ratio of the book value of total firm debt to the book value of total assets. The first column gives results from a standard leverage regression. Similar regressions have been run by numerous studies in empirical corporate finance (e.g. Rajan and Zingales (1995)). Our results are standard. `Leverage` is negatively related to `Profitability`, and positively related to `Tangibility` and firm size (measured by `Log Sales`). Also the positive relationship between book leverage and `Tobin's q` is in line with existing evidence (e.g. Frank and Goyal (2009), p. 22).

In column (2), we add the `OLD-Share` as an additional control variable. We note two results. First, the coefficients of the other explanatory variables are barely affected. This suggests that the `OLD-Share` adds genuinely new information to the statistical model. Second, the estimated coefficient of `OLD-Share` is positive and significant.

A one percentage point-increase in `OLD-Share` is associated to an increase in `Leverage` of 0.035 percentage points. The fact that this estimate is lower than the corresponding relationship in our parametrized model (an increase in leverage of 1.43 percentage points) is not too surprising given that there are many factors which affect `Leverage` and `OLD-Share` in the data but which are absent from our stylized model. Furthermore, other control variables (e.g. `Tobin's q`) are likely to pick up some of the explanatory power of `OLD-Share`. The standard deviation of `OLD-Share` across firms is 36 percentage points. Accordingly, a one-standard deviation increase in `OLD-Share` is associated to an increase in `Leverage` of 1.3 percentage points.

Our quantitative model predicts that the elasticity of `Leverage` with respect to `OLD-Share` is higher for firms with a high risk of default. To test this prediction, we split the sample using the Altman Z-score. A low Z-score is commonly used as an empirical indicator for a high risk of default. Column (3) reports results for firms with a Z-score below the sample median (i.e. high risk of default). We find that the estimated coefficient of `OLD-Share` is more than twice as large for the high-default-risk sample in column (3) than for the low-default-risk group in column (4).

[16] We remove year-fixed effects from all variables before creating the cross-section of firms. Lemmon, Roberts, and Zender (2008) document that the majority of empirical variation in capital structure is explained by between-firm variation of time-invariant target leverage ratios.

Table 3: Leverage

OLS-Regression	(1) Leverage	(2) Leverage	(3) Leverage (low Z-score)	(4) Leverage (high Z-score)
OLD-Share		0.0354***	0.0602***	0.0222*
		(3.33)	(3.56)	(2.51)
Tobin's q	0.0145***	0.0146***	0.0528***	0.00405
	(4.46)	(4.43)	(8.76)	(1.10)
Profitability	-0.215***	-0.218***	-0.0702**	-0.0770**
	(-11.60)	(-11.57)	(-3.05)	(-2.98)
Tangibility	0.214***	0.207***	0.252***	0.0608
	(6.91)	(6.51)	(6.56)	(1.75)
Firm Age	-0.00404***	-0.00414***	-0.00336**	-0.00692***
	(-6.91)	(-7.11)	(-2.91)	(-13.03)
Log Sales	0.0121***	0.0107***	0.0108***	0.0187***
	(8.83)	(7.52)	(4.74)	(10.39)
Constant	-0.0102***	-0.00635***	0.0713***	-0.101***
	(-8.54)	(-3.93)	(16.25)	(-33.87)
4-digit Industry FE	YES	YES	YES	YES
Adjusted R^2	0.233	0.235	0.202	0.283
Observations	9,398	9,398	4,548	4,564

t statistics in parentheses. Standard errors are clustered at the 4-digit industry level.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

5.2. Default

Moody's Default & Recovery Database contains information about firm default events. We create a dummy variable `Default` which takes the value of one if a firm defaults at least once during the sample period, and zero otherwise. There are 431 firms in our sample with at least one default event.[17] Since defaults are rare in certain industries, we include industry-fixed effects at the 1-digit level only.

Results are displayed in Table 4. The first column shows the estimated coefficients of a logit model using the same set of control variables as above (with the addition of `Leverage`). Firms that default have higher `Leverage` and lower values of `Tobin's q`.

[17]In the regressions we lose some default events because Compustat does not report all control variables for all firms.

Table 4: Default

Logit-Regression	(1) Default	(2) Default	(3) Default (low Z-score)	(4) Default (high Z-score)
OLD-Share		0.564***	1.241***	0.0491
		(5.09)	(6.69)	(0.17)
Leverage	3.224***	3.196***	2.784***	3.529***
	(13.26)	(13.06)	(8.76)	(5.53)
Tobin's q	-0.808***	-0.812***	-0.634**	-0.431
	(-5.21)	(-5.08)	(-3.03)	(-1.87)
Profitability	-0.393	-0.500	0.297	-3.023***
	(-1.33)	(-1.56)	(0.64)	(-3.61)
Tangibility	-0.0676	-0.133	-0.194	-0.592
	(-0.27)	(-0.53)	(-0.62)	(-0.98)
Firm Age	0.0247**	0.0238**	0.0109	0.0268
	(2.68)	(2.58)	(0.67)	(1.66)
Log Sales	0.317***	0.306***	0.336***	0.212***
	(12.81)	(12.19)	(10.47)	(4.19)
Constant	-3.571***	-3.516***	-3.342***	-3.258***
	(-16.08)	(-15.78)	(-11.14)	(-6.49)
1-digit Industry FE	YES	YES	YES	YES
Pseudo R^2	0.156	0.161	0.198	0.117
Observations	8,723	8,704	4,193	4,152

t statistics in parentheses. Robust standard errors.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

In column (2), we add the `OLD-Share`. The estimated coefficient for `OLD-Share` is significant and positive. This is true even though the control variable `Leverage` is likely to pick up some of the effect of debt dilution and debt overhang on default risk. A one percentage point-increase in `OLD-Share` is associated to an increase in the default probability of 0.01 percentage points. Again, this is smaller than the corresponding effect in the parametrized model (an increase in default risk of 0.24 percentage points). In this sample, the standard deviation of `OLD-Share` across firms is 43 percentage points.[18] This

[18]Since Moody's Default & Recovery Database has information about default events from 1988

implies that a one-standard deviation increase in `OLD-Share` corresponds to an increase in the probability of default of 0.43 percentage points. This is a sizeable amount given that the unconditional probability of having at least one default event during the sample period is about 4 percent.

