Growth, selection and appropriate contracts

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ABSTRACT

We study a dynamic model where growth requires both long-term investment and the selection of talented managers. When ability is not ex-ante observable and contracts are incomplete, managerial selection imposes a cost, as managers facing the risk of being replaced choose a sub-optimally low level of long-term investment. This generates a trade-off between selection and investment that has implications for the choice of contractual relationships and institutions. Our analysis shows that rigid long-term contracts sacrificing managerial selection may prevail at early stages of economic development and when heterogeneity in ability is low. As the economy grows, however, knowledge accumulation increases the return to talent and makes it optimal to adopt flexible contractual relationships, where managerial selection is implemented even at the cost of lower investment. Measures of investor protection aimed at limiting the bargaining power of managers improve selection under short-term contracts. Given that knowledge accumulation raises the value of selection, the optimal level of investor protection increases with development.© 2013 Elsevier Inc. All rights reserved.

1. Introduction

Economic growth requires both incentives to undertake projects that pay out in the future and an efficient mechanism to select the best managers to run them. There is no need to stress that avoiding myopic strategies is often crucial for economic success. To motivate long-term investment, it is therefore important that managers have sufficient prospects to be among those who will enjoy the future returns. At the same time, however, it is well documented that bad managerial quality can impose large costs. Having the flexibility to remove incompetent managers and workers may thus be essential too. An important role of contracts and institutions regulating production relationships is to strike a balance between these possibly conflicting goals. In this paper, we propose a model where ex-ante unobservability of talent and contract incompleteness give rise to a trade-off between selection and investment. We then study how this trade-off shapes the design of contracts and show why countries at early stages of economic development may choose rigid, long-term, contractual arrangements but eventually switch to more flexible, short-term, relationships.

Our analysis is motivated by both empirical and theoretical considerations. There is ample evidence that contractual institutions and production relationships differ markedly across countries and time. For example, state-owned and family firms, which are typically characterized by long-term relationships and very low managerial turnover, tend to prevail at earlier
stages of economic development. While some authors have emphasized the inefficiencies of such rigid arrangements, others have suggested that they may reflect the need for different institutional forms at various stages of development.1 In particular, Kuznets (1966, 1973), Gerschenkron (1962) and more recently Acemoglu et al. (2006), have forcefully stressed that economic growth is accompanied by a process of structural transformation that includes changes in production relationships and an increasing importance of skills.

The view that rigid institutions, maximizing investment at the expenses of managerial competition, may be beneficial at early stages of development seems consistent with several observations. For instance, several Latin American countries with highly-regulated markets were able to grow rapidly until the mid 1970s, but were then taken over by economies with more free-market policies, such as Hong Kong and Singapore. The cases of Korea and Japan are even more suggestive. Both countries achieved fast convergence thanks to heavy investments made by large conglomerates until the mid–1980s in Japan and the Asian crisis in Korea. Yet, economic growth resumed in Korea only once the country adopted reforms encouraging the hiring of professional managers and limiting the ability of families to retain control of conglomerates. Likewise, rapid growth in recent years in China has been associated with the dismantling of state-owned enterprises.2

In our model, firms and agents last for two periods and produce output by combining a broad form of knowledge capital (productivity) with managerial skills. In the first period, the owner hires a manager to run it. Managers differ in ability, which is initially unknown, and can invest in future productivity at the expense of current production. We assume that contracts are incomplete because outcomes are assumed to be non-verifiable and managerial compensation is determined through ex-post bargaining. Yet, owners and managers can agree to sign binding contracts specifying non-contingent actions, such as the duration (either one or two periods) of employment. At the end of the first period, agents learn the ability of the manager. Next, if the parties have signed a one-period contract, they may decide whether to confirm the manager or replace her with a new random draw. In the second period, past investment pays out and production takes place. In sum, managers choose long-term investment in order to maximize their own payoffs, which depend on output and the probability of not being fired, while owners would like to retain good managers, but without compromising too much ex-ante incentives to invest.

We first study the determinants of investment. Under flexible (one-period) contracts, two distortions induce managers to choose a sub-optimally low level of investment. First, the possibility of being fired implies that managers may not be able to enjoy the future rents and this reduces their expected benefit from investment. Second, there is a hold-up problem: increasing investment ex-ante weakens the bargaining position of the manager ex-post. Both distortions depend on the fact that managers face a non-zero probability of being removed. Hence, they represent the costs of selection. Its benefit, on the other hand, is that it ensures higher managerial ability.

