Abstract

This paper studies a model of endogenous bank opacity. Why do banks choose to hide their risk exposure from the public? And should policy makers force banks to be more transparent? In the model, bank opacity is costly because it encourages banks to take on too much risk. But opacity also reduces the incidence of bank runs (for a given level of risk taking). Banks choose to be inefficiently opaque if the composition of their asset holdings is proprietary information. In this case, policy makers can improve upon the market outcome by imposing public disclosure requirements (such as Pillar Three of Basel II). However, full transparency maximizes neither efficiency nor stability. The model can explain why empirically a higher degree of bank competition leads to increased transparency.

Keywords: bank opacity, bank runs, bank risk taking.

JEL classifications: G14, G21, G28.
1. Introduction

The risk exposure of banks is notoriously hard to judge for the public. Bank supervisors try to address this problem through public disclosure requirements which regulate how much information banks need to reveal about their investment behavior. Transparent bank balance sheets are supposed to allow financial markets to discipline bank risk taking.¹ During the recent financial crisis, public information about the risk exposure of individual banks appears to have been particularly scarce. Bank regulators in the U.S. and in Europe responded with the publication of bank stress test results. This information seems to have been valuable to the public.² Former Fed Chairman Ben Bernanke even called the 2009 U.S. Stress Test one of the critical turning points in the financial crisis (Bernanke, 2013).

Motivated by these observations, this paper seeks to identify the market failure which justifies the regulation of bank transparency. Should we force banks to be more transparent about their risk exposure than they choose to be? And what is the optimal level of transparency? Bank representatives regularly raise the concern that “disclosures have the de facto effect of compromising proprietary information of individual firms” (Group of Thirty (2003), p. 21). Information spillovers of this kind are one focus of my analysis.

In the model, banks are subject to roll-over risk, as some part of their funding is short-term debt. If short-term creditors refuse to roll over, there is a bank run and projects need to be liquidated prematurely. A bank chooses its risk exposure by selecting a portfolio of safe and risky investment projects. A bank chooses a level of transparency by selecting the probability that its portfolio choice becomes public information. Any value between zero (complete opacity) and one (full transparency) is possible.

In the model, transparency affects the risk of a bank run in three ways.

1. Transparency reduces risk taking. In the absence of transparency, even a benevolent bank has an incentive to deviate from a prudent portfolio choice and take on too much risk. It faces a credibility problem. Only a transparent bank can commit to take the risk of a bank run into account when it chooses its portfolio. I refer to this aspect of transparency as market discipline.

2. For a given level of risk taking, transparency may increase the incidence of bank runs. If banks’ risk exposure is unobservable, outsiders cannot disentangle weak banks from strong ones. Through opacity, a bank insures itself against the risk of picking the wrong portfolio. Transparency destroys this insurance mechanism. I


refer to this aspect of transparency as Hirshleifer effect.\textsuperscript{3}

3. A transparent portfolio reveals a bank’s private information. This helps the bank’s competitors to select a better portfolio and reduce their own risk of a bank run. Because of this information spillover, the competitors are able to offer a higher return to households at the expense of the transparent bank’s market share. For this reason, a bank’s portfolio choice is proprietary information in the presence of information spillovers.

The main result of the analysis is that banks choose to be too opaque if their portfolio choice is based on private information. By reducing transparency below the efficient level, a bank may increase its own risk of a bank run and thereby lower the expected return which it can offer to households. But because of the information spillover, the expected return offered by its competitors is reduced by even more. As a result, the bank increases its own market share.

Competing banks act as in a prisoner’s dilemma. Each of them ends up with an inefficiently low level of transparency and an inefficiently high amount of risk taking. Policy makers can improve upon the market outcome by imposing public disclosure requirements (such as the ones specified in Pillar Three of Basel II). But full transparency maximizes neither efficiency nor stability. Because of the Hirshleifer effect, the efficient level of transparency generally has an interior solution.

Existing models consider bank transparency as public information about a bank’s realized losses. In these models, bank losses are exogenous and banks do not choose the risk of their portfolio. Because there is no portfolio choice in these models, they do not address the concepts of market discipline or proprietary information. The key distinction of my model is that banks choose the risk of their portfolio. Transparency is public information about a bank’s portfolio choice. This is the kind of information which matters for market discipline and which is regulated by Pillar Three of Basel II. This is also the kind of information which generates information spillovers across competing banks. The trade-off between market discipline and proprietary information is discussed in the policy debate, but it has not yet been formally studied by the literature.\textsuperscript{4} The main contribution of my paper is to formalize banks’ trade-off between market discipline, the Hirshleifer effect, and information spillovers, and to study the welfare properties of the resulting equilibrium.

The model produces testable predictions. Strategic banks reduce transparency to avoid information spillovers to competitors. If the number of banks in a given market segment is increased, this reduces the exclusivity of the private information held by each individual bank. Information spillovers become less of a concern. This mechanism can rationalize the positive empirical relationship between bank competition and transparency found by Jiang, Levine, and Lin (2016). They document that the removal

\textsuperscript{3}Hirshleifer (1971) provides an early example of how public information reduces risk sharing opportunities among agents.

of regulatory impediments to bank competition by individual states in the U.S. has improved the informational content of banks’ financial statements.

In Section 2, I briefly survey some related literature. Section 3 describes the model environment and discusses key assumptions. Section 4 characterizes the equilibrium allocation. Its welfare properties are studied in Section 5. Concluding remarks follow. Formal proofs are deferred to the appendix.

2. Related Literature

A formal analysis of efficiency in the supply of public information about banks’ risk exposure is practically absent. This is surprising because a sound and consistent economic argument is needed to justify the observed regulatory interventions.

Cordella and Yeyati (1998), Matutes and Vives (2000), and Blum (2002) show that banks take on more risk if their portfolio choice is not publicly observable. But the level of transparency is exogenous in these studies and they do not address the question whether policy intervention is warranted. Another difference is that these models feature an agency problem between bank managers and the ultimate bearers of the risk. My model shows that transparency reduces inefficient risk taking even in the absence of agency conflicts of this kind. Sato (2014) studies the behavior of an investment fund whose portfolio is unobservable for outsiders. Again, transparency is not a choice in this model.5

Another strand of the literature addresses public disclosure of bank losses. In Chen and Hasan (2006), banks decide to delay the disclosure of losses in order to avoid efficient bank runs. Mandatory disclosure may be beneficial because it increases the probability of a bank run. This result is in contrast to the conventional wisdom that transparency should serve to reduce the likelihood of a banking crisis. Alvarez and Barlevy (2014) examine banks’ transparency choice in a network of interbank claims. They find that mandatory disclosure of bank losses may improve upon the equilibrium outcome because of contagion effects.

A number of contributions stresses the social benefits of limited disclosure. Opacity allows to pool investment risks. Liquidity insurance may be reduced if losses of investment projects become public. Examples of this mechanism are Kaplan (2006), or Dang, Gorton, Holmström, and Ordoñez (2017). These results rely on the Hirshleifer effect.6

Without exception, the models cited above study a bank’s incentive to hide losses from the public.7 Bank performance is largely exogenous and there is no or no interesting

5Sato (2014) shows that in his model fund managers would prefer to make their portfolio choice observable if they could. In Section 5 of his paper, he conjectures that fund managers might prefer to hide their portfolio choice if it was proprietary information.

6For the social costs of public information, see also Andolfatto and Martin (2013), Andolfatto, Berentsen, and Waller (2014), Gorton and Ordoñez (2014), and Monnet and Quintin (forthcoming). The benefits of symmetric ignorance relative to the case of asymmetric information are described by Gorton and Pennacchi (1990), Jacklin (1993), Pagano and Volpin (2012), and Dang, Gorton, and Holmström (2013).

7Dang, Gorton, Holmström, and Ordoñez (2017) show that a bank in their model has no incentive
role for the bank’s portfolio or risk choice. For this reason, these papers do not address
the concepts of market discipline or information spillovers. These two concepts feature
prominently in the policy debate about bank transparency. My model allows to study
the Hirshleifer effect together with market discipline and information spillovers.

My paper also relates to two contributions which study how an intermediary can
protect its informational advantage about investment projects from free-riding by com-
petitors. In Anand and Galetovic (2000), a dynamic game between oligopolistic banks
can sustain an equilibrium without free-riding on the screening of rivals. Breton (2011)
shows that intermediaries which fund more projects than they have actually screened
can appropriate more of the value created by their screening effort. In these two contrib-
utions, investment projects last for more than one period. Funding a project reveals
information about its quality to rivals which intensifies competition for projects at an
interim stage. In contrast, in my model projects require only initial funding. It is the
size of the investment in one project which reveals information about another project.
This intensifies competition among banks for funding at the initial stage and can be
counteracted by hiding the portfolio choice.

Other studies have examined stress tests and disclosure by bank regulators. These
papers do not address the question whether banks themselves might be able to supply
the efficient amount of transparency to the public. Recent examples of this literature
are Bouvard, Chaigneau, and de Motta (2015), or Goldstein and Leitner (2015).

In summary, the problem of a bank which deliberately hides its portfolio choice from
the public has not yet been formally studied. In contrast, informed trading on asset mar-
kets has been extensively analyzed building on the seminal contributions by Grossman
and Stiglitz (1980) and Kyle (1985). The key difference between that literature and my
model is that a bank’s informed portfolio choice is not reflected by publicly observable
asset prices.

Outside of the banking literature, Admati and Pfleiderer (2000) study disclosure de-
cisions by firms and find that transparency requirements may be beneficial if providing
public information has a direct resource cost. Since the authors model generic non-
financial firms, none of the three roles of transparency studied below (market discipline,
Hirshleifer effect, proprietary information) is present in the analysis.

Strategic information sharing among producers in goods markets has been studied
by Vives (1984) and Gal-Or (1985, 1986). In these models, oligopolistic firms share
information about fundamentals. Consumers care about information sharing only in so
far as it changes the resulting market price and quantities. In my model, households
have preferences about the level of transparency because it affects the expected return
on savings for given prices and quantities. The expected return depends on transparency
through banks’ portfolio choice and through the risk of a bank run. In Vives (1984) and
Gal-Or (1985, 1986), inefficiencies result from distorted quantities. In my model, the
size of the market is constant. It is the expected return on savings which is distorted by
strategic behavior.

to hide its portfolio choice from the public.
3. Model Setup

There are three periods \((t = 0, 1, 2)\) and two groups of agents: households and bankers. All agents are risk neutral and do not discount the future. All agents have rational expectations. During the major part of the analysis, I will assume that there are two bankers: banker \(A\) and banker \(B\). All variables and parameters are positive real numbers.

**Households.** There is a unit mass of small and identical households. At the beginning of period 0, households can invest an endowment \(w\) in equity and short-term debt sold by banker \(A\) and banker \(B\). Short-term debt matures in period 1. At this point, a household can demand repayment or he can lend his funds to the bank for another period (‘roll-over’). Short-term debt is served sequentially. Dividends are paid out to shareholders in period 2.

**Bankers.** Bankers do not have an endowment. At the beginning of period 0, banker \(j = A, B\) raises an amount \(k_j\) from households by issuing equity and short-term debt. She charges a fraction \(\tau_j\) of the period 2 value of assets under her management. Claims of bankers are senior to short-term debt, which is senior to the claims of shareholders.

**Projects.** Banker \(j\) divides her portfolio \(k_j\) between two investment projects: a safe and a risky one. Both projects are started at the end of period 0 and pay off in period 2. The safe project pays a return \(S \geq 1\) with certainty. The risky project has a higher marginal return \(R > S\). It is risky because the maximum project size \(\theta_j\) is uncertain. If the amount of resources invested by banker \(j\) in the risky project \(i_j\) is higher than \(\theta_j\), the surplus amount \(i_j - \theta_j\) is pure waste. Accordingly, the gross value of banker \(j\)’s asset portfolio at \(t = 2\) after the two projects have been completed is:

\[
V_j = S (k_j - i_j) + R \min\{ i_j , \theta_j \} .
\]

Each banker has exclusive access to one risky project. For simplicity, I assume that the random variables \(\theta_A\) and \(\theta_B\) are perfectly correlated. The maximum project size \(\theta_A = \theta_B = \theta\) follows a uniform distribution:

\[
\theta \sim \mathcal{U}(\mu - a, \mu + a), \quad \text{where } \mu - a > 0, \text{ and } \mu + a < \frac{w}{2} .
\]

**Short-term Debt.** In period 0, Banker \(j\) promises a total period 1 payment \(D^1_j\) to the holders of short-term debt. \(D^1_j\) is endogenous, but there is an exogenous upper limit \(M\) for the buffer between the period 2 payout of a “safe” portfolio (with \(i_j = 0\)) and short-term debt: \((1 - \tau_j)Sk_j - D^1_j \leq M\). In period 1, banker \(j\) promises a period 2 payment \(D^2_j\) to all creditors who roll-over their claim. \(D^2_j\) is endogenous.

**Premature Liquidation.** Investment projects are illiquid. If households do not roll-over their short-term debt claim in period 1, a banker needs to prematurely liquidate some of the investment projects under her management in order to generate cash. Premature liquidation is costly. For simplicity, I assume a fixed cost \(\Phi\). This fixed cost \(\Phi\) arises as soon as the first unit of the investment project is liquidated and is otherwise

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\(^8\)This constraint will make sure that banker \(j\) issues a positive amount of short-term debt in the initial period. See also the discussion of model assumptions below.
independent of the precise amount which is liquidated.

The net value of the bank’s asset portfolio is therefore equal to $V_j - \mathbb{1}_{\text{run}_j} \Phi$, where $\mathbb{1}_{\text{run}_j}$ is an indicator function with value one in case banker $j$ has liquidated early. I assume that the fixed cost of early liquidation is not too high relative to the return from the risky project:

$$\Phi < \min \{ R(\mu - a), R\mu - S(\mu + a) \}. \quad (9)$$

**Information.** Bankers’ choice of transparency is at the center of the analysis. At the beginning of period 0 when bankers sell equity and short-term debt to households, banker $j$ publicly chooses a level of transparency $\pi_j \in [0, 1]$. Bank transparency $\pi_j$ is the probability that banker $j$’s portfolio choice $i_j$ will become public information at the end of period 0.

After bankers have sold equity and short-term debt to households, each banker can screen her risky project. Screening is costless. With probability $p \in (0, 1)$, banker $j$ learns the true value of $\theta$ before she picks her portfolio. With probability $1 - p$ screening fails and the banker learns nothing. Success in screening is not observable by outsiders and statistically independent across the two bankers. The information derived from screening is private.

In the interim period $t = 1$, all agents learn the true realization of $\theta$.