In columns (3) and (4), we split the sample once more using the Altman Z-score. In the high-default-risk sample (with a low Z-score), there are 235 firms with at least one default event. In the low-default-risk sample (with a high Z-score), there are only 89 firms with a default event. This suggests that the Z-score is indeed a useful predictor of default. Comparing columns (3) and (4), we find that the positive relationship between the `OLD-Share` and default risk is much stronger in the high-default-risk sample than in the low-default-risk group. This is consistent with the convex policy function for the default rate calculated in Section 4.

5.3. Asset Growth

In Table 5 we use `Asset Growth` as the dependent variable. These regressions test the model predictions with respect to capital. `Asset Growth` is the median value of a firm's annual growth rate of total assets. As above, column (1) shows the results of a regression which does not include `OLD-Share` as an explanatory variable. `Asset Growth` is positively related to `Tobin's q` and `Profitability`. Older firms and firms with higher `Leverage` grow by less.

In column (2), we add the `OLD-Share`. The estimated coefficient is significant and negative. A one percentage point-increase in `OLD-Share` is associated to a decrease in `Asset Growth` of 0.07 percentage points. This compares to a 0.9 percent drop in capital in the parametrized model. As discussed above, some amount of the effect of debt dilution and debt overhang is likely to be picked up by other control variables. Nevertheless, a one-standard deviation increase in `OLD-Share` corresponds to a decrease in annual asset growth of 2.6 percentage points. This is a sizeable amount given that the median annual asset growth rate is 1.7 percent.

In columns (3) and (4), we repeat this regression separately for the high-default-risk and the low-default-risk sample. In line with our model, the negative relationship between the `OLD-Share` and `Asset Growth` is stronger for firms with a higher probability of default.

5.4. Discussion of Empirical Results

The empirical results are consistent with firm behavior in the dynamic model studied in Section 4. In the cross-section of firms, the `OLD-Share` is positively correlated with leverage and default risk, and negatively correlated with asset growth. In line with the model predictions, these correlations are stronger for firms with a higher default risk.[19]

onwards, we construct the cross-section of firms using data 1988-2015 for the `Default` regressions. The sample therefore differs from the one used in the `Leverage` and `Asset Growth` regressions.

[19]This result is consistent with Hennessy (2004) who estimates that debt overhang is more severe for firms with low credit ratings. He also finds that the negative relationship between existing debt

Table 5: Asset growth

OLS-Regression	(1) Asset Growth	(2) Asset Growth	(3) Asset Growth (low Z-score)	(4) Asset Growth (high Z-score)
OLD-Share		-0.0726***	-0.0981***	-0.0563***
		(-8.38)	(-8.21)	(-6.12)
Leverage	-0.0866***	-0.0802***	-0.0424*	-0.0530*
	(-6.33)	(-5.81)	(-2.28)	(-2.39)
Tobin's q	0.0502***	0.0499***	0.0416***	0.0451***
	(10.20)	(10.09)	(7.97)	(5.10)
Profitability	0.265***	0.271***	0.215***	0.240***
	(12.87)	(13.33)	(8.52)	(4.87)
Tangibility	0.0213	0.0340	0.0198	0.0358
	(1.17)	(1.88)	(0.83)	(1.21)
Firm Age	-0.00851***	-0.00826***	-0.00804***	-0.00713***
	(-9.70)	(-9.55)	(-6.26)	(-8.85)
Log Sales	0.00311	0.00591***	0.00922***	0.000217
	(1.95)	(3.74)	(3.90)	(0.11)
Constant	-0.0341***	-0.0420***	-0.0694***	-0.0143***
	(-18.35)	(-18.75)	(-15.89)	(-3.47)
4-digit Industry FE	YES	YES	YES	YES
Adjusted R^2	0.170	0.184	0.123	0.162
Observations	9,398	9,398	4,548	4,564

t statistics in parentheses. Standard errors are clustered at the 4-digit industry level.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We carry out a number of robustness checks. Results are qualitatively unchanged if we include additional control variables (e.g. dividend payments, non-debt tax shields), or if we exclude the control variable `Leverage` from the default and investment regressions.

The relationship between `Default` and `OLD-Share` is unchanged if we only consider severe default episodes (e.g. Chapter 11) or if we construct the cross-section of firms excluding all firm-year observations that follow a default episode. As an alternative

(more precisely: the recovery value of existing debt) and investment is too large to be explained by debt overhang alone.

to `Asset Growth`, we use the ratio of investment expenditures to firm assets as the dependent variable and obtain qualitatively identical results.

We also construct an alternative to the Z-score. In this exercise, we use the full panel of firm-year observations and regress a default dummy on a set of explanatory variables (excluding the `OLD-Share`). This gives us a statistical model which allows to relate observable firm characteristics to default probabilities. We use this statistical model to split the firms from the pure cross-section into two groups with high and low default risk. All of our results go through.

