Reserve Prices without Commitment

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When potential bidders in an auction have to incur a cost to prepare their bids and thus to learn their valuations, imposing a reserve price and announcing that in case no bid is submitted there will be another auction without a reserve price is both revenue and welfare improving. Reserve prices that induce less than maximum entry in the first auction may be optimal. Also, entry fees are not necessarily better instruments than reserve prices. Journal of Economic Literature Classification Number: D44. © 1996 Academic Press, Inc.

1. INTRODUCTION

In standard auction theory (with a fixed number of bidders) it is well established (see, for example, Myerson, 1981, or McAfee and McMillan, 1987a) that a well-chosen reserve price, strictly above the seller’s valuation of the item for sale, increases the seller’s expected revenues. That is, if the seller can credibly commit not to sell the object in case no bid exceeds the reserve price, the resulting increase in the aggressivity of the equilibrium bids compensates for the inefficiency caused by sometimes not selling the good when there are buyers with higher valuation than the seller’s.

The robustness of this result has been questioned from two directions. First, the use of reserve prices in auctions has often been criticized on the basis of skepticism about the ability of the seller to credibly commit not to eventually sell the good at a later moment, this time accepting lower bids. Indeed, if this commitment ability is lacking, the standard revenue equivalence results indicate that nothing can be gained by setting a reserve price (see, for example, Riley

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1 Throughout this paper we will talk about the bid-taker as a seller. Obviously, all our results remain unchanged if we consider a procurement auction instead.
and Samuelson, 1981)—in fact, with affiliated values it may even reduce the expected revenues.

Second, it has been observed that in practice the number of participating bidders in an auction depends crucially on its rules and especially on the reserve price that is set.\(^2\) Therefore, especially when designing “optimal” mechanisms, it is misleading to assume that the number of bidders is fixed exogenously. A number of theorists corrected for this “negligence,” arguing that actual bidders need to incur some cost in order to participate in the auction,\(^3\) and therefore the number of bidders who can profitably participate in an auction is limited endogenously. In such a setting, but assuming commitment to reserve prices, McAfee and McMillan (1987b), Engelbrecht-Wiggans (1987, 1993), and Levin and Smith (1994) show that reserve prices should only be used as an instrument to extract residual rents from the bidders: since the number of bidders is an integer, free entry alone fails to drive these rents to zero; then the optimal reserve price extracts this surplus without reducing the number of bidders. The decrease in competition that would result from reducing entry with a higher reserve price would outweigh the corresponding upward\(^4\) shift in the equilibrium bids of the participating bidders. Moreover, even this—relatively small—reserve price is not the best instrument to use. Entry fees also serve for rent extraction, while they yield \textit{ex post} efficient trade. Therefore, if entry fees are allowed, reserve prices (with commitment) are useless.

In contrast to these criticisms, reserve prices are commonly used in environments where bid preparation is costly, and where sometimes it is even explicitly announced(!) that in case no bid is submitted, there will be another auction, this time with lower or no reserve price in place.\(^5\) In this paper, we investigate the rationale for this policy. We restrict our attention to independent private values and first price sealed-bid auctions. We find that, even with reserve prices without commitment value beyond the present auction, if we take into account the endogeneity of bidder participation, we get back the standard results: (optimal) reserve prices do increase the seller’s expected revenues with respect to the standard, one-shot auction without reserve price. Thus, the first criticism given above

\(^2\) Following Albion (1961), Engelbrecht-Wiggans (1987) gives an interesting historical example. At the beginning of the 19th century the ports of Boston, New York, and Philadelphia were handling about equal quantities of imported goods, most of which were auctioned by the docks. When the port of New York abolished reserve prices, the trading volume in New York tripled the trading volumes of the other two ports in a few years.

\(^3\) This cost may be simply an opportunity cost or it may be the cost of preparing the bid (including finding out the object’s value). See Johnson (1979), French and McCormick (1984), Samuelson (1985), Engelbrecht-Wiggans (1987), and especially Harstad (1990, 1991), among others.

\(^4\) In case of a second price sealed-bid auction, there is no such shift, but the seller’s expected revenue still increases, since the reserve price may rise above the second price.

\(^5\) This is standard practice, for instance, in the Spanish administration when assets of a bankrupt firm are auctioned. Recently, the Picasso Towers, a future landmark of Madrid, were auctioned this way.
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loses its bite. Moreover, we show that sometimes high reserve prices are optimal even at the expense of reducing entry in the auction they apply to. Therefore, reserve prices are not used exclusively for extracting the bidders’ residual surplus, as has been argued in the endogenous entry literature. In addition, in our case reserve prices do not prevent efficient trade from taking place. As a result of this, entry fees are not necessarily better instruments. In fact, we provide an example where reserve prices dominate.