**Timing.** The timing contains an important stochastic element: one banker picks her portfolio before the other does. The Leader ($L$) picks her portfolio first. The Follower ($F$) moves second. Nature decides which of the two bankers is the Leader. With probability $1/2$, $L = A$ and $F = B$. With probability $1/2$, $L = B$ and $F = A$.

The timing of the setup is summarized below:

t=0 Banker $A$ and banker $B$ publicly choose transparency $\pi_A$ and $\pi_B$ and prices $\tau_A$ and $\tau_B$. They sell equity and short-term debt to households. The face value of short-term debt due in $t = 1$ is $D^1_A$ and $D^1_B$, respectively.

Each banker screens her respective risky project.

Nature decides who is the Leader. The Leader picks a portfolio $i_L$. The value of $i_L$ becomes public information instantaneously with probability $\pi_L$.

The Follower picks a portfolio $i_F$. The value of $i_F$ becomes public information instantaneously with probability $\pi_F$.

$t=1$ All agents observe $\theta$. Banker $j = A, B$ owes a period 1 payment of $D^1_j$ to the holders of short-term debt. She offers a period 2 payment $D^2_j$ to households who roll over their claim. If a positive mass of households decides not to roll over, she needs to liquidate some investment projects at the fixed cost $\Phi$.

$t=2$ Non-liquidated projects pay off. If all households have decided to roll over, they receive $\min \{ D^2_j, (1 - \tau_j)V_j \}$. Households who hold bank equity receive a dividend of $\max \{ 0, (1 - \tau_j)V_j - D^2_j \}$. Banker $j$ receives $\tau_j V_j$.

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9This assumption will make sure that the net value of the bank’s portfolio is always positive and its expected return is always higher than $S$. The latter is a sufficient condition to guarantee that more than one bank is active in equilibrium.
Discussion of Model Assumptions

As mentioned above, the problem of a bank which hides its portfolio choice from the public has not yet been formally studied. To formalize this problem, I modify standard assumptions in several ways. These modifications come at the cost of a certain loss of generality. The upside is that the setup described above allows to understand the equilibrium level of transparency as the result of a trade-off between three clearly identified economic forces: market discipline, the Hirshleifer effect, and information spillovers.

**Banker Compensation.** I assume that bankers charge a fraction $\tau_j$ of the period 2 net value of their portfolio and that banker compensation is senior to short-term debt. These two assumptions are designed to perfectly align the banker’s and households’ preferences with respect to the portfolio choice. Given this compensation contract, the banker chooses the portfolio to maximize the total bank value, that is, the sum of short-term debt and equity.\(^{10}\)

In the banking literature, often an agency conflict is assumed between bankers and the ultimate bearers of the risk. This agency conflict introduces a benefit of bank transparency, as only transparent bankers can be kept from acting on their own behalf. One result of my paper is that transparency is useful even if bankers’ portfolio choice maximizes the total bank value, that is, the sum of short-term debt and equity.\(^{11}\)

**Investment Projects.** Risky projects are modeled in an unconventional way. They are designed to generate a non-trivial portfolio choice problem for risk neutral agents with a simple solution in closed-form. The assumption of a random maximum project size $\theta$ has the additional benefit that it is easy to interpret in a banking context. The risky project may be thought of as a risky loan with a high interest rate. The interest rate is known, but it is uncertain if the loan will be fully paid back. By granting a larger loan amount, the risk increases that the banker can recover only some part of the loan.

Risky projects are bank-specific, but their uncertain project size is perfectly correlated. This assumption is warranted if one banker’s private information about a borrower is a close substitute for another banker’s private information about another borrower. This might be the case because both bankers are lending funds to firms in the same industry or to homeowners in the same region. The random variable $\theta$ captures a factor which all firms in the sector and all homeowners in the region are exposed to and which the private information acquired by a banker helps to predict.

**Short-term Debt.** There is an upper limit for the buffer between $(1 - \tau_j)Sk_j$ and short-term debt: $(1 - \tau_j)Sk_j - D_j^1 \leq M$. This is a lower limit for the promised period 1 payment $D_j^1$. To keep the analysis as simple as possible, I do not model a reason why banks should choose a positive amount $D_j^1$ besides the exogenous constraint $M$. The model does not explain why banks in practice choose to be fragile and engage in maturity transformation. Diamond and Dybvig (1983) motivate the mismatch between

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\(^{10}\)Through a few minor additional assumptions, this compensation structure could be uniquely identified as an optimal contract.

\(^{11}\)The assumption that banker compensation is senior to short-term debt implies that bank managers have a positive payout even in case of bankruptcy. Bebchuk, Cohen, and Spamann (2010) find that for Lehman Brothers this was indeed the case.
assets and liabilities by liquidity insurance. I take banks’ fragility as an empirical fact and discuss why transparency matters for fragile banks.\footnote{For U.S. banks, Shibut (2002) calculates that uninsured deposits account for 15 percent of overall liabilities. This ratio is very stable across size groups. She also documents the increasing importance of non-depository sources of credit. Beatty and Liao (2014) report that ‘non-core funding’ (which largely consists of short-term uninsured liabilities) accounts for roughly 20 percent of U.S. bank funding. For an empirical account of short-term funding of U.S. bank holding companies (including broker-dealers), see Hanson, Kashyap, and Stein (2011). An international view on bank liabilities is provided by the IMF (2013).}

**Liquidation Cost.** In line with the literature, I assume that premature liquidation of projects is costly. This assumption is based on the observation that long-term investments are partially irreversible. The assumption of a fixed cost of liquidation yields particularly tractable expressions.

**Bank Transparency.** At the beginning of the initial period, a banker publicly chooses transparency as the probability that her portfolio choice will become public information. In practice, banks choose the frequency and the level of detail of publicly disclosed information about their asset holdings. This disclosure becomes credible through external auditing. Bank regulators lend some credibility as well, since they know more about individual banks than the public.

In the model, bankers choose transparency ex-ante when they are still perfectly identical. I assume that transparency is chosen ex-ante to account for the fact that in a crisis situation an opaque bank cannot decide to become transparent instantaneously. When bad news hit the economy, each bank tries to convince the public that its own exposure to the bad shock is small. But it takes time to communicate this information in a credible way. Disclosed information needs to be verified by external auditors or bank supervisors. The “quickest” of the recent stress test exercises was the 2009 Supervisory Capital Assessment Program in the U.S., which took three months from the official announcement until the release of the results (Candelon and Sy (2015), Table 1).

**Timing.** In the model, one banker picks her portfolio before the other does. This assumption implies that a banker cannot disclose her portfolio to households without the possibility that her competitor benefits from information spillover. In practice, no clear ordering between Leaders and Followers exists because banks continuously adjust their portfolio in response to new information.

## 4. Equilibrium

The previous section has described the setup of a sequential move game between households and two bankers. This section describes the subgame perfect Nash equilibrium. In this model, the subgame perfect equilibrium can be determined using backward induction. Without loss of generality, I focus attention on banker \( A \). The problem of banker \( B \) is perfectly symmetric.
4.1. Roll-over Game

In period 2 all agents consume their entire wealth. The interim period at $t = 1$ is more interesting. All households are perfectly identical. Each of them holds a short-term debt claim of face value $D^1_A$ against banker $A$ which matures in $t = 1$. Banker $A$ offers a period 2 payment $D^2_A$ to each household who rolls over his claim.

Households already know the realization of the maximum project size $\theta$, but projects have not paid off yet. Banker $A$ holds a portfolio of size $k_A$. Households know $i_A$ if it is publicly observable. In this case, households also know the exact value of $V_A$. If households do not observe $i_A$, they have to form a belief about $i_A$ and $V_A$.

If an individual household considers banker $A$’s short-term debt as riskless, he is willing to roll over at the same conditions as before: $D^2_A = D^1_A$. If after the observation of $\theta$ a household assigns positive probability to the event that banker $A$’s portfolio value will not be sufficient to fully serve $D^1_A$, the household will demand a higher payment $D^2_A > D^1_A$ in $t = 2$ as compensation for the positive risk of default. But if the household’s expectation of banker $A$’s portfolio value is too low, no promise is high enough. In this case, the household will prefer immediate repayment of $D^1_A$ in $t = 1$.

An individual household has zero mass. Banker $A$’s portfolio is large relative to an individual household’s debt claim. If an individual household chooses not to roll over, banker $A$ is always able to pay. But if a positive mass of households chooses not to roll over, banker $A$ needs to prematurely liquidate projects in order to generate cash. I refer to this case as a bank run.

Bank runs are costly because of the cost of premature liquidation. Even though collectively households have no interest in a bank run, each household individually may find it optimal not to roll over. Banker $A$ gets a fraction $\tau_A$ of the net value of the bank’s portfolio: $\tau_A[V_A - 1_{\text{run}, A} \Phi]$. Because a bank run reduces her payoff, banker $A$ is willing to promise any amount $D^2_A$ to the holders of short-term debt to avoid a bank run.

Let $Q$ denote households’ information set at $t = 1$. If no household chooses to run, I refer to this case as roll-over. Lemma 4.1 states that roll-over is an equilibrium whenever households expect that the gross value of banker $A$’s portfolio $V_A$ is sufficient to cover both the compensation payment to banker $A$ and the period 1 short-term debt claims of households.

**Lemma 4.1.** Roll-over is a Nash equilibrium if and only if:

$$(1 - \tau_A)E[V_A|Q] \geq D^1_A.$$ 

Whenever the condition of Lemma 4.1 is not satisfied, a positive mass of households runs on the bank and demands immediate repayment in $t = 1$. As shown in the proof of Lemma 4.1 in the appendix, it may be that for certain realizations of $\theta$ both roll-over and a bank run are Nash equilibria. It will be useful to introduce a selection criterion which uniquely pins down a single Nash equilibrium for all realizations of $\theta$.

**Equilibrium Selection:** Roll-over occurs whenever it is a Nash equilibrium.
As Allen and Gale (1998), I assume that the Pareto-superior equilibrium is always selected. A bank run only occurs if it is the unique Nash equilibrium.\footnote{This assumption is made purely for analytical convenience. As in Diamond and Dybvig (1983), a lender of last resort could rule out the Pareto-inferior equilibrium at zero cost. Alternatively, equilibrium selection through sunspots could easily be accommodated. For models of sunspot-driven bank runs, see Cooper and Ross (1998), or Peck and Shell (2003).}

### Transparency

Consider the case that banker $A$’s portfolio choice $i_A$ is public information in $t = 1$. In this case, households directly observe both $i_A$ and $V_A$. For given values of $i_A$, $\tau_A$, $k_A$, and $D_A^1$, Lemma 4.1 (in combination with our equilibrium selection criterion) gives us a range of realizations of $\theta$ which trigger a bank run:

$$1_{\text{run}_A} = 1 \Leftrightarrow (1 - \tau_A) \left[ S(k_A - i_A) + R \min \{ i_A, \theta \} \right] < D_A^1. \quad (1)$$

I am interested in the case that an uninformed banker can reduce the risk of a bank run to zero by investing only in the safe project. This is the case if and only if the following condition is satisfied:

**\( (A0) \):**

$$ (1 - \tau_A) S k_A - D_A^1 \geq 0. $$

As shown below, the endogenous variables $\tau_A$, $k_A$, and $D_A^1$ will satisfy $(A0)$ in equilibrium. It follows from $(A0)$ that a bank run can only occur if banker $A$ has over-invested in the risky project: $i_A > \theta$. The probability of a bank run given the publicly observed portfolio choice $i_A$ is:

$$ \Pr_{o}(i_A) = \Pr \left\{ \theta < \frac{1}{R} \left[ Si_A - \left( S k_A - \frac{D_A^1}{1 - \tau_A} \right) \right] \right\}. \quad (2) $$

A bank run happens whenever $\theta$ turns out to be too low. A high value of $i_A$ increases the range of realizations of $\theta$ which trigger a bank run.

### Opacity

If $i_A$ is not public information at $t = 1$, households form a belief about $i_A$ and $V_A$. Consider the case that banker $A$ is the Leader. Households know that with probability $p$ banker $A$ has been successful in screening and has learned the true value of $\theta$. They understand that an informed banker sets $i_A = \theta$. Accordingly, their period 1 expectation of the gross value of banker $A$’s portfolio is:

$$ \mathbb{E}[V_A|Q] = p \left[ S k_A + (R - S)\theta \right] + (1 - p) \left[ S(k_A - \hat{i}_A) + R \min \{ \hat{i}_A, \theta \} \right], \quad (3) $$
where $\hat{i}_A$ is the portfolio choice which households at date 1 believe banker $A$ has selected if she did not know $\theta$. If (A0) is satisfied, this implies for the probability of a bank run:

$$
Pr_u(\hat{i}_A) = Pr\left\{ \theta < \frac{1}{R-pS} \left[ (1-p)S\hat{i}_A - \left( Sk_A - \frac{D_A^1}{1-\tau_A} \right) \right] \right\}.
$$

(4)

In contrast to the case of an observable portfolio choice, this probability does not directly depend on $i_A$ anymore but rather on $\hat{i}_A$ (creditors’ date 1 belief about $i_A$).

### 4.2. Portfolio Choice

I continue to proceed by backward induction and study banker $A$’s portfolio choice at the end of period 0. She can invest an amount $k_A$ in a portfolio of a risky and a riskless project. Her payoff is $\tau_A[V_A - 1_{\text{no run}}\Phi]$. Accordingly, her portfolio choice is shaped by two concerns: (1.) She wants to achieve a high expected portfolio value $V_A$, and (2.) she prefers a low probability of a bank run. I assume that banker $A$’s portfolio is large enough to rule out corner solutions:

(A1): $k_A \geq \mu + a$.

In a symmetric equilibrium with $k_A = k_B = w/2$, this condition is always satisfied because $w/2 > \mu + a$.

#### 4.2.1. Informed Portfolio Choice

Banker $A$ might know $\theta$ at the time when she picks her portfolio. An informed portfolio choice is simple. Clearly, $V_A$ is maximized by setting $i_A = \theta$. This portfolio choice also avoids a bank run in case it is publicly observed. If $i_A$ is not publicly observed, its actual value cannot affect households’ period 1 belief about $i_A$ and $V_A$. Whether a bank run occurs is independent of $i_A$ in this case. It follows that an informed banker always sets $i_A = \theta$.

#### 4.2.2. Uninformed Portfolio Choice

A portfolio choice under uncertainty is more difficult for the banker. We have seen that banker $A$’s actual portfolio choice $i_A$ only matters for the risk of a bank run if it is publicly observed. This happens with probability $\pi_A$. If banker $A$ does not know the true realization of $\theta$, she chooses $i_A$ by solving:

$$
\max_{i_A} \tau_A \left[ E[V_A] - \pi_A Pr_o(i_A) \Phi - (1-\pi_A)Pr_u(\hat{i}_A)\Phi \right],
$$

subject to: $V_A = S(k_A - i_A) + R \min\{i_A, \theta\}$.