The empirical results described above are not readily explained by alternative theoretical mechanisms. For instance, a model of liquidity risk (as in Diamond (1991)) may predict that firms with a high share of long-term debt face low refinancing risk which allows them to choose higher leverage. This can generate a positive relationship between `OLD-Share` and `Leverage`. This mechanism is plausible and it might be relevant in the data. However, it cannot rationalize the empirical result that firms with a high share of long-term debt (i.e. a high `OLD-Share`) face a higher risk of default and invest less than other firms.

6. The Cost of Debt Dilution and Debt Overhang

In Section 4, we studied firm behavior in an environment of debt dilution and debt overhang. As shown above, the model's predictions are in line with reduced-form firm-level evidence. The theoretical and empirical results suggest that the joint effect of debt dilution and debt overhang is economically significant. In this section, we use our model to isolate and quantify the respective effects of each of the two distortions. Which of them is more severe? Do they amplify or dampen one another?

To answer these questions, we compare the solution of our benchmark model from Section 4 with three alternative regimes in which either debt dilution, debt overhang, or both are eliminated. Debt dilution and debt overhang are the result of a commitment problem. In the model, creditors always break even on expectation so all costs which arise from future debt dilution and debt overhang are borne by today's firm which sells its long-term debt at a lower price. By allowing firms to choose either the total stock of debt, capital, or both with full commitment, we selectively eliminate the respective distortion.

We measure the effect of debt dilution and debt overhang using two variables: shareholder value of a firm without existing debt $V(0)$, and firm capital k. All future costs from debt dilution and debt overhang are included in $V(0)$ since firm debt is yet to be issued. Without existing debt ($b = 0$), shareholder value $V(0)$ is identical to firm value.

Our second variable of interest is firm capital k. Not all private costs of the firm are also social costs. Some part of liquidation costs consists of payments to lawyers and auditors. Floatation costs are paid to investment banks. Furthermore, also taxes and earnings shocks can be seen as purely redistributive. In such a world, financial frictions might matter to the social planner only insofar as they distort the allocation of capital.[20]

[20] To the extent that there are real social costs of liquidation, the effects of debt dilution and debt

6.1. No Debt Dilution

In this regime, we allow the firm to select its total stock of debt with full commitment. All other variables are chosen without commitment just as in the benchmark regime of Section 4. The optimal full-commitment level of debt is constant in our model. We choose the debt level which maximizes shareholder value $V(0)$. This is the level of debt which shareholders optimally commit to if there is no debt in place. Debt dilution is absent in this regime but debt overhang is still present.

Figure 4 shows the transition from our benchmark equilibrium derived in Section 4 to an equilibrium in which debt dilution is eliminated. From period 10 onwards, all firms set total debt to the full-commitment level and choose the remaining variables optimally (but without commitment).

The top left panel shows that firm value $V(0)$ increases by almost 2% after the elimination of debt dilution. The increase in aggregate capital is 1.7%. To get a sense of the economic significance of this change, we calculate the reduction in the corporate income tax rate τ which would generate an identical increase in capital in the benchmark economy from Section 4. An increase in aggregate capital of 1.7% corresponds to a reduction of τ by 0.75 percentage points.

The remaining four panels illustrate why firm value $V(0)$ increases after debt dilution is eliminated. The ability to commit to a level of total debt allows firms to strongly reduce $\tilde{b}^S + \tilde{b}^L$ from 20.4% of k^* in the benchmark economy to 15.7% in the economy without debt dilution. Both leverage and the default rate fall by substantial amounts. This lowers the credit spread which reduces the cost of capital and allows firms to increase investment.

Floatation costs fall as well because firms stop using short-term debt and only issue long-term bonds. In the benchmark model, the disadvantage of long-term debt is that it gives rise to debt dilution and debt overhang in the future. Debt dilution is now eliminated but debt overhang is still present. In our model, debt overhang alone is not sufficient to keep firms from exclusively issuing long-term debt. Given the parameter value $\gamma = 0.1283$, the corner solution for the share of long-term debt is 84.55%.

6.2. No Debt Overhang

In the second alternative regime, firms choose capital k with full commitment. Again, the optimal full-commitment level of k is constant in our model. We choose the capital level which maximizes shareholder value $V(0)$. As before, all other variables are chosen by the firm without commitment. This means that debt overhang is eliminated but debt dilution is still present.

Figure 5 shows the transition from the benchmark economy to the new regime. In the top left panel, we observe a surprising result. Firm value $V(0)$ *decreases* after the elimination of debt overhang. At the same time, aggregate capital increases strongly by 4.6%. This corresponds to a reduction of τ in the benchmark economy of 2.15 percentage points.

overhang on capital are a lower bound for the associated welfare effects.