The intuition behind these results is simple. When bidders have to pay a price for learning their valuation, the number of bidders that can profitably (ex ante) participate in an auction is such that a new entrant would not expect to recover the entry costs. Now, assume that a small reserve price is in effect and that when the auction is conducted no bid is submitted (and thus the good is not sold). If now there is a new auction (with no reserve price), a potential bidder who found it unprofitable to enter in the first auction can infer that the already informed bidders have observed low realizations of their valuations. Then, he expects these incumbent bidders not to be aggressive in the new auction. Thus, the (interim) profitability of obtaining information has increased, and therefore he could this time enter in the auction. This increase in competition can in turn result in more efficient trade, an efficiency gain that increases with the reserve price.

Of course, increasing the reserve price can diminish the profitability of obtaining information for the first auction and it can therefore reduce competition in that auction. Thus, the best policy with respect to reserve prices has to solve the trade-off of less competition in the first auction against more in the second. In particular, this trade-off need not be resolved in favor of maximum first period entry.

The paper is structured as follows. In the next section we present our model. Section 3 contains the analysis of a two-period auction in which the number of bidders in each period is determined exogenously, and there is a reserve price in the first auction. That is, at first, we disregard information costs and the endogenous determination of the number of bidders. This analysis is equivalent to setting a one period scheme in which some bidders are discriminated against (revelation principle). Then, we show that generically some small reserve price is always beneficial for the seller (as in Engelbrecht-Wiggans, 1993, in a one-shot auction). As a straightforward conclusion from the analysis in Section 3, Section 4 provides an algorithm for determining the optimal reserve price which, in general, implies less entry in period one than what would occur in the absence of reserve prices. To illustrate the algorithm and this latter result, an example with uniform distribution of values is examined. Then conclusions follow.

Engelbrecht-Wiggans (1988) analyzes an example in which bidders can sequentially buy different signals about their (private) value. In his model the bidders do not learn about others’ valuations, but their own signals can eventually make it unprofitable to continue buying information. Thus, the multistage aspect of the auction he studies is quite different from the one studied here.
2. THE MODEL

Assume there is a seller who owns an indivisible object which she values at zero. She is auctioning it off to a large number of potential buyers, who have independent private valuations for the good, described by $v_i \sim F : [0, 1] \rightarrow [0, 1]$. We assume that our problem is regular in the sense of Myerson (1981). That is, $J(v) = v - (1 - F(v))/f(v)$ is a strictly increasing function of $v$.\footnote{Note that this condition is satisfied by all the convex distribution functions (including the uniform distribution) and, for example, the (truncated) exponential distribution or the (truncated) normal distribution with sufficiently high variance.}

Potential buyer $i$ has to incur a cost $c$ if he decides to privately learn his valuation, $v_i$. We consider two alternative mechanisms for the sale of the good. The first is a standard first-price sealed-bid auction, while the second is a two-period procedure. It starts with a first-price sealed-bid auction with a reserve price, $r$, which—in case no bid exceeds the reserve price—is followed by a standard first-price sealed-bid auction. Note that the two-stage auction with a reserve price equal to zero is equivalent to the standard auction, since without a reserve price in the first auction the good always gets sold and the second stage never comes about. Therefore, we can dedicate our efforts exclusively to the two-stage auction.

3. THE ANALYSIS OF THE TWO-STAGE AUCTION

We start our analysis by abstracting from the endogeneity of the number of bidders. Thus, for the moment, let us disregard information costs, and therefore entry, and let us simply consider as given a two-stage mechanism with a fixed number of bidders in each of the stages. In its first stage there is a certain reserve price $r$ and $k$ bidders bid for the good. The object is assigned to the bidder who submits the highest bid above $r$, if there is any, at a price equal to his bid. If no bid is submitted, then $h$ more bidders are invited to submit bids, together with the previous $k$ bidders, this time without any reserve price. Again, the winner is the one who bids highest and the price is his bid. Let $K = \{1, 2, \ldots, k\}$ represent the set of first-stage bidders and $H = \{k + 1, k + 2, \ldots, k + h\}$ the set of second-stage bidders. Also, let $\mu(n; m)$ represent the expected value of the $n$th order statistic of $m$ realizations of the random variable described by $F$. The following lemmas partially characterize the potential symmetric, monotone equilibria of this game:

**Lemma 1.** Given $k, h$, and $r \in (0, \mu(1; k + h - 1))$, in every monotone, symmetric equilibrium there exists a cutoff point, $w \in (r, 1)$, such that bidder $i \in K$ submits a bid if and only if $v_i \geq w$. Moreover, if $v_i = w$, $i$ bids exactly $r$. 