(5)

(6)

For any $\tau_A \in (0,1)$, the preferences of banker $A$ and the households about the optimal portfolio choice are perfectly aligned. It is as if banker $A$ would choose $i_A$ to maximize household utility.
I am interested in an environment in which bank runs occur. If \( i_A \) is not publicly observed, bank runs happen if \( \theta \) turns out to be low and households’ belief about \( i_A \) is high. \( \Pr_u(i_A) \) is positive if and only if:

\[
\hat{i}_A > \bar{i}_A \equiv \frac{1}{(1-p)S} \left( (R - pS)(\mu - a) + S k_A - \frac{D_A^1}{1 - \tau_A} \right).
\] (7)

If \( \hat{i}_A \) is lower than \( \bar{i}_A \), even the lowest possible realization of \( \theta = \mu - a \) does not trigger a bank run. A high face value of short-term debt \( D_A^1 \) implies that the risk of a bank run is positive even for low values of \( \hat{i}_A \). If the probability to screen successfully \( p \) is high and \( i_A \) is not publicly observed, households’ expectation \( \mathbb{E}[V_A | Q] \) is high and a run is unlikely even if \( \hat{i}_A \) is large.

As shown in the proof of Lemma 4.2, a sufficient condition for a portfolio choice \( i_A^* > \bar{i}_A \) is that the return of the risky project \( R \) is high enough relative to the return of the safe project \( S \), the cost of early liquidation \( \Phi \), and \( \bar{i}_A \):

\[
(A2): \quad -S + R \left( \frac{2S}{2a} \right) - \frac{4S}{2aR} > 0.
\]

From now on, I will assume that conditions (A0)-(A2) hold. In this case, the optimal portfolio choice \( i_A^* \) has a simple solution in closed-form.

**Lemma 4.2.** The optimal portfolio choice is:

\[
i_A^* = \mu - a \left( \frac{2S}{R} - 1 \right) - \pi_A \frac{\Phi S}{R^2}.
\]

The objective function of an uninformed banker is strictly concave in \( i_A \). She knows that as long as \( i_A \) is smaller than \( \theta \), the marginal return of the risky project is higher than the safe return. But if \( \theta \) should turn out to be smaller than \( i_A \), the marginal return of the risky project is zero. An increase of \( i_A \) makes it more likely that the maximum project size \( \theta \) turns out to be smaller than \( i_A \). It follows that the expected marginal return of \( i_A \) is decreasing and the optimal portfolio choice generally has an interior solution.

The banker’s choice \( i_A^* \) is increasing in the expected maximum project size \( \mu \). An increase of uncertainty \( a \) decreases \( i_A^* \) if and only if \( S < R \leq 2S \). If \( R = 2S \), investing too much in the risky project and earning zero at the margin instead of the safe return \( S \) is just as costly as investing too little and missing out on \( R - S = S \). If \( R < 2S \), over-investment is more costly than under-investment. This is why an increase in uncertainty \( a \) lowers \( i_A^* \) in this case.

Importantly, the optimal portfolio choice \( i_A^* \) is falling in the level of transparency \( \pi_A \). Why is this the case? Consider a banker who knows that \( i_A \) will be public information: \( \pi_A = 1 \). If \( i_A \) is publicly observed, a higher value of \( \hat{i}_A \) increases the risk of a bank run. This reduces the expected marginal benefit of \( i_A \). A higher cost of early liquidation \( \Phi \) implies a lower choice \( i_A^* \).

Consider now a banker who knows that \( i_A \) will remain hidden: \( \pi_A = 0 \). Banker \( A \) is of course free to choose the value of \( i_A \) which is optimal under full transparency (\( \pi_A = 1 \)).
However, she has no reason to do so. If \( i_A \) is not publicly observed, the risk of a bank run is independent of \( i_A \). A bank run occurs if \( \theta \) is low and households’ expectation \( \hat{i}_A \) is high. If households do not observe \( i_A \), changes in \( i_A \) are not observed and have no effect on \( \hat{i}_A \). Banker \( A \) must take households’ expectation \( \hat{i}_A \) and the risk of a bank run \( \Pr_u(i_A) \) as given. This is why the cost of early liquidation \( \Phi \) does not affect \( i_A^* \) if \( \pi_A = 0 \). Only a transparent banker has an incentive to take the risk of a bank run into account when she selects her portfolio.

4.3. Bank Transparency

This is the central section of the paper. We have understood how the risk of a bank run depends on the banker’s portfolio choice, and how this portfolio choice depends on bank transparency. I continue to proceed by backward induction. At the beginning of period 0, banker \( A \) publicly chooses the probability that her portfolio choice becomes public information. At this point, banker \( A \) does not know whether she will be successful in screening the risky project, and whether she will be the Leader (who moves first) or the Follower.

When banker \( A \) chooses the optimal value of \( \pi_A \), she considers three distinct roles of bank transparency: (1.) market discipline, (2.) the Hirshleifer effect, and (3.) information spillovers. The first two of these roles describe how transparency affects the expected net value of banker \( A \)’s own portfolio.

Lemma 4.3. An increase in transparency \( \pi_A \) at \( t = 0 \) affects the expected net value of banker \( A \)’s portfolio according to:

\[
\frac{\partial \mathbb{E}[V_A - \mathbb{I}_{\text{run}_A} \Phi]}{\partial \pi_A} = \left(1 - \frac{p\pi_B}{2}\right) \left[ \frac{\Phi S^2}{R - pS} \frac{1 - \pi_A}{2aR^2} - \Phi \left( 1 - p \right) \Pr_u(i_A^*) - \Pr_u(i_A^*) \right].
\]

There are diminishing returns to transparency:

\[
\frac{\partial^2 \mathbb{E}[V_A - \mathbb{I}_{\text{run}_A} \Phi]}{\partial \pi_A^2} = - \left(1 - \frac{p\pi_B}{2}\right) \frac{(1 - p)\Phi S^2}{2aR^3} \frac{R + pS}{R - pS}.
\]

If banker \( A \) turns out to be the Follower (with probability \( 1/2 \)), she will have the chance to learn the realization of \( \theta \) from banker \( B \)’s portfolio choice \( i_B \). This happens if banker \( B \) is successful in screening her risky project (probability \( p \)) and if \( i_B \) is publicly observed (probability \( \pi_B \)). In this case, banker \( A \) observes \( i_B \neq i_B^* \) and concludes: \( i_B = \theta \). Banker \( A \) optimally sets \( i_A = \theta \). Also households observe \( i_B \neq i_B^* \). They understand that banker \( A \) has set \( i_A = \theta \). From (A0) it follows that the bank run risk is zero. In this case, transparency \( \pi_A \) does not matter for the expected payout of banker \( A \). This is why the marginal benefit of \( \pi_A \) is weighted with probability \( 1 - p\pi_B/2 \).

Whenever this is not the case, transparency matters for the expected payout of banker \( A \). The marginal benefit of transparency can be positive or negative but is always decreasing in \( \pi_A \). It can be decomposed into two distinct forces: (1.) market discipline, and (2.) the Hirshleifer effect.
4.3.1. Market Discipline

Consider the case that banker A fails at screening. This happens with probability $1 - p$. In this case, she will choose $i_A^*$ at the end of period 0. We know from the previous section that $i_A^*$ is falling in $\pi_A$. If banker A is more transparent, she will select a safer portfolio because there is a higher chance that her choice of $i_A$ will affect the risk of a bank run.

Through the reduction in $i_A^*$, transparency affects the expected net value of banker A’s portfolio. This effect is captured in Lemma 4.3 by the first term in square brackets:

$$(1 - p) \frac{1 - \pi_A}{R - pS} \frac{\Phi^2 S^2}{2aR^2} \geq 0.$$  

This term is positive. The expected net value of banker A’s portfolio increases. Evidently, transparency helps the banker to select a better portfolio. But if a reduction in $i_A^*$ is beneficial, why does a transparent banker carry out this reduction while an opaque banker does not?

Households have rational expectations. They understand banker A’s portfolio choice problem. This means that in equilibrium banker A is subject to the constraint $\hat{i}_A = i_A^*$ regardless if $i_A$ is observable or not. A fully transparent banker ($\pi_A = 1$) completely internalizes this constraint. She knows that her action $i_A$ influences households’ expectation $\hat{i}_A$. A fully opaque banker ($\pi_A = 0$) likewise would prefer a low value $\hat{i}_A$ because this reduces the risk of a bank run. However, when an opaque banker selects her actual portfolio $i_A$ she knows that changes in $i_A$ do not affect $\hat{i}_A$. An opaque banker must take households’ expectation $\hat{i}_A$ as given and chooses $i_A$ as a best response. Since the solution of this problem is different from the one of a transparent bank, the opaque banker faces a credibility problem. She cannot credibly commit to a prudent portfolio choice which takes the risk of a bank run into account. In equilibrium, this results in an inefficiently high probability of a bank run and a low expected net value of her portfolio.\(^\text{14}\)

As long as the banker is not fully transparent ($\pi_A < 1$) and as long as there is a chance that she must select a portfolio under uncertainty ($p < 1$), there is a benefit from an increase in $\pi_A$ which is transmitted through a reduction in $i_A^*$ and through a reduction in the equilibrium probability of a bank run.

4.3.2. Hirshleifer Effect

Consider the second effect of an increase in $\pi_A$ on the expected net value of banker A’s portfolio. This effect is captured in Lemma 4.3 by the second term in square brackets:

$$- \Phi \left[ (1 - p) \Pr_o(i_A^*) - \Pr_u(i_A^*) \right].$$  

This term measures the difference between the expected costs of a bank run if $i_A^*$ is observed and if $i_A^*$ is unobserved. This difference is generally different from zero.

\(^{14}\)The bank’s credibility problem is similar to the problem of time-inconsistency in Kydland and Prescott (1977). There is an important difference however: the bank’s portfolio choice does not depend on its timing, but on its observability.
Whether a bank run occurs, depends on households’ period 1 expectation $\mathbb{E}[V_A|Q]$. Consider some given realization of $\theta$. If $i_A$ is publicly observed, $\mathbb{E}[V_A|\theta, i_A]$ can take on two different values. If $i_A = \theta$, households observe the portfolio value of an informed banker $V_A^i(\theta) = Sk_A + (R - S)\theta$. This happens with probability $p$. If $i_A = i^*_A$, households observe the portfolio value of an uninformed banker $V_A^u(\theta) = S(k_A - i^*_A) + R \min \{i^*_A, \theta\}$. This happens with probability $1 - p$. If $i_A$ is not publicly observed, households’ period 1 expectation is:

\[
\mathbb{E}[V_A|\theta] = p \left[ Sk_A + (R - S)\theta \right] + (1 - p) \left[ S(k_A - i^*_A) + R \min \{i^*_A, \theta\} \right] \quad (10)
\]

\[
= p V_A^i(\theta) + (1 - p) V_A^u(\theta). \quad (11)
\]

For any $\theta$, we have: $V_A^u(\theta) < \mathbb{E}[V_A|\theta] < V_A^i(\theta)$. Depending on the realization of $i_A$, the observability of $i_A$ may be stabilizing or de-stabilizing for the bank.

1. $V_A^u(\theta) < \mathbb{E}[V_A|\theta] < \frac{D_A^i}{1 - \gamma_A} < V_A^i(\theta)$: Here $\theta$ is very low. A bank run is triggered even if $i_A$ remains hidden. The probability of being in this region is $Pr_u(i^*_A)$. Transparency is beneficial in this case because it allows to avoid a bank run with probability $p$.

2. $V_A^u(\theta) < \frac{D_A^i}{1 - \gamma_A} < \mathbb{E}[V_A|\theta] < V_A^i(\theta)$: Here $\theta$ is high enough to avoid a run if $i_A$ remains hidden but still too low to avoid a run if $i_A$ is observed and equal to $i^*_A$. The probability of this case is $Pr_o(i^*_A) - Pr_u(i^*_A)$. Opacity is beneficial here because it completely rules out all bank runs.

3. $\frac{D_A^i}{1 - \gamma_A} < V_A^u(\theta) < \mathbb{E}[V_A|\theta] < V_A^i(\theta)$: If $\theta$ is sufficiently high, the observability of $i_A$ does not matter for the risk of a bank run since it is always zero.

This explains the second effect of transparency. With probability $Pr_u(i^*_A)$, banker $A$ will find herself in region (1). An increase in $\pi_A$ lowers the risk of a bank run by a factor of $p$. With probability $Pr_o(i^*_A) - Pr_u(i^*_A)$, banker $A$ will find herself in region (2). An increase in $\pi_A$ raises the risk of a bank run by a factor of $1 - p$. On expectation, an increase in $\pi_A$ raises the risk of a bank run by:

\[
- Pr_u(i^*_A)p + [Pr_o(i^*_A) - Pr_u(i^*_A)](1 - p) = (1 - p)Pr_o(i^*_A) - Pr_u(i^*_A)
\]

\[
= \frac{p}{2a} \left[ \frac{(1 - p)S^2}{R(R - pS)} + \frac{R + (1 - p)S}{R(R - pS)} \right] \quad (12)
\]

The value of this expression depends on the efficiency of banker A’s screening technology.

**Lemma 4.4.** There exists a probability $\overline{p} \in [0, 1)$ such that:

\[
(1 - p)Pr_o(i^*_A) - Pr_u(i^*_A) > 0 \iff p > \overline{p}.
\]

If the probability that banker $A$ successfully screens her risky project is sufficiently high, then an increase in transparency $\pi_A$ raises the risk of a bank run (for a given
portfolio choice $i^*_A$). This is because it is more likely that banker $A$ will find herself in region (2) instead of region (1). In region (2), opacity is beneficial. An informed banker and an uninformed banker look alike if $i_A$ remains hidden. The possibility that an opaque banker is informed provides insurance for an uninformed banker. Informed bankers have no disadvantage from being pooled with uninformed bankers as long as a bank run is avoided. Lemma 4.4 states that opacity is valuable as a risk sharing mechanism as long as the probability mass of informed bankers $p$ is sufficiently high. Transparency precludes this risk sharing opportunity. The fact that public information can destroy risk sharing opportunities often is called the Hirshleifer effect. In this model, it creates a benefit of bank opacity which must be weighed against the associated reduction of market discipline.

4.3.3. Information Spillover

There is a third role for bank transparency in this model. In contrast to the first two, this third role describes how banker $A$’s choice of transparency $\pi_A$ affects the expected net value of her rival’s portfolio.