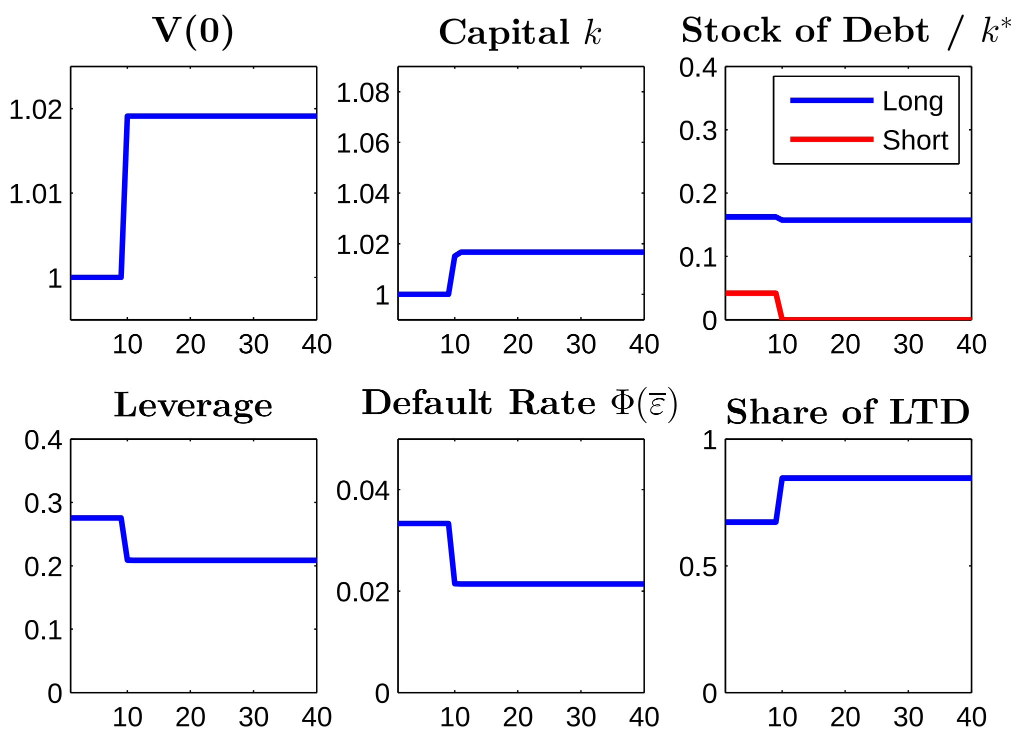

Figure 4: Eliminating Debt Dilution

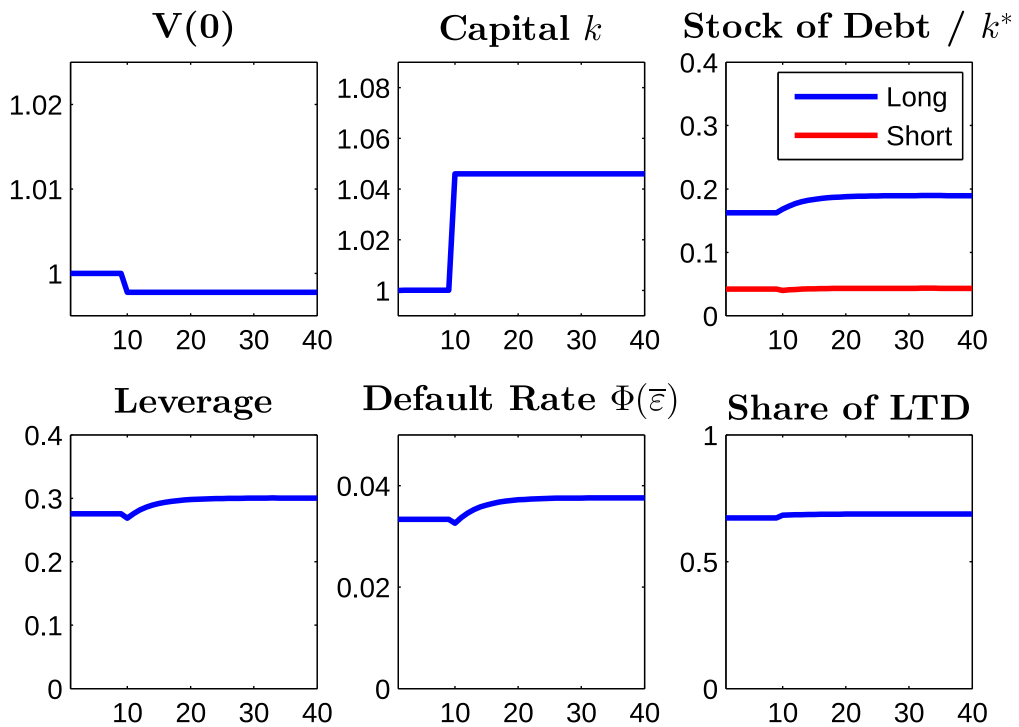

Figure 5: Eliminating Debt Overhang

Firm value $V(0)$ and aggregate capital are normalized by their benchmark values.

What explains that firm value $V(0)$ falls even though capital increases? The responses of leverage and of the default rate shed some light on this result. In the benchmark model, the firm realizes that issuing long-term debt instead of short-term debt increases the severity of debt overhang and debt dilution in the future. By eliminating debt overhang, we reduce the firm's cost of issuing long-term debt. The firm responds by issuing more long-term debt than in the benchmark model. In this regime, this has no effect on future debt overhang but it makes debt dilution more severe. As a result, leverage and the default rate are higher in the new regime. Higher credit spreads reduce the firm value. By eliminating one commitment problem (debt overhang) we increase the size of the second commitment problem (debt dilution). In other words, debt overhang helps to mitigate debt dilution.

With respect to $V(0)$, the effect of higher credit spreads outweighs the large increase in capital. Decreasing returns to production imply that the marginal unit of capital contributes nothing to shareholder value. Credit spreads are infra-marginal costs. Any changes to spreads therefore have sizeable effects on firm value.

Comparing capital across the two regimes, we notice that the increase in capital is much stronger if commitment applies to capital than if it applies to debt. Whereas debt overhang is more important for investment, debt dilution has a stronger effect on firm value $V(0)$. By mitigating debt dilution, debt overhang can even *increase* the firm value. This result has an interesting implication. If they had to choose, shareholders would prefer their firms to commit to debt, while a social planner might prefer that firms commit to capital.