\[
\mu(n; m) = \begin{cases} 
\frac{n - 1}{n}, & n < m \\
\frac{n}{m}, & n \geq m.
\end{cases}
\]
Proof. Any bidder with valuation lower than \( r \) does not bid in the first auction no matter what the others do (bidding, he would obtain negative expected profits). A bidder with valuation \( r \), on the other hand, would not bid (at least \( r \)) in the first stage either, since waiting to the next stage and bidding \( r/2 \) would report a profit of at least \( r/2 F(r/2)^k \) (no bidder with valuation lower than \( r/2 \) would bid more than \( r/2 \) in the second stage), which is positive. Now, assume that no bidder bids in the first stage in equilibrium. Then, we will have a standard auction in the second period with \( k + h \) bidders, and therefore a bidder with valuation \( v_i = 1 \) would bid \( \mu(1; k + h - 1) \) in the second period and obtain the good with probability 1. However, deviating and bidding \( r \) in the first period, he would also obtain the good with probability one and pay a price \( r \), lower than \( \mu(1; k + h - 1) \) by assumption. Therefore, in any monotone, symmetric equilibrium there is a cutoff point \( w \) in \((r, 1)\) such that bidder \( i \in K \) submits a bid if and only if \( v_i \geq w(r) \).

Finally, we prove that the buyer with valuation \( w \) will bid \( r \) in equilibrium, by contradiction. Assume that all bidders who bid, offer strictly more than \( r \). Then, a bidder with valuation \( w \) would benefit from deviating and bidding exactly \( r \) (the probability of winning is the same, and the price is strictly lower). Q.E.D.

Let \( B \) represent the bidding function in a standard (one-shot, with no reserve price) \( k + h \) bidder auction, and let \( r \) be a reserve price in \((0, \mu(1; k + h - 1))\).

**Lemma 2.** In a symmetric, monotone equilibrium for the two-stage auction,
(i) for values lower than \( w \) the second-period bidding function is common to all \( k + h \) bidders and coincides with \( B \).
(ii) as a function of \( r \), \( w \) is continuous and strictly increasing in \( r \), with \( w = 0 \) for \( r = 0 \) and \( w = 1 \) for \( r = \mu(1; k + h - 1) \).

**Proof.** Let \( B_1 \) represent the bidding function in the second auction of the bidders in \( K \), and let \( B_2 \) represent the bidding function of bidders in \( H \). Then, for \( v \) lower than both \( w \) and \( B_2^{-1}(B_1(w)) \), \( B_1 \) solves the differential equation (first order condition)
\[-B_1'(v) \frac{F(v)^{k-1} F(B_2^{-1}(B_1(v)))^h}{F(w)^{k-1}} + v - B_1(v) \frac{d F(v)^{k-1} F(B_2^{-1}(B_1(v)))^h}{d w} = 0,\]
whereas \( B_2 \) solves
\[-B_2'(v) \frac{F(v)^k F(B_2^{-1}(B_1(v)))^{k-1}}{F(w)^k} + v - B_2(v) \frac{d F(v)^k F(B_2^{-1}(B_1(v)))^{k-1}}{d v} = 0,\]
both with the same boundary condition \( B_1(0) = 0 \). Note that, setting \( B_1 = B_2 = B \), we have that both equations are the same as the one solved by \( B \). Therefore, \( w \) (the type which is indifferent between bidding \( r \) today or according to \( B \) tomorrow) is given by the solution to
\[w - r] F(w)^{k-1} = [w - B(w)] F(w)^{k+h-1}\]
or, rearranging terms,

\[ w[1 - F(w)^h] + B(w)F(w)^h = r. \]  

(1)

Now, the derivative of the left-hand side with respect to \( w \) is positive, since from the definition of \( B \),

\[ B'(w) = (k + h - 1)[w - B(w)]f(w)/F(w). \]

Therefore, as a function of \( r \), \( w \) is continuous and strictly increasing. Also, for \( w = 1 \), Eq. (1) becomes \( B(1) = \mu(1; k + h - 1) = r \), that is, for \( r = \mu(1; k + h - 1) \), \( w = 1 \). Q.E.D.

Equation (1) defines \( w \) as a function of both \( h \) and \( k \) (since \( B \) depends on \( k + h \)), and of \( r \). We will use this function, denoted by \( w(r, k, h) \), in what follows. Note that, given \( r \), \( w \) is a decreasing function of \( h \) and \( k \).

Now, existence of equilibrium can be shown by the same arguments as existence for the standard first-price auction. Indeed, for bidders in \( H \), first order conditions are sufficient in \([0, w(r, k, h)]\), since, given that all others use \( B \), \( B(x) \) is the only value that satisfies them for a bidder with valuation \( x \), and the corner alternatives are worse than that bid (notice that the bidder solves a differentiable maximization problem on a compact set). The same argument holds true for bidders in \( K \).