**Lemma 4.5.** An increase in transparency $\pi_A$ at $t = 0$ raises the expected net value of banker $B$’s portfolio by:

$$
\frac{\partial E[V_B - 1_{\text{run}_B}]}{\partial \pi_A} = \frac{p}{2} \left[ (1 - p) \left( E[V_B | i_B = \theta] - E[V_B | i_B = i^*_B] \right) + (1 - p) \pi_B \Pr_o(i^*_B) + (1 - \pi_B) \Pr_u(i^*_B) \right] > 0.
$$

If banker $A$ happens to be successful in screening her risky project (probability $p$) and banker $B$ should turn out to be the Follower (probability 1/2), banker $B$ benefits from observing $i_A$. With probability $1 - p$, banker $B$ will fail at screening. In this case, the expected net value of banker $B$’s portfolio benefits in two ways. (1.) $E[V_B]$ benefits from an informed portfolio choice, and (2.) the probability of a bank run if $i_B$ is publicly observed falls from $\Pr_o(i^*_B)$ to zero. But banker $B$ benefits even if she will screen successfully. If $i_B$ is not publicly observed, the probability of a bank run falls from $\Pr_u(i^*_B)$ to zero.

Information spillovers exist if and only if $p \in (0, 1)$. In this case, there is a chance that one banker has private information which is valuable for her rival and which is only revealed by her portfolio choice.

While $E[V_A - 1_{\text{run}_A}]$ is strictly concave in $\pi_A$, the expected net value of banker $B$’s portfolio $E[V_B - 1_{\text{run}_B}]$ is linear in $\pi_A$. This is because the expected net value of an uninformed banker’s portfolio is concave in her portfolio choice. As $\pi_A$ is increased, $i^*_A$ moves closer to the portfolio choice of a transparent bank. The closer $i^*_A$ gets to this optimum, the smaller becomes the marginal benefit of increasing $\pi_A$ further. In contrast, an increase in $\pi_A$ raises the probability that banker $B$ changes her portfolio choice $i_B$ from $i^*_B$ to $\theta$. Since this discrete change in $i_B$ is independent of the level of $\pi_A$, the marginal benefit from information spillovers is constant in $\pi_A$. 

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4.4. Bank Competition

In the beginning of period 0, banker $A$ and banker $B$ simultaneously choose $\pi_A$, $\pi_B$, $\tau_A$, $\tau_B$, $k_A$, $k_B$, $D^1_A$, and $D^1_B$. These choices are public information. Households allocate their endowment $w$ across the two banks. All agents take optimizing behavior at subsequent nodes of the game-tree (portfolio choice, roll-over game) as given. In equilibrium, bank transparency is determined by supply and demand in the capital market.

4.4.1. Supply of Capital

The representative household owns an endowment of quantity $w$. He can buy equity $e_j$ and short-term debt $d_j$ from banker $j$ ($j = A, B$):

$$\max_{e_A, d_A, e_B, d_B \geq 0} e_A r^e_A + d_A r^d_A + e_B r^e_B + d_B r^d_B$$

subject to: $e_A + d_A + e_B + d_B \leq w$,  

where $r^e_j$ and $r^d_j$ is the expected return on equity and short-term debt issued by banker $j = A, B$. The representative household invests all his wealth in the security with the highest expected return. If any two securities yield an identical expected return, the household is indifferent between the two.

4.4.2. Demand for Capital

We continue to focus on banker $A$. She collects capital by issuing equity and short-term debt: $k_A = e_A + d_A$. From households’ supply of capital, it follows that banker $A$ can only issue both equity and short-term debt if: $r^e_A = r^d_A \equiv r_A$. Hence:

$$e_A r^e_A + d_A r^d_A = (e_A + d_A) r_A = k_A r_A .$$

The total expected payout to households is:

$$k_A r_A = (1 - \tau_A) \mathbb{E} [V_A - 1_{run_A} \Phi] .$$

Banker $A$ sets $\pi_A$, $\tau_A$, $k_A$, $r_A$, and $D^1_A$ to maximize:

$$\max_{\pi_A, \tau_A, k_A, r_A, D^1_A} \tau_A \mathbb{E} [V_A - 1_{run_A} \Phi]$$

subject to: $k_A =\begin{cases} w & \text{if } r_A > r_B , \\ w - k_B & \text{if } r_A = r_B , \\ 0 & \text{if } r_A < r_B , \end{cases}$

$$r_A = \frac{(1 - \tau_A) \mathbb{E} [V_A - 1_{run_A} \Phi]}{k_A} , \quad r_B = \frac{(1 - \tau_B) \mathbb{E} [V_B - 1_{run_B} \Phi]}{k_B} ,$$

$$\left(1 - \tau_A\right) S k_A - D^1_A \leq M .$$
The first constraint is households’ supply of capital to banker $A$. It depends both on the expected return offered by banker $A$ and on the return offered by her competitor. The second constraint describes how these returns depend on the expected net value of the two bankers’ portfolios. The last constraint is the exogenous upper bound for the buffer between $(1 - \tau_A)S_{k_A}$ and $D^1_A$. We know from the previous analysis that the expected net value of banker $A$’s portfolio $\mathbb{E}[V_A - 1_{\text{run}_A}\Phi]$ is a function of $\pi_A$, $k_A$, and $D^1_A$. Importantly, also $\mathbb{E}[V_B - 1_{\text{run}_B}\Phi]$ depends on $\pi_A$ through its effect on banker $B$’s portfolio choice and the probability of a bank run.

4.4.3. Symmetric Equilibrium

Since the two bankers are perfectly identical ex-ante, it is natural to focus on a symmetric equilibrium. Proposition 4.6 characterizes transparency in a symmetric subgame perfect equilibrium. Until now, we have restricted our analysis to a parameter space which satisfies conditions (A0)-(A2). In a symmetric equilibrium, $k^*_A = k^*_B = w/2$. Because $w/2 > \mu + a$, it follows that (A1) is always satisfied. Furthermore, the constraint on the buffer between $(1 - \tau_A)S_{k_A}$ and $D^1_A$ is binding in equilibrium.\(^{15}\) From $M \geq 0$, it follows that (A0) is always satisfied. The last condition (A2) can now be re-formulated in terms of fundamentals only.

**Proposition 4.6.** Consider a set of parameter values which satisfies (A2):

\[
-S + \frac{R}{2a} (\mu + a - \bar{i}) - \frac{\Phi S}{2aR} > 0, \text{ where: } \bar{i} = \frac{1}{(1-p)S} \left[ (R - pS)(\mu - a) + \frac{M}{1 - \tau} \right].
\]

A symmetric subgame perfect Nash equilibrium exists and is characterized by:

\[
k^*_A = k^*_B = \frac{w}{2}, \text{ and: } (1 - \tau^*_A)S_{k_A}^* - D^1_A^* = (1 - \tau^*_B)S_{k_B}^* - D^1_B^* = M.
\]

The two bankers charge:

\[
\tau^*_A = \tau^*_B = \frac{\rho - S}{\rho - \frac{S}{2}}, \text{ where: } \rho = \frac{\mathbb{E}[V_A - 1_{\text{run}_A}\Phi]}{k^*_A} = \frac{\mathbb{E}[V_B - 1_{\text{run}_B}\Phi]}{k^*_B}.
\]

An interior solution for transparency $\pi^*_A = \pi^*_B \in (0, 1)$ satisfies the following first order condition:

\[
\frac{\partial \mathbb{E}[V_A - 1_{\text{run}_A}\Phi]}{\partial \pi_A} - (1 - \tau^*_B) \frac{\partial \mathbb{E}[V_B - 1_{\text{run}_B}\Phi]}{\partial \pi_A} = 0.
\]

The two bankers try to attract capital from households by offering a higher expected return than the rival bank. This can be done in two ways: (1.) A banker can increase

\(^{15}\)Without the exogenous upper limit on $M$, banker $A$ would never choose a level of $D^1_A$ which implies a positive risk of a bank run. In this model, there is no benefit from having a maturity mismatch between assets and liabilities. The model does not explain why banks engage in maturity transformation. I take banks’ fragility as an empirical fact and discuss why transparency matters for fragile banks. See also the discussion of model assumptions in Section 3.
the expected net value of her own portfolio $\mathbb{E}[V_A - 1_{\text{run}_A}\Phi]$, or (2.) she can reduce the expected net value of her rival’s portfolio $\mathbb{E}[V_B - 1_{\text{run}_B}\Phi]$. Proposition 4.6 describes the optimal weight which banker $A$ assigns to each of these two options. For a given return $r_A$ offered to households, an increase in the expected net value of her own portfolio is fully captured by banker $A$. At the same time, a reduction in the expected net value of her rival’s portfolio benefits banker $A$ only to the extent that it reduces $r_B$. This is why in the first order condition of $\pi_A^*$, the reduction of $\mathbb{E}[V_B - 1_{\text{run}_B}\Phi]$ is weighted by the fraction $(1 - \tau^*_B)$ which is passed on to households by banker $B$.$^{16}$

We know from Lemma 4.3 that the expected net value of banker $A$’s own portfolio is strictly concave in $\pi_A$. By Lemma 4.5, her rival’s portfolio value is linear in $\pi_A$. It follows that the equilibrium level of transparency may have an interior solution. Banker $A$ increases $\pi_A$ as long as the benefit of market discipline outweighs the costs of the Hirshleifer effect and information spillovers to her rival.$^{17}$

Importantly, banker $A$ does not choose transparency to maximize the expected net value of her portfolio. On the contrary, she reduces this value if this hurts her rival more than it hurts herself. The two bankers interact as in a prisoner’s dilemma. In equilibrium, both end up with an expected net portfolio value (and an expected payoff) which is lower than it could be.

### 4.5. More than two Banks

The two banks studied above act strategically. They reduce transparency to gain a competitive advantage over their competitor. In this section, I extend the analysis to any number $N = 3, 4, 5, \ldots$ of banks. Apart from this, the environment essentially remains unchanged. Each banker has access to a riskless project and a bank-specific risky project. The portfolio choice always has an interior solution: $\mu + a < w/N$. As before, there are Leaders who move first, and Followers who move second. Half of the bankers become Leaders, and half of them become Followers.$^{18}$

Compared to the case of two banks, each Follower is more likely now to learn the realization of $\theta$ from one of the Leaders who screens successfully and whose portfolio choice is public information. Only if this is not the case, the Follower’s bank run risk is positive and her choice of transparency matters for market discipline and the Hirshleifer effect. Information spillovers are a concern for Leaders only if $\theta$ is not revealed by some other Leader.

**Proposition 4.7.** The equilibrium level of transparency $\pi^*$ is increasing in $N$. For each given bank, the probability of a bank run is falling in $N$.

$^{16}$Proposition 4.6 characterizes the equilibrium if (A2) is satisfied. If (A2) does not hold, $\Pr_u(i^*_A)$ is not strictly positive anymore for all values of $\pi_A$. Even $\Pr_o(i^*_A)$ might be zero in this case. If the risk of a bank run is zero (for instance, because $M$ is large), market discipline and the Hirshleifer effect become irrelevant. Information spillovers are still a concern though.

$^{17}$Note that the Hirshleifer effect and information spillovers vanish as $p \to 0$. These forces are only relevant if bankers may have private information. Market discipline still matters though even if $p = 0$.

$^{18}$In case of an uneven number of bankers, the last banker becomes a Leader or a Follower with probability one half.
An increase in $N$ affects banker $A$’s trade-off in choosing $\pi_A$. As shown in the proof of Proposition 4.7, the probability that a given competitor observes $i_A$ before selecting a portfolio largely remains unchanged. But with $N/2$ Leaders, there are $N/2$ potentially informative signals about $\theta$. As the number of signals is increased, it becomes less likely that $i_A$ is the only informative signal about $\theta$ which the competitor observes. Each individual signal becomes less pivotal for the information set of other bankers. This allows banker $A$ to put less weight on information spillovers which has a positive effect on transparency.\footnote{The result that $\pi^*$ is increasing in $N$ is not specific to the setup chosen here. For instance, it is straightforward to show that $\pi^*$ is also increasing in $N$ if (1) there is one Leader and $N - 1$ Followers, or if (2) there is one Follower and $N - 1$ Leaders.}

The positive effect of bank competition on transparency is interesting for two reasons. First, this prediction is in line with the empirical evidence. Jiang, Levine and Lin (2016) estimate that the removal of regulatory impediments to bank competition by individual states in the U.S. has improved the informational content of banks’ financial statements. Proposition 4.7 suggests that this might have been driven by a reduction of strategic concerns as regional markets became more competitive.

Secondly, Proposition 4.7 states that for any given bank the risk of a bank run is falling in $N$. As the equilibrium level of transparency increases in $N$, market discipline improves and information spillovers contribute to stability. A higher number of banks increases financial stability in the model. This is in contrast to the widely held view of a clear-cut trade-off between bank competition and financial stability (e.g. Keeley (1990)).

5. Efficiency

In this model, bank transparency is chosen by strategic banks who compete for households’ funds. The previous section has described the symmetric subgame perfect Nash equilibrium of this game. In equilibrium, the expected net value of bankers’ portfolios is not maximized. Bankers interact as in a prisoner’s dilemma. This suggests that there is room for policy.

There are various sources of inefficiency in the model: banks have market power, their portfolio choice is subject to a credibility problem, and costly bank runs occur in equilibrium. In this paper, I focus on the question whether the equilibrium level of transparency is efficient. Can a policy-induced change of bank transparency increase efficiency?

I use a notion of constrained efficiency. For the case of two banks, the social planner solves:

$$\max_{\pi_A, \pi_B} \mathbb{E}\{V_A - \mathbbm{1}_{\text{run}_A} \Phi + V_B - \mathbbm{1}_{\text{run}_B} \Phi\}. \tag{21}$$

The planner maximizes the expected value of aggregate consumption. She cannot directly change banks’ portfolio choice or households’ behavior in the roll-over game. She
can only control the two banks’ levels of transparency $\pi_A$ and $\pi_B$ set in the beginning of period 0.

**Proposition 5.1.** The constrained-efficient level of transparency is unique and symmetric: $\pi_A^P = \pi_B^P$. This value is higher than the equilibrium value: $\pi_A^P > \pi_A^*$. 

The first order condition for a constrained efficient choice of transparency $\pi_A^P$ reads as:

$$
\frac{\partial \mathbb{E} [V_A - 1_{\text{run}_A} \Phi]}{\partial \pi_A^P} + \frac{\partial \mathbb{E} [V_B - 1_{\text{run}_B} \Phi]}{\partial \pi_A^P} = 0. 
$$

(22)

This first order condition is different from the one of banker A’s equilibrium choice of transparency described in Proposition 4.6. Just as in Proposition 4.6, information spillovers matter for transparency. But while these spillovers reduce the *private* value of transparency, they increase its *social* value. For the planner, they are a reason to choose a higher level of transparency. She increases $\pi_A$ as long as the benefits of market discipline and information spillovers outweigh the cost of the Hirshleifer effect.