6.3. No Debt Dilution & No Debt Overhang

The third and final regime allows firms to choose both debt and capital with full commitment. Total firm debt and capital maximize firm value $V(0)$. This leaves the maturity choice as the only variable chosen without commitment. But in a regime without debt dilution or debt overhang, firms' maturity choice is trivial. In the benchmark model, the only disadvantage of long-term debt is that it gives rise to debt dilution and debt overhang in the future. In the absence of these two distortions, firms optimally issue only long-term debt.

Figure 6 shows how firms react to the elimination of debt dilution and debt overhang. Firm value $V(0)$ increases by 2%. This is only slightly higher than its level after the elimination of debt dilution alone. The same is true for leverage and the default rate. They are higher now because the elimination of debt overhang reduces the cost of (long-term) debt. In the absence of debt dilution, clearly debt overhang only has a minor impact on credit spreads and firm value.

Note that the effect of debt dilution and debt overhang on investment depends on whether the other distortion is present as well. In an economy with debt dilution, we showed above that eliminating debt overhang leads to an increase in investment equivalent to a reduction in the corporate income tax of 2.15 percentage points. If debt dilution is absent, eliminating debt overhang raises capital from 101.7% of its benchmark value to 108.5%. This corresponds to a reduction in τ of 2.75 percentage points.

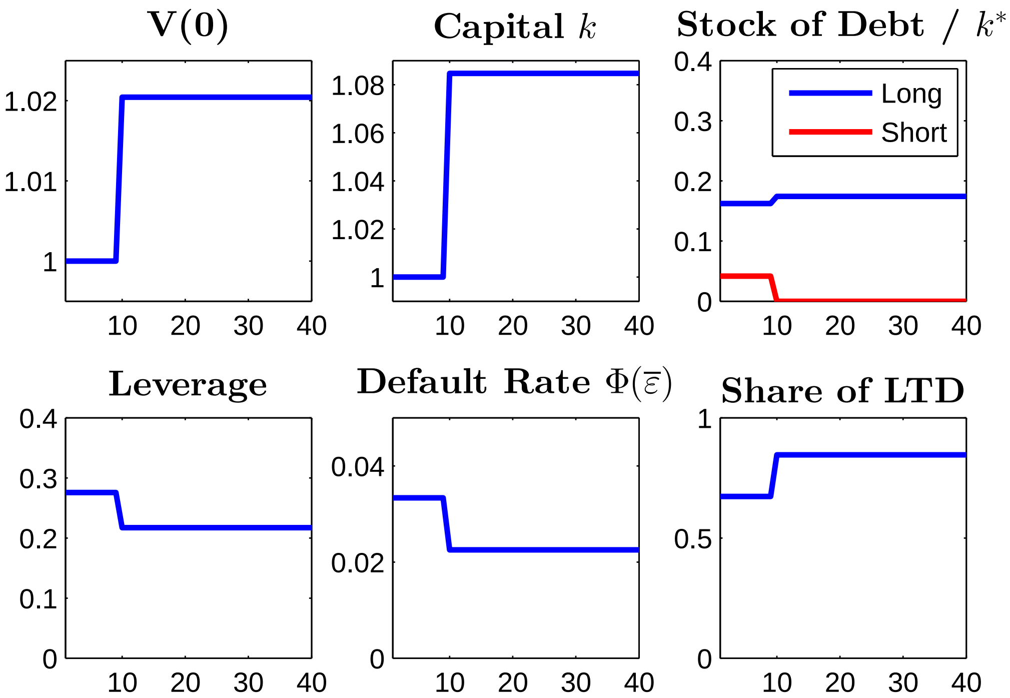

Figure 6: Eliminating Debt Dilution and Debt Overhang

Firm value $V(0)$ and aggregate capital are normalized by their benchmark values.

The effect of debt dilution on investment is sizeable as well. In an economy with debt overhang, eliminating debt dilution increases capital by as much as a reduction in τ of 0.75 percentage points. If debt overhang is absent and debt dilution is eliminated, capital rises from 104.6% to 108.5%. This corresponds to a reduction in τ of 1.35 percentage points.

Table 6 summarizes the simulation results across the different regimes. We conclude that debt dilution matters more for firm value than debt overhang, and that its effect on investment is economically significant.

Table 6: The Cost of Debt Dilution and Debt Overhang

	$V(0)$	k	$\tilde{b}^S$	$\tilde{b}^L$	Lev.	$\Phi(\bar{\varepsilon})$	LTD	$\Delta\tau$
Benchmark	1	1	0.04	0.16	0.28	3.34%	0.67	-
No Debt Dilution (DD)	1.019	1.017	0	0.16	0.21	2.14%	0.85	-0.75
No Debt Overhang (DO)	0.998	1.046	0.04	0.19	0.30	3.76%	0.69	-2.15
No DD & No DO	1.020	1.085	0	0.17	0.22	2.25%	0.85	-3.50

The aggregate stock of short-term debt $\tilde{b}^S$ and the aggregate stock of long-term debt $\tilde{b}^L$ are normalized by k^*. *LTD* is the average share of long-term debt. $\Delta\tau$ is the change of τ in the benchmark economy (in percentage points) which is necessary to obtain an identical change in capital with respect to its benchmark level.