The following result is the basic explanation of why a reserve price with only temporary commitment value can increase the revenues of the seller:

**Proposition 1.** Given \( k \) and \( h \), the seller’s expected revenue is strictly increasing in the reserve price \( r \). The expected rents of the bidders in \( K \) are decreasing in \( r \) and \( k \), and the expected rents of the bidders in \( H \), conditional on reaching the second stage, are also decreasing in \( r \) and \( k \).

**Proof.** Following Myerson (1981, Lemma 3) we can write the expected revenue of the seller as

\[ \Pi = \int \sum_{i \in K \cup H} J(v_i)p_i(v)f(v)dv, \]  

(2)

where \( J(x) = x - (1 - F(x))/f(x) \) and where \( p_i(v) \) denotes the probability that \( i \) wins the auction if the vector of valuations is \( v \). By Lemmas 1 and 2, in our case, \( p_i(v) = 1 \) for \( i \in K \) if either \( v_i \) is the highest component of \( v \), or \( v_i \) is above \( w(r, k, h) \) and it is the highest of the first \( k \) components of \( v \) (the ones that correspond to \( K \)). If \( i \in H \), \( p_i(v) \) is one if \( v_i \) is the highest component of \( v \) and \( v_j < w(r, k, h) \) for all \( j \in K \). Otherwise, \( p_i(v) \) is zero. Then, (2) can be
written as
\[
\Pi(w(r, k, h), k, h) = \int_{w(r, k, h)}^{1} J(x)kf(x)F^{k-1}(x) \, dx \\
+ F^k(w(r, k, h)) \int_{w(r, k, h)}^{1} J(x)hf(x)F^{h-1}(x) \, dx \\
+ \int_{0}^{w(r, k, h)} J(x)(k + h) f(x)F^{k+h-1}(x) \, dx.
\] (3)

Using the fact that, by the regularity assumption, \( J(x) \) is increasing in \( x \), it is straightforward to see that \( \Pi \) is increasing in \( w(r, k, h) \) (and therefore in \( r \)).

Using Lemmas 1 and 2 and Myerson (1981) again, the expected rents of \( i \in K \) can be written as
\[
R_K(w(r, k, h), k, h) = \int_{w(r, k, h)}^{1} (x - J(x)) f(x)F^{k-1}(x) \, dx \\
+ \int_{0}^{w(r, k, h)} (x - J(x)) f(x)F^{k+h-1}(x) \, dx,
\]
while those of \( i \in H \), conditional on reaching stage 2, as
\[
R_H(w(r, k, h), k, h) = \int_{w(r, k, h)}^{1} (x - J(x)) f(x)F^{h-1}(x) \, dx \\
+ F^{-k}(w(r, k, h)) \\
\times \int_{0}^{w(r, k, h)} (x - J(x)) f(x)F^{k+h-1}(x) \, dx.
\]

It is straightforward to check that the derivatives of both \( R_K \) and \( R_H \) with respect to \( w(r, k, h) \), and therefore with respect to \( r \), are positive while the derivatives with respect to \( k \) are negative. Q.E.D.

The purpose of this article is, of course, to study the optimality of setting a reserve price \( r \) with endogenous determination of \( h \) and \( k \), which we will do in the next section. However, Proposition 1 has an immediate corollary for the generic optimality of some positive reserve price. Indeed, consider the generic case in which there exists an integer, \( n \), s.t. \( R_K(0, n, \cdot) > c > R_K(0, n + 1, \cdot) \).

Then we can say the following:

**Corollary.** There exists a reserve price \( r > 0 \) such that the seller’s revenues in the two-stage auction are higher than in the standard auction without reserve price.

**Proof.** By continuity of \( R_H \) and \( R_K \) in \( w(r, k, h), k = n \) for \( w(r, k, h) \) (and \( r \)) small enough. Also, \( \Pi \) is right continuous in \( w \) and independent of \( h \) at \( w = 0 \). Therefore, by Proposition 1, the corollary follows. Q.E.D.
In other words, when this is possible, a small reserve price that does not reduce first period entry (with respect to the standard auction) increases the seller’s revenues. However, the optimal reserve price could, in principle, imply fewer first-stage participants than in the standard auction. In the next section we provide the algorithm to calculate this optimal reserve price and show that indeed it may be optimal to set a reserve price high enough so that first-period entry is less than maximal.