This model rationalizes the puzzling observation that real-world households and other investors choose to lend money to opaque banks while many commentators argue at the same time that bank opacity is higher than it should be. Information spillovers create an environment in which the market rewards banks which set an excessive level of opacity.

If a policy maker has the option to increase $\pi_A$ above its equilibrium value, for instance through minimum public disclosure requirements or through periodic and standardized public stress tests, this is beneficial. Market discipline is improved and risk taking $i_A^*$ is reduced. This mechanism provides a justification for the observed policy interventions such as the public disclosure requirements specified in Pillar Three of Basel II. Interestingly though, the social planner’s objective (aggregate expected consumption) is strictly concave in $\pi_A$. Because of the Hirshleifer effect, the constrained-efficient level of transparency $\pi_A^P$ generally has an interior solution. Imposing a maximum amount of transparency ($\pi_A = \pi_B = 1$) does not necessarily maximize efficiency.

A similar result holds for financial stability.

**Proposition 5.2.** The value of transparency which minimizes the frequency of bank runs is unique and symmetric: $\pi_A^R = \pi_B^R$. This value is higher than the equilibrium value: $\pi_A^R > \pi_A^*$. 

Increasing transparency improves market discipline and fosters information spillovers across banks. These two channels always contribute to a reduction in the frequency of bank runs. But through the Hirshleifer effect, an increase in transparency might eventually cause the opposite. As shown in the proof of Proposition 5.2, the frequency of bank runs is strictly convex in $\pi_A$ and $\pi_B$. Because of the Hirshleifer effect, $\pi_A^R$ might have an interior solution.
6. Conclusion

This paper studies a model of endogenous bank transparency. It identifies three distinct forces which determine banks’ equilibrium choice of transparency: (1.) market discipline, (2.) the Hirshleifer effect, and (3.) information spillovers. This third force drives a wedge between the market outcome and the constrained-efficient level of transparency. A social planner would choose a higher value of transparency than the market outcome. However, full transparency maximizes neither efficiency nor stability.

The model environment differs from others used in the literature. It is designed to provide a tractable framework which allows to clearly identify the three described roles of bank transparency. The fundamental roles of market discipline, the Hirshleifer effect, and information spillovers are unlikely to change if individual model assumptions are modified to move closer to existing models. For instance, I assume a fixed cost of early liquidation. A proportional cost would change the particular solution to banks’ portfolio choice. But it would not change the fact that banks’ portfolio choice is subject to a credibility problem. The same is true for the particular type of portfolio choice which banks face in this model (uncertainty about the maximum project size).

One interesting result of the model is that market discipline is useful even though bankers and investors agree about the optimal portfolio choice. Many models in the literature assume an agency problem between bank managers and the ultimate bearers of the risk (e.g. shareholders, deposit insurance agency, etc.). A possible extension of the model is to introduce a similar agency conflict. This might affect the equilibrium choice of transparency and thereby add a fourth role of transparency besides the three forces identified above.

Transparency matters for financial stability because banks have a maturity mismatch between assets and liabilities. In the model, this mismatch arises because of an exogenous constraint which obliges banks to issue short-term debt. In practice, this mismatch arises for different reasons. It is possible that banks’ choice of transparency interacts with their choice of maturity transformation. Endogenizing banks’ financing choice (short-term vs long-term debt or equity) could allow to study this interaction within the framework set out above.

One last possible extension is to endogenize banks’ screening intensity. Because of information spillovers, an opaque bank is likely to exert more effort in screening than a transparent bank. This relationship could introduce a second social cost of transparency besides the Hirshleifer effect. The question of how endogenous screening affects the socially optimal level of transparency I leave for future research.
A. Proofs and Derivations

Proof of Lemma 4.1

Consider banker $A$ at date $t = 1$. She receives a payoff $\tau_A V_A$ if a bank run is avoided, and $\tau_A [V_A - \Phi]$ otherwise. Clearly, she is willing to promise any amount $D_A^2$ to the holders of short-term debt to avoid a bank run.

Now consider an individual atomistic household at $t = 1$. He observes the value of $V_A$ if $i_A$ is public information. Otherwise he has to form a belief about $V_A$. He holds a short-term debt claim of value $D_A^1$ against banker $A$. Banker $A$ promises an amount $D_A^2$ in exchange for roll-over. If in period 2 banker $A$ should be unable to fully service $D_A^2$, dividends will be zero and the holders of short-term debt become the residual claimants of the bank.

If everyone else rolls over, roll-over by an individual household yields:

$$\min\{ D_A^2, (1 - \tau_A)V_A \}.$$  \hspace{1cm} (23)

Banker $A$’s portfolio is large relative to an individual household’s debt claim. If an individual household chooses not to roll over, banker $A$ is always able to pay without the need to liquidate investment projects. If everyone else rolls over, an individual household who deviates and demands immediate repayment gets $D_A^1$.

Now assume that a positive mass $\lambda \in (0, 1]$ of households decides not to roll over. Banker $A$ needs to liquidate investment projects to generate cash. Short-term debt is served sequentially. If an individual household rolls over, he is repaid only if banker $A$ can fully serve all other households at $t = 1$. He receives:

$$\min \left\{ D_A^2, \max \left\{ \frac{1}{1 - \lambda} \left[ (1 - \tau_A)(V_A - \Phi) - \lambda D_A^1 \right], 0 \right\} \right\}.$$  \hspace{1cm} (24)

If $\lambda D_A^1 > (1 - \tau_A)(V_A - \Phi)$, banker $A$ cannot generate enough cash in $t = 1$ to fully serve all households which run. In this case, the payout of an individual household who runs depends on his position in line. Some households are first in line and are served fully. Others come late and receive nothing. On expectation, a household who runs receives:

$$\min \left\{ D_A^1, \frac{1}{\lambda} (1 - \tau_A)(V_A - \Phi) \right\}.$$  \hspace{1cm} (25)

We introduce the following indicator function:

$$X_D = \begin{cases} 1 & \text{if } \lambda D_A^1 \leq (1 - \tau_A)(V_A - \Phi), \\ 0 & \text{otherwise}, \end{cases}$$  \hspace{1cm} (26)

and we define: $q(\lambda) \equiv \Pr(X_D = 1 | Q)$. This is the probability which households in $t = 1$ assign to to event that $(1 - \tau_A)(V_A - \Phi)$ is high enough to fully service the short-term debt claims of a mass $\lambda$ of households.
Banker $A$ is willing to promise any amount $D^2_A$ to the holders of short-term debt to avoid a bank run. Assume an extreme case: $D^2_A = \infty$. In this case, dividends in $t = 2$ are always zero. Households who roll over become the residual claimants of the bank.

We can distinguish three cases. The first two cases are not mutually exclusive.

1. $(1 - \tau_A)\mathbb{E}[V_A | Q] \geq D^1_A$: If all other households roll over, roll-over by an individual household yields an expected payoff $(1 - \tau_A)\mathbb{E}[V_A | Q]$. If an individual household deviates and demands immediate repayment, he gets $D^1_A$. It follows that roll-over is a Nash equilibrium.

2. $q(1) = 0$: Conditional on households’ period 1 information set, the probability that $(1 - \tau_A)[V_A - \Phi]$ is higher than $D^1_A$ is zero. If all other households run, roll-over yields 0 with certainty. Running like everyone else yields $(1 - \tau_A)[V_A - \Phi]$. It follows that a bank run is a Nash equilibrium.

3. $(1 - \tau_A)\mathbb{E}[V_A | Q] < D^1_A$ and $q(1) > 0$: Roll-over is not an equilibrium. Some agents would deviate and demand early payment. Banker $A$ needs to liquidate some investment projects at cost $\Phi$. But neither a bank run is an equilibrium in pure strategies. Given $D^2_A = \infty$, some agents would deviate and bet on the positive chance that they become the residual claimants of the bank in $t = 2$.

The only Nash equilibrium is one in mixed strategies. An individual household must be indifferent between roll-over and running:

$$q(\lambda) \times \frac{1}{1 - \lambda} \left[(1 - \tau_A)\mathbb{E}[V_A - \Phi | X_D = 1; Q] - \lambda D^1_A\right] + \left[1 - q(\lambda)\right] \times 0$$

$$= q(\lambda) \times D^1_A + \left[1 - q(\lambda)\right] \times \frac{1}{\lambda} (1 - \tau_A)\mathbb{E}[V_A - \Phi | X = 0; Q].$$

Assume that $\lambda \to 0$. Nearly everyone rolls over. Roll-over yields $(1 - \tau_A)\mathbb{E}[V_A - \Phi | Q]$ in this case; running yields $D^1_A$. Since we have $(1 - \tau_A)\mathbb{E}[V_A | Q] < D^1_A$, it follows that $\lambda \to 0$ cannot hold in equilibrium because running is strictly preferred to roll-over.

Assume now the opposite case: $\lambda \to 1$. Nearly everyone runs. Since we have $q(1) > 0$, it follows that roll-over yields an expected payout of $\infty$; running yields a finite payout. It follows that $\lambda \to 1$ cannot hold in equilibrium because roll-over is strictly preferred to running.

The payoffs of roll-over and running are both continuous in $\lambda$. It follows from Nash’s Existence Theorem that at least one Nash equilibrium in mixed strategies exists. A strictly positive mass of households runs in this equilibrium.

**Proof of Lemma 4.2**

By changing $i_A$, banker $A$ affects the risk of a bank run only if $i_A$ is observable to households at $t = 1$. This happens with probability $\pi_A$. If (A0) holds, the marginal
The benefit of $i_A$ is given as:

$$
\frac{\partial \tau_A}{\partial i_A} \left[ \mathbb{E}[V_A] - \pi_A \text{Pr}_o(i_A) \Phi - (1 - \pi_A) \text{Pr}_u(\hat{i}_A) \Phi \right] = \begin{cases} 
-S + R \frac{\mu + a - i_A}{2a}, & \text{if } i_A \leq Sk_A - \frac{D_A}{1 - \tau_A} + \frac{R}{S}(\mu - a), \\
-S + R \frac{\mu + a - i_A}{2a} - \pi_A \frac{FS}{2aR}, & \text{otherwise}.
\end{cases}
$$

(28)

The certain return $S$ of the safe asset is given up in exchange for a return $R$ which happens with probability:

$$
\text{Pr}[i_A \leq \theta] = \frac{\mu + a - i_A}{2a}.
$$

(29)

Importantly, the probability that $i_A$ is smaller than $\theta$ is decreasing in $i_A$. For this reason, the expected payout of banker $A$ and the expected net value of her portfolio are strictly concave in $i_A$:

$$
\frac{\partial^2 \tau_A}{\partial i_A^2} \left[ \mathbb{E}[V_A] - \pi_A \text{Pr}_o(i_A) \Phi - (1 - \pi_A) \text{Pr}_u(\hat{i}_A) \Phi \right] = -\frac{R}{2a}.
$$

(30)

This means that the optimal portfolio choice generally has an interior solution.

If condition (A2) is satisfied, the marginal benefit of increasing $i_A$ above $\bar{i}_A$ is positive. This implies:

$$
i^*_A > \bar{i}_A > Sk_A - \frac{D_A}{1 - \tau_A} + \frac{R}{S}(\mu - a).
$$

(31)

The first order condition gives:

$$
i^*_A = \mu - a \left( \frac{2S}{R} - 1 \right) - \pi_A \frac{FS}{R^2}.
$$

(32)

**Proof of Lemma 4.3**

There are two possible situations for banker $A$’s portfolio choice: (1.) banker $A$ is the Leader, or (2.) banker $A$ is the Follower. If banker $A$ is the Leader, she knows $\theta$ if and only if she has screened successfully. In this case, she might still face a bank run if $i_A$ is not public information. If $i_A$ is public information and $i_A = \theta$, the risk of a bank run is zero.

If banker $A$ is the Follower, she knows $\theta$ if she has screened successfully or if she learns $\theta$ from the observation of $i_B$. The latter case occurs if and only if (1.) $i_B$ is public information, and (2.) banker $B$ has screened successfully. Whenever banker $A$ observes $i_B \neq i^*_B$, she knows that $i_B = \theta$. If $i_B$ is public information and $i_B \neq i^*_B$, households know that $i_A = i_B = \theta$ independently of whether $i_A$ is public information. In this case, the bank run risk is zero for both bankers.
Ex-ante, before screening takes place, the unconditional expected net value of banker A’s portfolio is equal to:

\[ E[V_A - 1_{\text{run}_A} \Phi] = \]
\[ \frac{1}{2} p \left[ E[V_A | i_A = \theta] - (1 - \pi_A) \Phi \Pr_u(i^*_A) \right] + \frac{1}{2} (1 - p) \left[ E[V_A | i_A = i^*_A] - \pi_A \Phi \Pr_o(i^*_A) - (1 - \pi_A) \Phi \Pr_u(i^*_A) \right] + \frac{1}{2} p \pi_B \left[ E[V_A | i_A = \theta] \right] + \frac{1}{2} (1 - p \pi_B) \left[ E[V_A | i_A = \theta] - (1 - \pi_A) \Phi \Pr_u(i^*_A) \right] + \frac{1}{2} (1 - p \pi_B)(1 - p) \left[ E[V_A | i_A = i^*_A] - \pi_A \Phi \Pr_o(i^*_A) - (1 - \pi_A) \Phi \Pr_u(i^*_A) \right]. \] (33)

The first two lines on the right hand side of the equation above give banker A’s payoff if she is the Leader. She might screen successfully (first line) or not (second line). Households have rational expectations. They understand banker A’s portfolio choice problem: \( \hat{i}_A = i^*_A \) and \( \Pr_u(\hat{i}_A) = \Pr_u(i^*_A) \). The third line gives her payoff if she is the Follower and learns \( \theta \) from the observation of \( i_B \). The fourth line is her payoff as Follower if she does not learn \( \theta \) from \( i_B \) but screens successfully. The fifth line is her payoff as Follower if she does not learn \( \theta \) before picking her portfolio.