6.4. Discussion of Quantitative Results

Our model is designed to be as simple as possible in order to derive clear and transparent results. The exact magnitude of the quantitative effects described above depends on the structure of the model. It is an open question how the introduction of equity issuance costs and persistent firm-level shocks (as in Cooley and Quadrini (2001)) would affect our quantitative results. These additional model elements would generate rich heterogeneity across firms. This would allow studying the cross-sectional differences of debt dilution and debt overhang in a less transparent but arguably more policy-relevant environment.

Another interesting property of a model with equity issuance costs is that a given firm optimally adjusts leverage over time even in the absence of debt dilution or debt overhang. Such an environment could provide the appropriate laboratory to study the trade-off between commitment through debt covenants (e.g. maximum leverage ratios) and the associated costs of reduced flexibility in the firm's choice of capital or debt. This analysis could shed light on the questions of why the empirical use of debt covenants is limited (see Appendix A), and to what extent these covenants are able to reduce debt dilution and debt overhang.

7. Conclusion

We have studied two distortions of investment which are absent from standard models of one-period debt: debt dilution and debt overhang. A model which takes these two distortions into account generates novel testable predictions for firm behavior which are in line with reduced-form firm-level evidence. Using our model, we decompose the joint effect of debt dilution and debt overhang and study the interaction between the two distortions.

Our model highlights two distortions which emerge naturally in models of risky long-term debt. While our model deliberately abstracts from several potentially relevant extensions, the described roles of debt dilution and debt overhang will continue to be present in richer environments. In this paper, we put the focus on identifying the two distortions in a clean and transparent way. In addition, we provided a first measure of their joint quantitative significance and interaction. Our results can serve as the basis for future research in firm dynamics, corporate finance, and government policies directed at firm investment. They may also prove useful in empirical attempts to disentangle the respective effects of debt dilution and debt overhang.

Finally, an important area of additional research is the role of debt dilution and debt overhang for the amplification and propagation of cyclical fluctuations (Gomes et al. (2016)). This line of research may yield novel results which are relevant for optimal monetary and fiscal stabilization policies.

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A. Empirical Evidence on Debt Covenants

Billett et al. (2007), Table III, p. 707, provide an overview of the empirical usage of different kinds of debt covenants. Typical covenants are cross-default provisions which trigger default as soon as a firm defaults on another liability.

Myers (1977) argues that covenants which restrict a firm's dividend policy might partially address debt overhang. Debt dilution can be mitigated by borrowing limits.

Nash et al. (2003) find that 15.66% of 364 investment grade bond issues in 1989 and in 1996 feature restrictions on additional debt. 8.24% include restrictions of the firm's dividend policy.[21] In a sample of 100 bond issues between 1999-2000, Begley and Freedman (2004), Table 2, p. 24, report that 9% contain additional borrowing restrictions. The percentage for dividend restrictions is identical (9%). Billett et al. (2007), Table III, p. 707, calculate that 22.8% of 15,504 investment grade bond issued between 1960 and 2003 had a covenant which restricts future borrowing of identical (or lower) seniority. 17.1% had a covenant which restricts dividend policy. Reisel (2014), Table 4, p. 259, finds in a sample of 4,267 bond issues from 1989 - 2006 that 5.9% of investment grade bonds feature covenants which restrict additional borrowing or the firm's dividend policy.

It is very common for covenants to limit the issuance of additional secured debt with priority over existing debt. As shown above, covenants which limit the issuance of additional unsecured debt with identical (or lower) seniority (e.g. through general leverage limits or minimum interest coverage ratios) are far less common. For junk bonds, these debt covenants are more frequent than for investment grade bonds.

In summary, the empirical corporate finance literature finds that less than 25% of investment grade bonds include covenants which address debt dilution, and less than 20% feature restrictions with the potential to limit debt overhang.

B. Proofs of Analytical Results

Proof of Proposition 3.1

1. The first order condition (14) associated to $\bar{\varepsilon}$ is:

$$[1-\Phi(\bar{\varepsilon})](1-\tau)k\frac{\tau c}{1+(1-\tau)c} - \varphi(\bar{\varepsilon})(1+c)\left(\frac{k+(1-\tau)\left[f(k)-\delta k+\bar{\varepsilon}k\right]}{1+(1-\tau)c} - b\right) = 0\,. \tag{27}$$

   The marginal benefit of $\bar{\varepsilon}$ is increasing in b. As long as $\bar{\varepsilon}$ has an interior solution, it follows that this solution increases in b.

[21] Of the 496 bonds considered in their Compustat sample, 120 feature additional debt restrictions (Table 3, p. 218). Of those, 57 bonds are investment grade (Table 4, p.220). It follows that out of a total of 364 investment grade bonds (Table 2, p.216), 15.66% feature additional debt restrictions. Out of the full sample, 99 bonds include restrictions of the firm's dividend policy (Table 3, p. 218). Of those, 30 bonds are investment grade (Table 4, p.220). It follows that 8.24% of the investment grade bonds in the sample feature dividend restrictions.