4. THE OPTIMAL RESERVE PRICE

We return to our original problem, in which potential bidders have to decide whether to incur the information cost \( c \) or not, i.e., whether to participate in the auction or not. The owner announces a reserve price \( r \) which will be in effect during the first auction. It is common knowledge that, in case nobody submits bids in this auction, there will be a second auction with no reserve price (and then, another opportunity for additional entrants). Assume \( k \) bidders have acquired information (and this is common knowledge), but no one has submitted a bid in the first auction. Also, assume those \( k \) bidders used a monotone bidding strategy with a cutoff point at \( w \). Finally, assume that \( h \) more bidders buy information (enter) in the second period. We are then in the situation analyzed in the previous section. The bidding functions in the second auction are as described in Lemma 2, and so are the rents for the seller and the bidders. Now, \( r \) is selected by the seller, whereas \( k \) and \( h \) are the result of equilibrium choices by the potential bidders. A direct approach to obtaining the optimal reserve price would then involve computing \( k \) and \( h \) as a function of \( r \) and then maximizing \( \Pi(w(r, k, h), k(r), h(k, r)) \) in \( r \). However, there are two difficulties with this approach: first, obtaining the equilibrium values for \( k \) and \( h \) for each reserve price \( r \) is, to say the least, involved; second, \( h \) and \( k \) are, by definition, discrete variables, so the solution to the maximization problem would not be given by a first order condition.

Next, we provide a simple algorithm to solve the above problem. This algorithm is based on two straightforward points. First, the set of possible equilibrium combinations \((k, h)\) is finite. Second, at the optimal reserve price either the first period or the second period bidders must be making zero expected profits. Otherwise, the seller could slightly increase the reserve price without changing the number of bidders in any stage, thus increasing her revenue.

These two ideas, plus the information contained in Lemmas 1 and 2 and Proposition 1, are all we need to construct the algorithm. Let us denote by \( k(0) \) the number of bidders that would participate in a standard auction without a reserve price:

\[
R_K(0, k(0), \cdot) \geq c > R_K(0, k(0) + 1, \cdot).
\]
Now note that we have that for any \( r, k(0) \geq h \geq k \). Indeed, the second inequality can be checked by simple inspection of the definitions of \( R_K \) and \( R_H \) in Proposition 1. Also, for each \( h \), \( R_H \) is decreasing in \( r \) and \( k \), and at its maximum \( (r = k = 0) \), \( R_H(0, 0, h) = R_K(0, h, 0) \), and thus \( h \) must be no more than \( k(0) \). Moreover, observe that \( h + k \geq k(0) \). To see this, note that given any \( r \) and \( k \),

\[
R_H(w(r, k, h), k, h) \geq \int_0^1 (x - J(x)) f(x) F_{k+h}^{k+1}(x) \, dx = R_K(0, k + h, \cdot),
\]

and therefore \( h \geq k(0) - k \), since otherwise more than \( h \) bidders would enter in the second period given that \( k + h + 1 \leq k(0) \) implies \( R_H(w(r, k, h), k, h + 1) \geq R_K(0, k(0), \cdot) \geq c \). Finally, note that any \( r \) that gives rise to an entry of \( k + h = k(0) \) is dominated by \( r = 0 \) (which implies \( k = k(0) \)). Indeed, for \( h = k(0) - k \), from Eq. (3) we have

\[
\Pi(w, k, h) = \int_0^1 J(x) k f(x) F_{k-1}^{k-1}(x) \, dx - F_k(w)
\]

\[
\times \int_0^w J(x) k f(x) F_{k(0)-1}^{k(0)-1}(x) \, dx
\]

\[
+ F_k(w) \int_0^w J(x) k f(x) F_{k(0)-1}^{k(0)-1}(x) \, dx
\]

\[
+ \int_0^w J(x) k f(x) F_{k(0)-1}^{k(0)-1}(x) \, dx \leq \Pi(0, k(0), \cdot)
\]

for any \( w \geq 0 \). Therefore we only have to consider pairs \((k, h)\) such that \( k + h > k(0) \). Given these preliminaries, the following algorithm finds the optimal reserve price:

**Proposition 2 (The Algorithm for Optimal Reserve Prices).** Solve the following algorithm:

1. For each \((k, h)\) with \( k \leq h \leq k(0) \) and \( k + h \geq k(0) \):
2. If \( R_H(w, k, h) \geq c \) and \( R_H(w, k, h + 1) < c \) for the \( w \) given by \( R_K(w, k, h) = c \), select that \( w \); otherwise select the \( w \) given by \( R_H(w, k, h) = c \).
3. For the \( w \) selected, compute \( r^* \) such that \( w(r^*, k, h) = w \).
4. Given \( r^* \), for each \((h', k')\) such that \( k(0) \geq h' > k' > k \), check if \( w' = w(r^*, k', h') \) satisfies \( R_H(w', k', h') \geq c \) and \( R_K(w', k', h') < c \).
5. For all triples \((w, k, h)\) selected, at most \( k^2/(4) \), compute \( \Pi(w, k, h) \). Select the \((w^*, k^*, h^*)\) that maximizes it. Then, the reserve price that maximizes the seller’s revenues, \( r^* \), is given by \( w^* = w(r^*, k^*, h^*) \).