Several elements of this expression are affected by a change in \( \pi_A \) through its impact on \( i^*_A \):

1. First I evaluate the effect of \( \pi_A \) on the gross value of banker A’s portfolio if she does not know \( \theta \).

\[ \frac{\partial E_{\theta} [V_A | i_A = i^*_A]}{\partial \pi_A} = \frac{\partial E_{\theta} [V_A | i_A = i^*_A]}{\partial i^*_A} \frac{\partial i^*_A}{\partial \pi_A} = \left[ -S + R \frac{\mu + a - i^*_A}{2a} \right] \left( -\frac{\Phi S}{R^2} \right). \] (34)

Applying Lemma 4.2, we derive:

\[ \frac{\partial E_{\theta} [V_A | i_A = i^*_A]}{\partial \pi_A} = \frac{\pi_A \Phi S}{2aR} \left( -\frac{\Phi S}{R^2} \right) = -\frac{\pi_A \Phi^2 S^2}{2aR^3}. \] (35)

The gross value of banker A’s portfolio is reduced as banker A’s choice \( i^*_A \) falls in \( \pi_A \).

2. I continue to consider the case that banker A does not know \( \theta \). Now I assess the
impact of $\pi_A$ on $Pr_o(i_A^*)$:

$$Pr_o(i_A^*) = \Pr\left\{ \theta < \frac{1}{R} \left[ Si_A^* - \left( Sk_A - \frac{D_A}{1 - \tau_A} \right) \right] \right\}$$

$$= \frac{Si_A^* - \left( Sk_A - \frac{D_A}{1 - \tau_A} \right)}{2aR} - \frac{\mu - a}{2a}.$$  \hspace{1cm} (36)

It follows that:

$$\frac{\partial Pr_o(i_A^*)}{\partial \pi_A} = \frac{S}{2aR} \left( - \frac{\Phi S}{R^2} \right) = - \Phi S^2 \frac{2a}{2aR^3}.$$  \hspace{1cm} (37)

If $i_A$ is publicly observed, the probability of a bank run is reduced as banker A’s choice $i_A^*$ falls in $\pi_A$.

3. If $i_A$ remains hidden, the probability of a bank run depends on $\hat{i}_A$: households’ expectation of banker A’s portfolio choice in case she does not know $\theta$. I assume that households have rational expectations. They understand banker A’s portfolio choice problem: $\hat{i}_A = i_A^*$ and $Pr_u(\hat{i}_A) = Pr_u(i_A^*)$.

$$Pr_u(i_A^*) = \Pr\left\{ \theta < \frac{1}{R - pS} \left[ (1 - p) Si_A^* - \left( Sk_A - \frac{D_A}{1 - \tau} \right) \right] \right\}$$

$$= \frac{(1 - p) Si_A^* - \left( Sk_A - \frac{D_A}{1 - \tau} \right)}{2a(R - pS)} - \frac{\mu - a}{2a}.$$  \hspace{1cm} (38)

It follows that:

$$\frac{\partial Pr_u(i_A^*)}{\partial \pi_A} = \frac{(1 - p) S}{2a(R - pS)} \left( - \frac{\Phi S}{R^2} \right) = - \frac{(1 - p) \Phi S^2}{2aR^2(R - pS)}.$$  \hspace{1cm} (39)

Even if $i_A$ is not publicly observed, the probability of a bank run is reduced as households correctly expect that banker A’s choice $i_A^*$ falls in $\pi_A$.

Applying these results, we find that an increase in $\pi_A$ at $t = 0$ affects the expected net value of banker A’s portfolio according to:

$$\frac{\partial E[V_A - 1_{\text{run}_A} \Phi]}{\partial \pi_A} = \left(1 - \frac{p\pi_B}{2}\right) \left(1 - p\right) \left( \frac{\partial E[V_A | i_A = i_A^*]}{\partial \pi_A} - \pi_A \Phi \frac{\partial Pr_o(i_A^*)}{\partial \pi_A} \right)$$

$$- (1 - \pi_A) \Phi \frac{\partial Pr_u(i_A^*)}{\partial \pi_A} - \Phi \left[ (1 - p) Pr_o(i_A^*) - Pr_u(i_A^*) \right]$$

$$= \left(1 - \frac{p\pi_B}{2}\right) \left(1 - p\right) \frac{1 - \pi_A}{R - pS} \frac{\Phi^2 S^2}{2aR^2} - \Phi \left[ (1 - p) Pr_o(i_A^*) - Pr_u(i_A^*) \right].$$  \hspace{1cm} (40)
There are diminishing returns to transparency:

$$
\frac{\partial^2 E[V_{\text{A}} - 1_{\text{run, A}} \Phi]}{\partial \pi_{\text{A}}^2} = - \left(1 - \frac{p\pi_{\text{B}}}{2}\right) \frac{(1 - p)\Phi^2 S^2}{2aR^3} \frac{R + pS}{R - pS}.
$$

(41)

**Proof of Lemma 4.4**

There exists a value $\hat{p} \in [0, 1)$ which is high enough such that the risk of a bank run is close to zero as long as $i_{\text{A}}$ remains hidden: $\Pr_u(i_{\text{A}}^*) = \epsilon$. Since $\hat{p} < 1$, for this value we have:

$$
(1 - \hat{p}) \Pr_o(i_{\text{A}}^*) - \Pr_u(i_{\text{A}}^*) > 0.
$$

(42)

Furthermore, we know that $(1 - p)\Pr_o(i_{\text{A}}^*) - \Pr_u(i_{\text{A}}^*) > 0$ if and only if the term in square brackets in equation (12) is strictly positive. This term is strictly increasing in $p$:

$$
\frac{\partial}{\partial p} \left[ \mu - a - \frac{(1 - p)S^2}{R(R - pS)} i_{\text{A}}^* + \frac{R + (1 - p)S}{R(R - pS)} \left( Sk_{\text{A}} - \frac{D_{\text{A}}^1}{1 - \tau_{\text{A}}} \right) \right] > 0.
$$

(43)

There are two possibilities:

1. $\bar{p} = 0$: If the term in square brackets in equation (12) is positive for all values of $p \in [0, 1]$, then:

$$
(1 - p) \Pr_o(i_{\text{A}}^*) - \Pr_u(i_{\text{A}}^*) > 0 \Leftrightarrow p > 0.
$$

(44)

2. $\bar{p} > 0$: If the term in square brackets in equation (12) is negative for some value $\hat{p} \in (0, 1)$, then there exists a value $\bar{p} \in (\hat{p}, \bar{p})$ such that:

$$
(1 - \bar{p}) \Pr_o(i_{\text{A}}^*) - \Pr_u(i_{\text{A}}^*) > 0 \Leftrightarrow p > \bar{p}.
$$

(45)

**Proof of Lemma 4.5**

Consider the expression for $E[V_{\text{A}} - 1_{\text{run, A}} \Phi]$ in the proof of Lemma 4.3. The first derivative of $E[V_{\text{A}} - 1_{\text{run, A}} \Phi]$ with respect to $\pi_{\text{B}}$ is:

$$
\frac{\partial E[V_{\text{A}} - 1_{\text{run, A}} \Phi]}{\partial \pi_{\text{B}}} = \frac{p}{2} \left[ (1 - p) \left( E[V_{\text{A}} | i_{\text{A}} = \theta] - E[V_{\text{A}} | i_{\text{A}} = i_{\text{A}}^*] \right) 
+ (1 - p) \pi_{\text{A}} \Pr_o(i_{\text{A}}^*) \Phi + (1 - \pi_{\text{A}}) \Pr_u(i_{\text{A}}^*) \Phi \right] > 0.
$$

(46)

Lemma 4.5 follows from symmetry.

**Proof of Proposition 4.6**

The proof proceeds in four steps.
1. \((1 - \tau^*_A)S_A^* - D^{1*}_A = (1 - \tau^*_B)S_B - D^{1*}_B = M\). I describe an equilibrium in which condition (A2) is satisfied. This means that \(\Pr_o(i_A^*)\) and \(\Pr_u(i_B^*)\) are strictly positive. Both the expected return offered to households \(r_A\) and banker \(A\)’s expected payoff \(\tau_A\mathbb{E}[V_A - 1_{\text{run}_A}\Phi]\) are strictly decreasing in \(D^{1*}_A\). It follows that banker \(A\) can benefit by reducing \(D^{1*}_A\) until the upper limit on the buffer between \((1 - \tau_A)S_A\) and \(D^{1*}_A\) is binding. By symmetry, the same holds for banker \(B\).

2. \(r_A = r_B\): Assume otherwise. For instance: \(r_A < r_B\). In this case, banker \(A\) has a payoff of zero. Banker \(B\) collects \(k_B = w\) and offers an expected return \(r_B\) to households:

\[
r_B = (1 - \tau_B)\frac{\mathbb{E}[S(w - i_B) + R \min\{i_B, \theta\} - 1_{\text{run}_B}\Phi]}{w}.
\]  

(47)

The two bankers’ problems are perfectly symmetric. By choosing \(\pi_A = \pi_B, \tau_A = \tau_B,\) and \(k_A = w/2\), banker \(A\) can offer an expected return:

\[
r_A = (1 - \tau_A)\frac{\mathbb{E}[S\left(\frac{w}{2} - i_A\right) + R \min\{i_A, \theta\} - 1_{\text{run}_A}\Phi]}{w}.
\]  

(48)

Using expressions from the proof of Lemma 4.3, we find that the risk of a bank run for banker \(A\) depends on the exogenous upper bound for the buffer between \((1 - \tau_A)S_A\) and \(D^{1*}_A\):

\[
\Pr_o(i_A^*) = \frac{S_A^* - \frac{M}{1 - \tau_A} - \frac{\mu - a}{2a}}{2aR}, \quad \Pr_u(i_A^*) = \frac{(1 - p)S_A^* - \frac{M}{1 - \tau_A} - \frac{\mu - a}{2a}}{2a(R - pS)}.
\]  

(49)

In equilibrium, both \(\Pr_o(i_A^*)\) and \(\Pr_u(i_A^*)\) are independent of the size of banker \(A\)’s portfolio \(k_A\) (as long as (A2) holds). Furthermore, having a portfolio size of \(w/2\) instead of \(w\) does not affect banker \(A\)’s portfolio choice because the support of \(\theta\) is bounded: \(\mu + a < w/2\).

It follows that the risk of a bank run for banker \(A\) is not affected by having a smaller portfolio than banker \(B\). Neither is her portfolio choice. We have:

\[
\mathbb{E}\left[-S_A + R \min\{i_A, \theta\} - 1_{\text{run}_A}\Phi\right] = \mathbb{E}\left[-S_B + R \min\{i_B, \theta\} - 1_{\text{run}_B}\Phi\right].
\]  

(50)

It follows that \(r_A\) is strictly higher than \(r_B\) if and only if:

\[
\mathbb{E}\left[S\left(\frac{w}{2} - i_A\right) + R \min\{i_A, \theta\} - 1_{\text{run}_A}\Phi\right] > \mathbb{E}\left[S(w - i_B) + R \min\{i_B, \theta\} - 1_{\text{run}_B}\Phi\right]
\]

\[
\iff \mathbb{E}\left[-S_A + R \min\{i_A, \theta\} - 1_{\text{run}_A}\Phi\right] > 0.
\]  

(51)

We know that banker \(A\) and banker \(B\) choose \(i_A\) and \(i_B\) in an attempt to maximize the expected net value of their portfolios. If there was not a credibility problem, this alone would imply that the inequality above always holds. But we know
that $i_A$ and $i_B$ are higher than their optimal value if there is less than perfect transparency ($\pi_A = \pi_B < 1$). Consider an extreme case: $i_A = \mu + a$. Even in this case, the inequality above holds since I have assumed that the fixed cost of early liquidation $\Phi$ is not too high relative to the return from the risky project $R$:

$$-S(\mu + a) + R\mu - \Phi > 0. \quad (52)$$

Even if $i_A = \mu + a$, the expected net return of banker $A$’s portfolio is still larger than $S$. It follows that banker $A$ can always attract a positive amount of capital and earn a positive expected payoff. Since banker $A$ prefers this to a payoff of zero, it must be true in equilibrium that $r_A = r_B$.

3. From $r_A = r_B$, it follows:

$$\frac{(1 - \tau_A)E[V_A - 1_{\text{run}_A} \Phi]}{k_A} = \frac{(1 - \tau_B)E[V_B - 1_{\text{run}_B} \Phi]}{k_B}. \quad (53)$$

Together with $k_A + k_B = w$, this implies:

$$\tau_A E[V_A - 1_{\text{run}_A} \Phi] = E[V_A - 1_{\text{run}_A} \Phi] - \frac{k_A}{w - k_A} (1 - \tau_B) E[V_B - 1_{\text{run}_B} \Phi]. \quad (54)$$

Holding fixed $\pi_A$, the first derivative of banker $A$’s objective with respect to $\tau_A$ is given as:

$$\frac{\partial \tau_A E[V_A - 1_{\text{run}_A} \Phi]}{\partial \tau_A} = \frac{\partial k_A}{\partial \tau_A} \left( \frac{\partial E[V_A - 1_{\text{run}_A} \Phi]}{\partial k_A} \right) - (1 - \tau_B) \left( \frac{w}{(w - k_A)^2} E[V_B - 1_{\text{run}_B} \Phi] + \frac{k_A}{w - k_A} \frac{\partial E[V_B - 1_{\text{run}_B} \Phi]}{\partial k_A} \right). \quad (55)$$

Since $\mu + a < w/2$, we have:

$$\frac{\partial E[V_A - 1_{\text{run}_A} \Phi]}{\partial k_A} = S, \quad \text{and:} \quad \frac{\partial E[V_B - 1_{\text{run}_B} \Phi]}{\partial k_A} = -S. \quad (56)$$

Furthermore, we derive from $r_A = r_B$:

$$\frac{\partial k_A}{\partial \tau_A} = \frac{E[V_A - 1_{\text{run}_A} \Phi]}{(1 - \tau_A)S - (1 - \tau_B) \left( \frac{w}{w - k_A} S E[V_B - 1_{\text{run}_B} \Phi] - \frac{k_A}{w - k_A} S \right)} \neq 0. \quad (57)$$

It follows that a first order condition for an optimal choice of $\tau_A$ is:

$$S - (1 - \tau_B) \left( \frac{w}{(w - k_A)^2} E[V_B - 1_{\text{run}_B} \Phi] - \frac{k_A}{w - k_A} S \right) = 0. \quad (58)$$

Evaluated at a value $\tau_A^*$ which satisfies this first order condition, the second deriva-
tive of banker A’s objective with respect to $\tau_A$ is:

$$\frac{\partial^2 \tau_A E [V_A - \mathbb{1}_{\text{run}_A} \Phi]}{\partial \tau_A^2} = -2(1 - \tau_B) \frac{w}{(w - k_A)^2} \left( \frac{\partial k_A}{\partial \tau_A} \right)^2 \frac{E [V_B - \mathbb{1}_{\text{run}_B} \Phi] - S(w - k_A)}{w - k_A} < 0. \quad (59)$$

Here we use again the fact that the expected net return of a banker’s portfolio is always higher than $S$: $E [V_B - \mathbb{1}_{\text{run}_B} \Phi] > S(w - k_A)$. It follows that banker A’s objective is strictly concave in $\tau_A$. There is a unique optimal choice $\tau_A^*$ (for given values of $\pi_A$, $\pi_B$, and $\tau_B$).