2. Consider the marginal benefit of k as given on the left hand side of first order condition (13). If $\bar{\varepsilon}$ does not respond to the change in b, neither does k. But we know from Proposition 3.1 that $\bar{\varepsilon}$ is increasing in b. A marginal increase of $\bar{\varepsilon}$ affects the benefit of increasing k according to:

$$-\frac{1+c}{1+r}\varphi(\bar{\varepsilon})\frac{1+(1-\tau)[f'(k)-\delta+\bar{\varepsilon}]}{1+(1-\tau)c}+[1-\Phi(\bar{\varepsilon})]\frac{1-\tau}{1+r}\frac{\tau c}{1+(1-\tau)c}\,. \tag{28}$$

We consider a change in $\bar{\varepsilon}$ which is caused by an increase in b. Since $\bar{\varepsilon}$ is chosen optimally, the first order condition (14) holds:

$$[1-\Phi(\bar{\varepsilon})]\frac{1-\tau}{1+r}\frac{\tau c}{1+(1-\tau)c} = \varphi(\bar{\varepsilon})(1+c)\frac{\tilde{b}-b}{k(1+r)}\,. \tag{29}$$

It follows that an increase in b raises the benefit of increasing k if and only if:

$$\frac{1+c}{1+r}\varphi(\bar{\varepsilon})\left[\frac{1+(1-\tau)\left[\frac{f(k)}{k}-\delta+\bar{\varepsilon}\right]}{1+(1-\tau)c}-\frac{b}{k}-\frac{1+(1-\tau)[f'(k)-\delta+\bar{\varepsilon}]}{1+(1-\tau)c}\right]>0\,. \tag{30}$$

This is the case if and only if:

$$\frac{(1-\tau)\left[\frac{f(k)}{k}-f'(k)\right]}{1+(1-\tau)c}>\frac{b}{k}\,. \tag{31}$$

3. By equation (9), leverage is:

$$\frac{\tilde{b}}{k}=\frac{1+(1-\tau)\left[\frac{f(k)}{k}-\delta+\bar{\varepsilon}\right]}{1+(1-\tau)c}\,. \tag{32}$$

We know from Proposition 3.1 that $\bar{\varepsilon}$ is increasing in b. If k is falling in b, it follows from diminishing returns that $f(k)/k$ is increasing.

Proof of Proposition 3.2

1. Consider the effect of b on k. The marginal benefit of k responds to an increase in b by $-dp/dk$.

2. Because $\tilde{b}$ is fixed, the effect of b on leverage $\tilde{b}/k$ directly follows from the effect of b on k.

3. It follows from equation (15) that:

$$\frac{d\bar{\varepsilon}}{dk}<0 \quad\Leftrightarrow\quad 1+(1-\tau)[f'(k)-\delta+\bar{\varepsilon}]>0. \tag{33}$$

C. Derivation of Model Variables

Table 7 defines key model variabels. In the following, we derive some of the expressions from Table 7 in more detail.

The total amount of debt is the present value of future debt payments:

$$
\begin{aligned}
D &= \frac{1+c}{1+r}\,\tilde{b}^S + \frac{\gamma+c}{1+r}\,\tilde{b}^L + (1-\gamma)\frac{\gamma+c}{(1+r)^2}\,\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\,\tilde{b}^L + ... \\
&= \frac{1+c}{1+r}\,\tilde{b}^S + \frac{\gamma+c}{1+r}\,\tilde{b}^L \sum_{j=0}^{\infty}\left(\frac{1-\gamma}{1+r}\right)^j = \frac{1+c}{1+r}\,\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\,\tilde{b}^L\,. \qquad (34)
\end{aligned}
$$

The long-term debt share is the present value of debt payments due more than one year from today divided by D:

$$
\begin{aligned}
&\frac{1}{D}\left((1-\gamma)\frac{\gamma+c}{(1+r)^2}\,\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\,\tilde{b}^L + ...\right) \\
&= \frac{1}{D}\,\frac{\gamma+c}{\gamma+r}\,\frac{1-\gamma}{1+r}\,\tilde{b}^L\,. \qquad (35)
\end{aligned}
$$

The short-term spread compares the gross return (in the absence of default) from buying a short-term bond with the riskless rate:

$$
\frac{1+c}{p^S} - (1+r)\,. \qquad (36)
$$

The long-term spread compares the gross return (in the absence of default and assuming p^L is constant) from buying a long-term bond with the riskless rate:

$$
\frac{\gamma+c+(1-\gamma)p^L}{p^L} - (1+r) = \frac{\gamma+c}{p^L} + 1 - \gamma - (1+r)\,. \qquad (37)
$$

The Macaulay duration is the weighted average term to maturity of the cash flows from a bond divided by the price:

$$
\mu = \frac{1}{p_r^L}\sum_{j=1}^{\infty} j\,(1-\gamma)^{j-1}\,\frac{c+\gamma}{(1+r)^j} \;=\; \frac{c+\gamma}{p_r^L}\,\frac{1+r}{(\gamma+r)^2}\,, \qquad (38)
$$

where p_r^L is the price of a riskless long-term bond:

$$
p_r^L = \sum_{j=1}^{\infty}(1-\gamma)^{j-1}\frac{c+\gamma}{(1+r)^j} = \frac{c+\gamma}{r+\gamma}\,. \qquad (39)
$$

Table 7: Key Variables

Capital-output ratio	$\frac{k}{y}$
Total Debt	$D \equiv \frac{1+c}{1+r}\,\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\,\tilde{b}^L$
Leverage: Debt / Assets	$\frac{D}{k}$
Default Rate	$\Phi(\bar{\varepsilon})$
Stock of Long-term Debt	$\tilde{b}^L$
Share of Long-term Debt	$\frac{1}{D}\,\frac{\gamma+c}{\gamma+r}\,\frac{1-\gamma}{1+r}\,\tilde{b}^L$
Issuance Long-term Debt	$\tilde{b}^L - b$
Share of Old Debt	$\frac{b}{D}$
Short-term Spread	$\frac{1+c}{p^S} - (1+r)$
Long-term Spread	$\frac{\gamma+c}{p^L} + 1 - \gamma - (1+r)$
Macaulay Duration	$\frac{1+r}{\gamma+r}$

It follows for the Macaulay duration:

$$\mu = \frac{1+r}{\gamma+r}\,. \tag{40}$$

D. Data Appendix

In this section, we describe the construction of our data set used in Section 5. We use firm-level balance sheet data from Compustat and information on default events from Moody's Default & Recovery Database.