**Proof.** First we show that for each \( r \) in the interval \([0, \mu(1; k(0) - 1)]\) there exists an equilibrium for the game induced by the two-step auction mechanism
with endogenous entry, such that \( k(0) \geq h \geq k \). Indeed, given \( 0 \leq k \leq k(0) \) and \( r \), we can compute \( h \) as a function of \( w \) using the condition

\[
R_H(w, k, h) \geq c > R_H(w, k, h + 1).
\]

(4)

This is a decreasing (step) function with \( h = k(0) \) for \( w = 0 \) and \( h = k(0) - k \) for \( w = 1 \). Also, given the same \( k \) and \( r \), Eq. (1) gives us \( w \in [0, 1] \) as a function of \( h \), for each \( h \) in the range \([k(0) - k, k(0)]\). Indeed, notice that Eq. (1) can be written as

\[
w + [F(w)]^{h}[B(w) - w] = r.
\]

(5)

The left-hand side is decreasing in \( h \) and increasing in \( w \), and for \( h = k(0) - k \) and \( w = 1 \), this left-hand side is \( B(1) = \mu(1; k(0) - 1) \geq r \), so that for that value of \( h \) the solution for \( w \) is \( w \leq 1 \), whereas for \( h = k(0) \) and \( w = 0 \), the left-hand side is \( 0 \leq r \), so that for that value the solution for \( w \) is \( w \geq 0 \). Then it is easy to see that there exists at least one pair \((w, h)\) with \( w \in [0, 1] \) and \( h \) in the range \([k(0) - k, k(0)]\) that satisfy both conditions (4) and (5)—that is, the two “curves” cross. Take the pair with highest \( h \). Equilibrium is then given by the highest value of \( k \) such that \( R_K(w, k, h) \geq c \), or \( k = 0 \) if \( R_K(w, k, h) < c \) for all \( k < k(0) \) (for \( k > k(0) \), \( R_K < c \) for all \( r \)).

Second, we show that the optimal \( r \) has to satisfy either \( R_H = c \) or \( R_K = c \). Indeed, both functions are continuous in \( w \), and \( w(r, k, h) \) is continuous in \( r \), given \( k \) and \( h \). Therefore if both \( R_k < c \) and \( R_H > c \), increasing \( r \) would not change \( h \) and \( k \) and would, consequently, increase \( \Pi \).

Finally, we show that any four tuple \((r, w, k, h)\) that satisfies step 1 constitutes an equilibrium if and only if step 3 is also satisfied. Indeed given \( k \) and \( r \), (2) is simply Eq. (1), whereas (1) guarantees that \( h \) is the equilibrium entry given \( w \). Moreover, if 1 and 3 are satisfied then \( k \) is the maximum entry in the first period. On the other hand, if step 3 is not satisfied we do not have an equilibrium, since then \( k' > k \) first period entrants could cover costs—using a bidding strategy defined by \( w' \) and expecting \( h' \) second period entrants. It follows that \( r'' \) is the optimal reserve price. Q.E.D.

**An example with uniformly distributed values.** Assume \( v_i \) is uniformly distributed in \([0, 1]\), and assume \( c = \frac{1}{11} \). First, recall that the equilibrium bidding function for a standard first-price auction with \( n \) bidders is given by

\[
B(x) = \frac{n - 1}{n} x.
\]

(6)

We will use (6) to compute \( r \) for a given triple \((h, k, w)\). Now, we can calculate \( k(0) \) from \( R_K(0, k(0), \cdot) \geq c > R_K(0, k(0) + 1, \cdot) \), which gives us \( k(0) = 3 \) \((R_K(0, 3, \cdot) = \frac{1}{11}, R_K(0, 4, \cdot) = \frac{1}{22})\). Then we have to run our algorithm for the following \((k, h)\) pairs: \((1, 3)\); \((2, 2)\); \((2, 3)\); and \((3, 3)\). In step 1 for \((1, 3)\) we
select $w = 0.475$ from $R_H = c$ (the $w$ given by $R_K = c$ violates the conditions on $R_H$). This corresponds to a value $r = 0.463$, in step 2. But then step 3 with $k' = h' = 2$ (with $w' = 0.492$) eliminates this case. We then turn to the pair $(2, 2)$, for which step 1 selects $w = 0.591$ (from $R_K = c$). Step 2 then gives us $r = 0.539$, and step 3 is passed. The same is done with the pairs $(2, 3)$ and $(3, 3)$ with $w = 0.4$ and $r = 0.395$ for the pair $(2, 3)$ and $w = 0.292$ and $r = 0.291$ for the pair $(3, 3)$.