4. Given the optimal choice $\tau_A^*$, the first derivative of banker A’s objective with respect to $\pi_A$ is given as:

$$\frac{\partial \tau_A E [V_A - \mathbb{1}_{\text{run}_A} \Phi]}{\partial \pi_A} = \frac{\partial E [V_A - \mathbb{1}_{\text{run}_A} \Phi]}{\partial \pi_A} - \frac{k_A}{k_B} (1 - \tau_B) \frac{\partial E [V_B - \mathbb{1}_{\text{run}_B} \Phi]}{\partial \pi_A}. \quad (60)$$

In response to changes in $\pi_A$, $k_A$ and $k_B$ ($= w - k_A$) respond to maintain the equality between $r_A$ and $r_B$. But from the optimal choice of $\tau_A^*$, it follows from the envelope theorem that the marginal effect of a change in $k_A$ on banker A’s objective is zero.

The second derivative with respect to $\pi_A$ is:

$$\frac{\partial^2 \tau_A E [V_A - \mathbb{1}_{\text{run}_A} \Phi]}{\partial \pi_A^2} = \frac{\partial^2 E [V_A - \mathbb{1}_{\text{run}_A} \Phi]}{\partial \pi_A^2} - \frac{k_A}{k_B} (1 - \tau_B) \frac{\partial^2 E [V_B - \mathbb{1}_{\text{run}_B} \Phi]}{\partial \pi_A^2}. \quad (61)$$

We know from Lemma 4.3 and Lemma 4.5:

$$\frac{\partial^2 E [V_A - \mathbb{1}_{\text{run}_A} \Phi]}{\partial \pi_A^2} < 0 \quad \text{and} \quad \frac{\partial^2 E [V_B - \mathbb{1}_{\text{run}_B} \Phi]}{\partial \pi_A^2} = 0. \quad (62)$$

It follows that banker A’s objective is strictly concave in $\pi_A$. There is a unique solution $\pi_A^*$ (given $\tau_A^*$, $\tau_B$, and $\pi_B$).

5. A symmetric equilibrium exists: This part of the proof consists of two steps.

   a) Transparency: Banker A’s first order condition for transparency implicitly defines a unique optimal choice $\pi_A^* \in [0, 1]$ for any given value $\pi_B \in [0, 1]$. This choice function $\pi_A^* = f(\pi_B)$ is continuous. By Brouwer’s fixed-point theorem, there must be at least one value $x \in [0, 1]$ such that: $f(x) = x$. Now assume that $\tau_A^* = \tau_B^*$. Since the two bankers’ problems are completely symmetric, $\pi_A^* = \pi_B^* = x$.

   b) Price: Banker A’s first order condition for $\tau_A$ implicitly defines a unique optimal choice $\tau_A^* \in [0, 1]$ for any given value $\tau_B \in [0, 1]$. This choice function
\[ \tau_A^* = g(\tau_B) \] is continuous. By Brouwer’s fixed-point theorem, there must be at least one value \( y \in [0,1] \) such that: \( g(y) = y \). If \( \pi_A = \pi_B \), the two bankers’ problems are completely symmetric. In this case, \( \tau_A^* = \tau_B^* = y \).

It follows that there exists a symmetric subgame perfect Nash equilibrium with \( \pi_A^* = \pi_B^* \) and \( \tau_A^* = \tau_B^* \).

6. From \((1 - \tau_A)Sk_A^* - D^*_A = (1 - \tau_B)Sk_B^* - D^*_B = M\), \( \pi_A^* = \pi_B^* \), \( \tau_A^* = \tau_B^* \), and \( r_A = r_B \), it follows: \( k_A^* = k_B^* = w/2 \). Applying this to the first order condition for \( \tau_A \) in (58), we derive:

\[
\tau_A^* = \frac{\mathbb{E}[V_A - 1_{\text{run}_A}] - S}{\mathbb{E}[V_A - 1_{\text{run}_A}] - S}. \tag{63}
\]

**Proof of Proposition 4.7**

The proof starts out along the lines of the proof of Proposition 4.6.

1. Each banker \( j \in \{1,2,...N\} \) chooses \((1 - \tau_j^*)Sk_j^* - D^*_j = M\). Just as in the case of two bankers, banker \( j \) can always benefit by reducing \( D^*_j \) until the upper limit on the buffer between \((1 - \tau_j)Sk_j \) and \( D^*_j \) is binding.

2. \( r_j = r_l \) for any \( j,l \in \{1,2,...N\} \): Assume otherwise: \( r_j < r_l \) for some \( j,l \in \{1,2,...N\} \). In this case, banker \( j \) has a payoff of zero. Assume that banker \( l \) collects \( k_l = w/(N - 1) \). The proof holds a fortiori if there is a banker who offers \( r_l \) and has a bigger portfolio then \( w/(N - 1) \).

Banker \( l \) offers an expected return \( r_l \) to households:

\[
r_l = (1 - \tau_l) \frac{\mathbb{E}[S \left( \frac{w}{N - 1} - i_l \right) + R \min\{i_l, \theta\} - 1_{\text{run}_l}]}{\frac{w}{N - 1}}. \tag{64}
\]

By choosing \( \pi_j = \pi_l \), \( \tau_j = \tau_l \), and \( k_j = w/N \), banker \( j \) can offer an expected return:

\[
r_j = (1 - \tau_j) \frac{\mathbb{E}[S \left( \frac{w}{N} - i_j \right) + R \min\{i_j, \theta\} - 1_{\text{run}_j}]}{\frac{w}{N}}. \tag{65}
\]

For \( \pi_j = \pi_l \), \( \tau_j = \tau_l \), \( k_j = w/N \), and \( k_l = w/(N - 1) \), we have:

\[
\mathbb{E}[-Si_j + R \min\{i_j, \theta\} - 1_{\text{run}_j} \Phi] = \mathbb{E}[-Si_l + R \min\{i_l, \theta\} - 1_{\text{run}_l} \Phi]. \tag{66}
\]

It follows that \( r_j \) is strictly higher than \( r_l \) if and only if:

\[
\frac{\mathbb{E}[S \left( \frac{w}{N} - i_j \right) + R \min\{i_j, \theta\} - 1_{\text{run}_j} \Phi]}{\frac{w}{N}} > \frac{\mathbb{E}[S \left( \frac{w}{N - 1} - i_l \right) + R \min\{i_l, \theta\} - 1_{\text{run}_l} \Phi]}{\frac{w}{N - 1}} \iff \mathbb{E}[-Si_j + R \min\{i_j, \theta\} - 1_{\text{run}_j} \Phi] > 0. \tag{67}
\]
We know: \(-S(\mu + a) + R\mu - \Phi > 0\). The expected net return of banker \(j\)’s portfolio is always higher than \(S\). Banker \(j\) can always attract a positive amount of capital and earn a strictly positive expected payoff. Since banker \(j\) prefers this to a payoff of zero, it must be true in equilibrium that \(r_j = r_l\) for any \(j, l \in \{1, 2, ..., N\}\).

3. From \(r_j = r_l\), it follows:

\[
\frac{(1 - \tau_j)E[V_j - 1_{\text{run}_j}\Phi]}{k_j} = \frac{(1 - \tau)E[V_l - 1_{\text{run}_l}\Phi]}{k_l}.
\] (68)

In a symmetric equilibrium, this gives:

\[
\tau E[V_j - 1_{\text{run}_j}\Phi] = E[V_l - 1_{\text{run}_l}\Phi] - \frac{k_j}{w - k_j} (1 - \tau) \frac{1}{N-1} E[V_l - 1_{\text{run}_l}\Phi].
\] (69)

Holding fixed \(\pi_j\), the first derivative of banker \(j\)’s objective with respect to \(\tau_j\) is given as:

\[
\frac{\partial \tau_j E[V_j - 1_{\text{run}_j}\Phi]}{\partial \tau_j} = \frac{\partial k_j}{\partial \tau_j} \left[ \frac{\partial E[V_j - 1_{\text{run}_j}\Phi]}{\partial k_j} \right] - (1 - \tau_j) \left( \frac{w}{N-1} \frac{E[V_l - 1_{\text{run}_l}\Phi]}{w - k_j} \frac{k_j}{N-1} \frac{\partial E[V_l - 1_{\text{run}_l}\Phi]}{\partial k_j} \right).
\] (70)

Since \(\mu + a < w/N\), we have:

\[
\frac{\partial E[V_j - 1_{\text{run}_j}\Phi]}{\partial k_j} = S, \text{ and: } \frac{\partial E[V_l - 1_{\text{run}_l}\Phi]}{\partial k_j} = -\frac{S}{N-1}.
\] (71)

Proceeding as in the case of two bankers, we derive the following first order condition for \(\tau_j\):

\[
S - (1 - \tau_l) \left( (N - 1) \frac{w}{(w - k_j)} E[V_l - 1_{\text{run}_l}\Phi] - \frac{k_j}{w - k_j} S \right) = 0.
\] (72)

4. Given the optimal choice \(\tau_j^*\), the first derivative of banker \(j\)’s objective with respect to \(\pi_j\) is given as:

\[
\frac{\partial \tau_j E[V_j - 1_{\text{run}_j}\Phi]}{\partial \pi_j} = \frac{\partial E[V_j - 1_{\text{run}_j}\Phi]}{\partial \pi_j} - \frac{k_j}{k_l} (1 - \tau) \frac{\partial E[V_l - 1_{\text{run}_l}\Phi]}{\partial \pi_j}.
\] (73)
The second derivative with respect to \( \pi_j \) is:

\[
\frac{\partial^2 \tau_j \mathbb{E} [V_j - \mathbbm{1}_{\text{run}_j} \Phi]}{\partial \pi_j^2} = \frac{\partial^2 \mathbb{E} [V_j - \mathbbm{1}_{\text{run}_j} \Phi]}{\partial \pi_j^2} - \frac{k_j}{k_i} (1 - \tau_i) \frac{\partial^2 \mathbb{E} [V_i - \mathbbm{1}_{\text{run}_i} \Phi]}{\partial \pi_i^2}.
\] (74)

Let \( MD \) and \( HE \) denote market discipline and the Hirshleifer effect, respectively:

\[
MD + HE \equiv (1 - p) \frac{1 - \pi_j \Phi^2 S^2}{R - p S^2 a R^2} - \Phi \left[ (1 - p) \Pr_o(i_j^*) - \Pr_u(i_j^*) \right].
\] (75)

Then we can write the first derivative of the expected net value of banker \( j \)'s portfolio for the case of \( N \) bankers as:

\[
\frac{\partial \mathbb{E} [V_j - \mathbbm{1}_{\text{run}_j} \Phi]}{\partial \pi_j} = \frac{1}{2} [MD + HE] + \frac{1}{2} F(N) [MD + HE],
\] (76)

where \( F(N) \) is the probability that \( \theta \) is not revealed by any of the Leaders. If there are \( N/2 \) Leaders with a symmetric level of transparency \( \pi \), \( F(N) = (1 - p \pi)^{N/2} \).

With probability 1/2, banker \( j \) is a Leader and \( MD + HE \) matters. But with probability 1/2, banker \( j \) is a Follower. In this case, \( MD + HE \) only matters as long as \( \theta \) has not been revealed by any of the Leaders.

Let \( IX \) denote information spillovers:

\[
IX \equiv p \left[ (1 - p) \left( \mathbb{E} [V_i | i_i = \theta] - \mathbb{E} [V_i | i_i = i_j^*] \right) + (1 - \pi_l) \Pr_o(i_j^*) \Phi + (1 - \pi_l) \Pr_u(i_j^*) \Phi \right].
\] (77)

Then we can write the first derivative of the expected net value of banker \( l \)'s portfolio (\( l \neq j \)) as:

\[
\frac{\partial \mathbb{E} [V_l - \mathbbm{1}_{\text{run}_l} \Phi]}{\partial \pi_j} = \frac{1}{2} \frac{1}{N - 1} \tilde{F}(N) (1 - \tau_i) [IX].
\] (78)

With probability 1/2, banker \( j \) is a Leader. In order to have information spillovers, banker \( l \) must be a Follower. Conditional on banker \( j \) being a Leader, this happens with probability 1/2 \( \times N/(N - 1) \). \( \tilde{F}(N) \) is the probability that \( \theta \) is not revealed by any of the Leaders besides banker \( j \). If there are \( N/2 \) Leaders with a symmetric level of transparency \( \pi \), \( \tilde{F}(N) = (1 - p \pi)^{N/2 - 1} \).

5. Applying the same reasoning as in the case of two bankers, it can be shown that a symmetric subgame perfect Nash equilibrium exists with \( \pi_j^* = \pi_l^* \) and \( \tau_j^* = \tau_l^* \). From \( (1 - \tau_j^*)Sk_j^* - D_j^{1*} = (1 - \tau_l^*)Sk_l^* - D_l^{1*} = M \), \( \pi_j^* = \pi_l^* \), \( \tau_j^* = \tau_l^* \), and \( r_j = r_l \).
it follows: \( k_j^* = k_l^* = w/N \). It follows from equation (72):

\[
\tau_j^* = \frac{\mathbb{E}[V_{j-1 \text{ run}_j} \Phi]}{\frac{w}{N}} - S. \tag{79}
\]

6. 1-\( \tau_j^* \) is falling in \( N \): Consider the share of banker \( j \)'s net portfolio value which is paid out to households:

\[
1 - \tau_j^* = 1 - \frac{\mathbb{E}[V_{j-1 \text{ run}_j} \Phi]}{\frac{w}{N}} - S = \frac{S - \frac{S}{N}}{\frac{N}{w} \mathbb{E}[V_j - \mathbb{I}_{\text{ run}_j} \Phi] - \frac{S}{N}}. \tag{80}
\]

I study how 1-\( \tau_j^* \) responds to an increase in \( N \). For the sake of the argument, I assume for a moment that \( N \) is a continuous variable:

\[
\frac{\partial (1 - \tau_j^*)}{\partial N} = \frac{S}{(\frac{N}{w} \mathbb{E}[V_j - \mathbb{I}_{\text{ run}_j} \Phi] - \frac{S}{N})^2} \left[ \frac{1}{N^2} \left( \frac{N}{w} \mathbb{E}[V_j - \mathbb{I}_{\text{ run}_j} \Phi] - \frac{S}{N} \right) - \frac{N - 1}{N} \left( \frac{\mathbb{E}[V_j - \mathbb{I}_{\text{ run}_j} \Phi]}{w} - \frac{S}{N} + \frac{S}{N^2} \right) \right]. \tag{81}
\]

Here I have used that:

\[
\frac{\partial \mathbb{E}[V_j - \mathbb{I}_{\text{ run}_j} \Phi]}{\partial N} = \frac{\partial k_j^*}{\partial N} \frac{w}{N} S = \frac{\partial w}{\partial N} \frac{S}{N} = -\frac{w}{N^2} S. \tag{82}
\]

It follows that 1-\( \tau_j^* \) is falling in \( N \) if and only if:

\[
\left( \frac{\mathbb{E}[V_j - \mathbb{I}_{\text{ run}_j} \Phi]}{w} - \frac{S}{N^2} \right) - (N - 1) \left( \frac{\mathbb{E}[V_j - \mathbb{I}_{\text{ run}_j} \Phi]}{w} - \frac{S}{N} + \frac{S}{N^2} \right) < 0. \tag{83}
\]

This is the case if and only if:

\[
(N - 2) \left( \frac{S}{N} - \frac{\mathbb{E}[V_j - \mathbb{I}_{\text{ run}_j} \Phi]}{w} \right) < 0. \tag{84}
\]

This is true in a symmetric equilibrium since \( N > 2 \) and:

\[
\frac{S}{N} - \frac{\mathbb{E}[V_j - \mathbb{I}_{\text{ run}_j} \Phi]}{w} < 0 \iff S < \frac{\mathbb{E}[V_j - \mathbb{I}_{\text{ run}_j} \Phi]}{w}. \tag{85}
\]

This is true because the expected return on banker \( j \)'s portfolio is always higher than \( S \).
$1 - \tau^*_j$ is falling in $N$ because the equilibrium price $\tau^*_j$ is increasing in $N$. As $N$ grows, bank $j$’s portfolio shrinks. This increases the average return of bank $j$’s portfolio. But it also means that bank $j$’s average return becomes more sensitive to a given change in $k_j$.