D.1. Firm Sample

We use annual Compustat data from 1984 to 2015. Moody's default data is available from 1988 onwards. Compustat includes firms listed on three U.S. exchanges: NYSE, AMEX, and Nasdaq. We exclude firms without a U.S. incorporation code and remove financial firms (SIC codes 6000-6999) and utilities (SIC codes 4900-4949). Following Covas and Den Haan (2011), we exclude firms that were part of a large merger or acquisition, delete four big U.S. companies which were strongly affected by an accounting change in 1988 (General Motors, General Electric, Ford, and Chrysler), firm-years with missing `Total Assets`, firm-years which violate the accounting identity (*total assets = equity + liabilities*) by more than 10 percent, and firm-years with less than \$500'000 in `Total Assets`. We restrict our analysis to firms which have at least three consecutive observations of `OLD-Share` and `Leverage`. All balance sheet variables are winsorized at one percent. This leaves us with 10,071 firms and 120,536 firm-years.

From Moody's Default & Recovery Database, we obtain information about default events of debt issues. This information includes a firm identifier, the time of default, and a brief description of the type of default (e.g. 'Missed interest payment', 'Chapter 11', or 'Distressed exchange'). We build a panel of default events and merge it with the balance-sheet data from Compustat. Because naming conventions differ, we employ an algorithm that computes the Levenshtein distance between firm names in Compustat and Moody's to facilitate matching. Out of $1,716$ firm-year observations of default events in Moody's, we can match 820 events involving 687 unique firms to the Compustat database. After cleaning the firm sample in the way described above, in our final sample we are left with 431 different firms with one or more default event during the sample period.

D.2. Variable Definitions

The empirical variables are constructed from the firm panel. We regress all variables on year-fixed effects and keep the residuals. This makes sure that year-fixed effects do not influence results in the pure cross-section of firms. In the panel, a given variable for a given firm is recorded as a time series. For each variable and each firm, we keep the median of this time series and thereby reduce the panel to a cross-section.

The $\texttt{OLD-Share}_t$ of a given firm in year t is defined as:

$$\texttt{OLD-Share}_t = \frac{\texttt{debt\_long}_{t-1}}{\texttt{debt\_tot}_t}\,.$$

$\texttt{debt\_long}_{t-1}$ is the stock of firm debt in year $t-1$ with remaining term to maturity above one year. $\texttt{debt\_tot}_t$ is the stock of total firm debt in year t.

Leverage is debt over assets at book value:

$$\texttt{Leverage}_t = \frac{\texttt{debt\_tot}_t}{\texttt{at}_t}\,,$$

where $\texttt{at}_t$ is the book value (at historical cost) of total firm assets.

Default events are from Moody's. For a given firm, the dummy variable `Default` is equal to one if Moody's records at least one default event for this firm during the sample period.

The variable $\texttt{Asset Growth}_t$ is constructed as first differences in log total firm assets:

$$\texttt{Asset Growth}_t = \log\left(\texttt{at}_t\right) - \log\left(\texttt{at}_{t-1}\right)\,.$$

As a robustness check we use the ratio of investment expenditures (i.e. capital expenditures $\texttt{capx}_t$) to total firm assets $\texttt{at}_t$.

As control variables, we use $\texttt{Tobin's q}_t$, $\texttt{Profitability}_t$, $\texttt{Tangibility}_t$, $\texttt{Firm Age}_t$, and $\texttt{Log Sales}_t$. We calculate $\texttt{Tobin's q}_t$ as:

$$\texttt{Tobin's q}_t = \frac{\texttt{me}_t + \texttt{liab\_tot}_t}{\texttt{at}_t},$$

where $\texttt{liab\_tot}_t$ is total liabilities and $\texttt{me}_t$ is the market value of equity. We calculate it as: $\texttt{me}_t = \texttt{csho}_t \cdot \texttt{prcc\_f}_t + \texttt{pstkl}_t$, where $\texttt{csho}_t$ is common shares outstanding, $\texttt{prcc\_f}_t$ is the firm's stock price at the end of year t, and $\texttt{pstkl}_t$ is the liquidating value of preferred stock. $\texttt{Profitability}_t$ is $\texttt{ib}_t/\texttt{at}_t$, that is, income before extraordinary items over total assets. $\texttt{Tangibility}_t$ is $\texttt{ppent}_t/\texttt{at}_t$, where $\texttt{ppent}_t$ is tangible fixed property (at historical cost) less accumulated depreciation. $\texttt{Firm Age}_t$ is the number of years since the firm's entry into the Compustat sample. We proxy firm size by $\texttt{Log Sales}_t$, i.e. the natural logarithm of net sales.

The Altman Z-score is computed as:

$$\text{Z-score} = 1.2 \cdot \frac{\texttt{work}_t}{\texttt{at}_t} + 1.4 \cdot \frac{\texttt{ret}_t}{\texttt{at}_t} + 3.3 \cdot \frac{\texttt{ebit}_t}{\texttt{at}_t} + 0.6 \cdot \frac{\texttt{me}_t}{\texttt{liab\_tot}_t} + 1.0 \cdot \frac{\texttt{sales}_t}{\texttt{at}_t},$$

where $\texttt{work}_t$ is working capital, $\texttt{ret}_t$ is retained earnings, $\texttt{ebit}_t$ is earnings before interest and taxes, and $\texttt{sales}_t$ is net sales.