The corresponding expected rents of the seller turn out to be

\[
\Pi(0.292, 3, 3) = 0.53, \\
\Pi(0.4, 2, 3) = 0.49, \text{ and} \\
\Pi(0.591, 2, 2) = 0.54.
\]

Therefore, the optimal reserve price we obtain is $r^o = 0.539$, which implies an entry of two bidders in the first auction and two more in the second, if it ever takes place.

Figure 1 shows $\Pi$ as a function of $w$ for this example. The example we have just presented is of the "generic type," that is, a case in which $R_K(0, k(0), \cdot) > c$. However, the convenience of reserve prices is not limited to these cases. Indeed, assume $c = \frac{1}{12}$. In this case any positive reserve price will induce less entry in the first period, but, as Fig. 2 shows, the efficient reserve price is still positive ($w(r^o) = 0.55, r^o = 0.51$).

Notice that in this last case ($c = \frac{1}{12}$), nothing can be gained by (only) setting entry fees. Indeed, any positive entry fee induces less entry (as any positive reserve price). The seller’s revenues (for a candidate to optimal entry fee) will be given by the expected valuation of the winner minus information costs: 0.50 if
two bidders enter and 0.42 if only one bidder enters, both lower than the revenues expected setting \( r^o = 0.51 \).

**Remark 1.** The same arguments used to show that a scheme with reserve price in a first stage and no reserve price in the second is usually better than a standard auction could be extended to more stages. Indeed, there could be examples in which additional increases in the seller’s revenue would be obtained by setting a second reserve price in the second auction, again knowing that the seller would then offer the good later in case no bid is submitted. To illustrate this point we have worked out our uniform example with information costs \( c = \frac{1}{12} \). Remember that the best two-stage scheme is the one that sets a reserve price equal to 0.539, which then has two entrants in each stage and implies an expected revenue for the seller equal to 0.54. Now, assume the seller sets a reserve price \( r_1 = 0.53 \) for the first period and \( r_2 = 0.46 \) for the second. Then it can be checked that two bidders enter in the first period, each bidding if and only if their value is above \( w_1 = 0.55 \), and two more bidders would enter in the second, if it ever takes place. The four of them would then bid if and only if their valuation is above \( w_2 = 0.48 \). In case the good is not sold in the second period either, still two more bidders would enter in the final no-reserve price auction. The seller’s expected revenues are in this case 0.56, higher than in the best of the two-stage schemes.

**Remark 2.** Second-price auctions are equivalent to first-price auctions, even when considering information costs and noncommitment reserve prices. Indeed, given \( w \) (the cutoff point for bidding in the first stage), both mechanisms fare equally. Moreover, to each \( w \) in the interval \((0, 1)\) there exists a reserve price that makes bidders act according to this cutoff point, and the same number of
bidders participate in each auction. Therefore, the usual revenue equivalence results apply, and thus the choice between using first- or second-price auctions is immaterial.

**Remark 3.** As could be expected, the revenues for the seller are decreasing with the cost of acquiring information. Indeed, given $k$ and $h$, the buyers’ profit function is continuous in $w$. Therefore, the values obtained from the algorithm (setting $R_K$ or $R_H$ equal to $c$) change continuously and inversely with $c$. Then, lower costs mean that a higher reserve price can be set without altering the number of bidders in each stage. But since, for given $k$ and $h$, $\Pi$ is increasing in $r$, the seller’s revenues increase when $c$ decreases. Note that this implies the seller has an incentive to credibly reveal value relevant information to the potential bidders.

The way the optimal reserve price changes with $c$ is ambiguous. Generically (that is, when the optimal reserve price is strictly the best), an infinitesimal reduction in $c$ implies an increase in $w$. The reason is as before: a reduction in $c$ means that a higher reserve price is still compatible with the same number of entrants. Since (for given $k$ and $h$) the bidder’s profit functions are continuous, the optimal reserve price is still in the same region (same number of entrants) and therefore the optimal reserve price is higher. However, further reductions can make the optimal reserve price jump (the number of entrants change). Indeed, consider the uniform case above. When the cost of acquiring information changed from $\frac{1}{13}$ to $\frac{1}{12}$, the optimal reserve price changed from 0.54 to 0.51 (in both cases, with $k = h = 2$). However, as Fig. 3 shows, a further increase of $c$ to $\frac{1}{9}$ means a higher reserve price of 0.55 (with $k = 1$ and $h = 2$ and $w = 0.62$).

**Remark 4.** The analysis above could be adjusted to encompass situations in which potential bidders are asymmetric with respect to their cost of acquiring information. In some instances, this feature could only strengthen our results in the sense that the ordering in cost efficiency would determine the identity of bidders entering in each stage. Something similar would happen when bidders are asymmetric with respect to the range of their (ex ante) valuation.