Now consider banker $j$’s choice of $\tau_j$. Given some market return $r_l$, banker $j$ has to decrease $\tau_j$ in order to increase $k_j$. If the return of her portfolio is more sensitive to changes in $k_j$, she has to lower $\tau_j$ by more for a given increase in $k_j$. Increasing the market share becomes more expensive for the banker. This is why bankers compete less aggressively as the number of bankers grows.

7. The equilibrium level of transparency is increasing in $N$: Consider banker $j$’s first order condition for transparency:

$$\frac{\partial \tau \mathbb{E}[V_j - 1_{\text{run}_j} \Phi]}{\partial \pi^*_j} = \left( \frac{1}{2} + \frac{1}{2} F(N) \right) [MD + HE] - \frac{1}{4N-1} \tilde{F}(N)(1 - \tau^*_l) [IX] = 0. \quad (86)$$

$1 - \tau^*_l$ is strictly decreasing in $N$. This increases the marginal benefit of $\pi_j$. It is therefore sufficient to show that the marginal benefit of $\pi_j$ is also increasing in $N$ for some constant value $(1 - \tau_l)$.

For any given value $\pi_j$, we can subtract the left-hand-side of banker $j$’s first order condition for the case of $N + 1$ bankers from the one for the case of $N$ bankers:

$$\frac{1}{2} [F(N + 1) - F(N)][MD + HE] - \frac{1}{4} \left[ \frac{N + 1}{N} \tilde{F}(N + 1) - \frac{N}{N - 1} \tilde{F}(N) \right] (1 - \tau_l) [IX]. \quad (87)$$

There are two cases. With probability $1/2$, the number of Leaders remains unchanged after $N$ is increased to $N + 1$. With probability $1/2$, there is one more Leader.

a) The number of Leaders remains unchanged: $F(N + 1) = F(N)$ and $\tilde{F}(N + 1) = \tilde{F}(N)$. The difference above becomes:

$$- \frac{1}{4} \tilde{F}(N + 1) \left[ - \frac{1}{N(N - 1)} \right] (1 - \tau_l) [IX] > 0, \quad \text{for } N \geq 2. \quad (88)$$

The banker’s first order condition is falling in $\pi$. It follows that the equilibrium level of transparency must increase in $N$.

b) There is one more Leader: $F(N + 1) = F(N)(1 - p\pi)$ and $\tilde{F}(N + 1) = \tilde{F}(N)(1 - p\pi)$.

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\( \tilde{F}(N)(1 - p\pi) \). The difference above becomes:

\[
- \frac{p\pi}{2} F(N)[MD + HE] - \frac{1}{4} \tilde{F}(N) \left[ \frac{N+1}{N} (1 - p\pi) - \frac{N}{N-1} \right] (1 - \tau_l) [IX] \\
= - \frac{p\pi}{2} F(N)[MD + HE] - \frac{1}{4} \tilde{F}(N) \left[ -\frac{1}{N(N-1)} \right] (1 - \tau_l) [IX] \\
+ \frac{p\pi}{4} \tilde{F}(N) \frac{N+1}{N} (1 - \tau_l) [IX]. \quad (89)
\]

This difference is positive if and only if:

\[
- \frac{1}{2} F(N)[MD + HE] - \frac{1}{4p\pi} \tilde{F}(N) \left[ -\frac{1}{N(N-1)} \right] (1 - \tau_l) [IX] \\
+ \frac{1}{4} \tilde{F}(N) \frac{N+1}{N} (1 - \tau_l) [IX] \geq 0. \quad (90)
\]

Now consider the value \( \pi^* \) which satisfies each individual banker’s first order condition for the case of \( N \) bankers:

\[
- \frac{1}{2} F(N)[MD + HE] = - \frac{1}{4} \frac{N}{N-1} \tilde{F}(N)(1 - \tau_l) [IX] + \frac{1}{2}[MD + HE]. \quad (91)
\]

Evaluated at \( \pi^* \), the difference from above becomes:

\[
\frac{1}{2} [MD + HE] + \left( \frac{1}{p\pi^*} - 1 \right) \frac{1}{4} \frac{F(N)}{N(N-1)} (1 - \tau_l) [IX]. \quad (92)
\]

We know that in equilibrium, \( [MD + HE] \) is always positive. It follows that, for \( N \geq 2 \), the equilibrium level of transparency must increase in \( N \).

8. For each given bank, the probability of a bank run is falling in \( N \): The risk of a run on bank \( j \) is given as:

\[
\left( \frac{1}{2} + \frac{1}{2} F(N) \right) \left[ \pi_j (1 - p) \text{Pr}_o(i^*_j) + (1 - \pi_j) \text{Pr}_u(i^*_j) \right]. \quad (93)
\]

This risk is affected by an increase of \( N \) in three ways:

a) \( N \) increases. This lowers \( F(N) \) and contributes to a decrease in the risk of a bank run on bank \( j \).

b) All other bankers increase \( \pi \). This lowers \( F(N) \) and contributes to a decrease in the risk of a bank run on bank \( j \).

c) Bank \( j \) increases \( \pi_j \). A marginal increase in \( \pi_j \) affects the risk of a run on
bank \( j \) according to:

\[
\left( \frac{1}{2} + \frac{1}{2} F(N) \right) \frac{1}{\Phi} \left( -\pi_j(1-p) \frac{\Phi^2 S^2}{2 \alpha R^3} - [MD + HE] \right). 
\]

(94)

If the banker's first order condition holds, \([MD + HE]\) is strictly positive. Furthermore, \([MD + HE]\) is strictly positive for every value \( \pi_j < \pi^* \). It follows that the derivative above with respect to \( \pi_j \) is strictly negative for all \( \pi_j \leq \pi^* \). Consider now the equilibrium level \( \pi^* \) for the case of \( N+1 \) bankers. The fact that banker \( j \) has increased \( \pi_j \) with respect to the case of \( N \) bankers has lowered the risk of a bank run.

**Proof of Proposition 5.1**

The expected net value of banker \( A \)'s portfolio is strictly concave in \( \pi_A \). The expected net value of banker \( B \)'s portfolio is linear in \( \pi_A \). It follows that the sum \( \mathbb{E}\{V_A - 1_{\text{run}_A} \Phi + V_B - 1_{\text{run}_B} \Phi \} \) has a unique maximum in \( \pi_A \) (for a given value of \( \pi_B \)). Since the two bankers’ problems are perfectly symmetric, the constrained-efficient choice of \( \pi_A \) and \( \pi_B \) must be symmetric.

It remains to be shown that \( \pi^*_A < \pi^*_B \). By comparing banker \( A \)'s first order condition for transparency from Proposition 4.6 with the social planner’s first order condition, it is clear that there is one symmetric equilibrium with \( \pi^*_A < \pi^*_B \). Can there be another symmetric equilibrium with \( \pi^{**}_A > \pi^{**}_A \)?

We know that banker \( A \)'s marginal benefit of transparency is negative for \( \pi_A = \pi^*_A \) and \( \pi_B = \pi^*_B \). A necessary condition for a symmetric equilibrium with \( \pi^{**}_A > \pi^{**}_A \) is that banker \( A \)'s marginal benefit of transparency grows as \( \pi_B \) is increased from \( \pi^*_B \) to \( \pi^{**}_B \). Banker \( A \)'s marginal benefit of transparency falls in \( \pi_B \) through the term \((1 - p \pi_B)/2\). It follows that banker \( A \)'s marginal benefit of transparency can increase in \( \pi_B \) only if the cost of information spillovers is smaller for \( \pi^{**}_A \) and \( \pi^{**}_B \) than for \( \pi^*_A \) and \( \pi^*_B \). Consider now the marginal cost of information spillovers:

\[
(1 - \tau_B^*) \left[ \left( 1 - p \right) \left[ \mathbb{E}[V_B \mid i_B = \theta] - \mathbb{E}[V_B \mid i_B = i^*_B] \right] 
+ \left( 1 - p \right) \pi_B \Pr \left( i_B = i^*_B \right) \Phi + \left( 1 - \pi_B \right) \Pr \left( i_B = i^*_B \right) \Phi \right]. \tag{95}
\]

This term can be smaller for \( \pi^{**}_A \) and \( \pi^{**}_B \) than for \( \pi^*_A \) and \( \pi^*_B \) only if at least one of the following cases applies:

1. \( (1 - \tau_B^*) \) is lower. This implies that \( \tau_B^* \) is larger. This is the case only if \( \mathbb{E}[V_A - 1_{\text{run}_A} \Phi] \) and \( \mathbb{E}[V_B - 1_{\text{run}_B} \Phi] \) are larger for \( \pi^{**}_A \) and \( \pi^{**}_B \) than for \( \pi^*_A \) and \( \pi^*_B \). But this cannot be true.

2. The term in square brackets is smaller. This is the case if and only if the following...
term is larger:

\[(1 - p) \mathbb{E} [V_B | i_B = i_B^*] - (1 - p) \pi_B \Pr_o(i_B^*) \Phi - (1 - \pi_B) \Pr_u(i_B^*) \Phi. \quad (96)\]

This is true if and only if the following term is larger for \(\pi_{A}^{**}\) and \(\pi_B^{**}\) than for \(\pi_A^P\) and \(\pi_B^P\):

\[p \mathbb{E} [V_B | i_B = \theta] + (1 - p) \mathbb{E} [V_B | i_B = i_B^*]
- (1 - p) \pi_B \Pr_o(i_B^*) \Phi - (1 - \pi_B) \Pr_u(i_B^*) \Phi. \quad (97)\]

This term is the expected value of banker B’s portfolio conditional on being a Leader. This value is strictly concave in \(\pi_B\). It is maximized for some value \(\tilde{\pi}_B < \pi_B^P < \pi_B^{**}\).

It follows that there is no symmetric equilibrium with \(\pi_A^{**} > \pi_A^P\).

**Proof of Proposition 5.2**

Consider the expected number of bank runs \(E\). This variable depends on \(\pi_A\) and \(\pi_B\):

\[E(\pi_A, \pi_B) \in [0, 2].\]

\[
E(\pi_A, \pi_B) = \frac{1}{2} \left[ (1 - p) \pi_A \Pr_o(i_A^*) + (1 - \pi_A) \Pr_u(i_A^*) 
+ (1 - p) \pi_B \Pr_o(i_B^*) + (1 - \pi_B) \Pr_u(i_B^*) 
+ \frac{1}{2} \left[ (1 - p) \pi_B \Pr_o(i_B^*) + (1 - \pi_B) \Pr_u(i_B^*) 
+ (1 - p) \pi_A \Pr_o(i_A^*) + (1 - \pi_A) \Pr_u(i_A^*) \right] \right]. \quad (98)
\]

The term in square brackets in the first two lines of the equation above gives the expected number of bank runs in case banker A happens to be the Leader and banker B is the Follower. The term in square brackets in the third and fourth line gives the expected number of bank runs in case banker B is the Leader. This expression is perfectly symmetric in \(\pi_A\) and \(\pi_B\).

\[
\frac{\partial E(\pi_A, \pi_B)}{\partial \pi_A} = \left(1 - \frac{p \pi_B}{2}\right) \left[(1 - p) \Pr_o(i_A^*) - \Pr_u(i_A^*) \right] 
- (1 - p) \frac{(1 - \pi_A) \Phi S^2}{2 a R^2 (R - p S)} 
- (1 - p) \frac{\pi_A \Phi S^2}{2 a R^3} 
- \frac{1}{2} p \left[(1 - p) \pi_B \Pr_o(i_B^*) + (1 - \pi_B) \Pr_u(i_B^*) \right]. \quad (99)
\]
The expected number of bank runs is strictly convex in $\pi_A$:

$$\frac{\partial^2 E(\pi_A, \pi_B)}{\partial \pi_A^2} = \left(1 - \frac{p\pi_B}{2}\right) \frac{1 - p}{R - pS} \frac{p \Phi S^3}{aR^3}. \quad (100)$$

The unique value $\pi_A^R$ which minimizes $E(\pi_A, \pi_B)$ satisfies the following first order condition:

$$\left(1 - \frac{p\pi_B}{2}\right) \left[-(1 - p) Pr_u(i_A^*) \Phi + Pr_u(i_A^*) \Phi + (1 - p) \frac{(1 - \pi_A) \Phi^2 S^2}{2aR^2(R - pS)} + (1 - p) \frac{\pi_A \Phi^2 S^2}{2aR^3}\right]
+ \frac{1}{2} p [(1 - p) \pi_B Pr_u(i_B^*) \Phi + (1 - \pi_B) Pr_u(i_B^*) \Phi] = 0. \quad (101)$$

A comparison with banker A’s first order condition from Proposition 4.6 shows that at $\pi_A = \pi_A^*$, a marginal increase of $\pi_A$ lowers $E(\pi_A, \pi_B)$. 

41
References