**Remark 5.** We could also extend our analysis to situations in which only entry costs are present, that is, situations in which the valuation is costlessly known to the bidders, but there is a cost in preparing the bids. This model has been analyzed by Samuelson (1985). In this case, there is a break even point below which bidders do not submit a bid. In Samuelson’s article, the seller (buyer in his case) has an outside option with known value, which avoids the problem of what happens if nobody is above the break even point, and therefore the good is not sold. Now, assume this outside option is not present. Then, even without reserve price in the first stage, the seller could choose to perform a second auction when the good is not sold in the first. Potential bidders that chose not to bid in the first stage now learn that all competitors have bad realizations.
Thus, the expected profits from preparing a bid are improved, exactly as in our model. Reserve prices can then be used to determine the optimal location of the break even point, which in our case we denoted by \( w(r, k, h) \).

Remark 6. The previous analysis opens the question of how efficient the optimal (from the seller’s point of view) auction mechanism is compared to the one without reserve price. On the one hand, entry costs can increase or decrease, since the first-stage entry can be reduced at the expense of higher entry in the second stage. On the other hand, the surplus obtained from the good can also be higher or lower for exactly the same reason. We do not have a general answer to the previous question. However, for the generic case it can be shown easily that the derivative of the net surplus with respect to \( w \) at \( w = 0 \) is positive. Therefore the corollary to Proposition 1 holds if we substitute net surplus for seller’s revenue. (In the uniform example above, the gross surplus— without taking information costs into account—is higher with the optimal reserve price—0.77 versus 0.75 with no reserve price—and the expected information costs are lower—0.21 versus 0.23. Thus, the net surplus increases. Also, we have found cases in which either of these two inequalities reverse, although not the inequality regarding net surplus.)

Remark 7. We have assumed independent private values. In the other polar case, the common value hypothesis, knowing that all rivals have bad realizations is bad news for a bidder. But, assume that given a certain reserve price, the equilibrium bidding strategy for the entrants is characterized by a cutoff point \( w \). Then, in case nobody bids in the first period the \textit{ex ante} valuation of the good is lower. However, there are still chances that all rivals have simply had bad luck, and therefore there is an opportunity to obtain the good for a bargain. The
trade-off between these two effects could still be resolved in favor of a positive reserve price.

Remark 8. The outcome of the algorithm given in Proposition 2 may be affected by multiplicity of equilibria. While it is true that the algorithm determines both the optimal \(w\) and \(r\) (generically) uniquely, once \(r\) is announced, there can be more than one \(w\) (and \(h\)) compatible with that reserve price. (Note that since step 3 of our algorithm ensures maximum entry in the first period, \(k\) is defined uniquely.) As an example, consider the uniform case studied above when \(c = \frac{1}{12}\). In this case the optimal \(w\) is 0.55, which implies \(k = h = 2\). This can be obtained by setting \(r = 0.51\). However, as Fig. 4 illustrates, there is another equilibrium configuration compatible with \(r = 0.51\). Indeed, this same reserve price corresponds to the case \(k = 2, h = 1, and w = 0.57\). That is, if the two first-stage entrants expected only one more entrant tomorrow, they would only bid whenever their valuation exceeded 0.57. But then only one more bidder would find it profitable to buy information tomorrow. Both equilibria are equally plausible. The only hope to single out the "good" equilibrium is by a focal point argument. Indeed, the seller could announce that a preregistration period would be open tomorrow until two more bidders sign up as eligible bidders (before committing to really spend resources on information acquisition). However, still nothing can prevent first stage bidders to expect only one bidder tomorrow and then bid only above 0.57. Therefore, nothing can prevent prospective new entrants to believe that, either.

\[8\] The also possible case in which the "wrong" equilibrium involves more entry can more easily be solved. It suffices to announce that no more than \(h\) bidders will be eligible as bidders in the future.
5. CONCLUDING REMARKS

We have analyzed the use of reserve prices in auctions when the seller cannot commit not to sell the good in the future in case no bid is submitted. We have found that when bidders have to incur costs in order to find out their own valuation, the use of reserve prices could increase the seller’s revenue. The intuition behind this result is that no bidding in an auction with certain reserve price is a signal of bad realizations for the first-period (already informed) bidders, which therefore improves the rents that prospective bidders expect from buying information in the second stage. All in all, competition (and efficiency) could be increased with such reserve prices.

We have performed the analysis in the context of private, independent values, and have provided an algorithm to calculate the optimal reserve price. The same effects of reserve prices are present, however, in case bidders know their valuation before incurring the cost (of preparing a bid) and in the common value case. Also, the optimal length of the sequence of auctions can be more than two stages. That is, there could be gains from setting a new reserve price in the second auction, and then opening the possibility of a third auction, etc.

REFERENCES