Next, we turn to study how this trade-off shapes the choice of contracts. We find that rigid contracts are preferred by the parties when ability is concentrated and the productive capacity of the economy is low. These are cases in which selection is not very useful, while investment is relatively more valuable. The model thus suggests long-term contracts to prevail in developing countries with low levels of physical and human capital.3 Yet, as the productive capacity of the economy grows endogenously, managerial ability, which is a complementary (and scarce) input, becomes relatively more important so that short-term contracts are chosen.

We also study some normative implications of our model. We show that, in general, the equilibrium choice of contract is not the one that maximizes the expected discounted value of firms. More precisely, the switch to short-term contracts tends to occur too late because of an appropriability problem. Under short-term contracts, the owner and the manager do not internalize the part of the surplus appropriated by a new manager in case of replacement. This reduces the value of short- relative to long-term contracts. We next ask whether measures of investor protection aimed at limiting the bargaining power of managers may improve incentives. The effect of such measures is to increase selection. Since the value of selection rises with accumulation, the optimal degree of investor protection increases as well.

These results are broadly consistent with a number of empirical observations and can help understand the relationship between contracting institutions and economic prosperity. We review a number of cross-country and cross-firm empirical studies suggesting that firms become more selective where investor protection and corporate governance are stronger. Perhaps more importantly, the prediction that selection matters more at later stages of development is consistent with the finding in Aguirre (2012) that investor protection – which increases selection – has a positive effect on GDP growth in advanced economies, but not in developing countries.

This paper contributes to the theoretical literature, still in its infancy, on appropriate institutions and growth.4 The evolution of contractual relationships along the process of development has so far received little attention. One exception is Acemoglu et al. (2006). In their model, selecting talent becomes more useful as countries get closer to the technology frontier because skill is assumed to be more important for innovation than for adoption. Our analysis is complementary and has a different focus. First, we provide a micro-foundation for the trade-off between investment and selection. Second, we

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1 On the role of family firms see, for example, Burkart et al. (2003), Caselli and Gennaioli (2013) and references therein. Freeman (2010) summarizes the debate on whether labor market regulations promote or hinder efficiency in developing countries.
2 Acemoglu et al. (2006) provide more systematic evidence that barriers to competition may be beneficial when countries are far from their steady state.
3 However, long-term contracts may also be optimal in societies that are very homogeneous. Japan may provide an interesting example.
4 This literature has been pioneered by the works of Douglas North (see, for example, North, 1994). Among others, recent contributions focusing on economic institutions are Rodrik (2007), Acemoglu et al. (2006) and Aghion and Howitt (2006).
study its implications for the choice between contracts of different rigidity, instead of the effect of competition policy at various stages of development. The trade-off between selection and investment is also the subject of Aghion et al. (2013), arguing that institutional ownership reduces managerial turnover in case of bad performance and promotes investment in innovation. However, their paper does not study the optimality of institutional ownership.5

Our paper is also related to the relatively small literature on incomplete contracts and growth. Acemoglu et al. (2007), and Francois and Roberts (2003) study how contractual frictions affect technology adoption and innovation. Hemous and Olsen (2010) argue that repeated interaction may help to overcome the static costs associated with limited contractibility, but at the cost of dynamic inefficiencies. Differently from these papers, we are interested in studying the endogenous choice of contracts and its interaction with economic growth.

Next, the literature on law and economics documents the prevalence of family firms and rigid contractual relationships in developing countries. Theoretical works explaining this fact argue that family firms arise in the presence of weak institutions (see Mork et al., 2005, for a survey). None of the existing papers, however, studies the endogenous evolution of optimal contractual arrangements. We instead abstract from enforcement problems and issues related to firm ownership and organization.

Finally, the corporate finance literature studies various aspects of the contracts between managers and shareholders and the instruments to align their interests (e.g., Clementi et al., 2006; Gabiax and Landier, 2008; Edmans et al., 2009; Benmelech et al., 2010, and references therein). Our aim is to embed some of these ideas into a growth model and study how the choice of contracts changes with economic development. For this reason, we depart from this literature by focusing on simple and incomplete contracts that are more likely to be used in developing countries.

The rest of the paper is organized as follows. Section 2 lays down a model illustrating the main trade-off between selection and investment under alternative contractual arrangements. Section 3 solves for the equilibrium choice of contracts (long- versus short-term contracts) and shows how it varies with the level of development and other parameters. Section 4 focuses on normative questions and shows how optimal investor protection and labor market institutions should vary with development. Section 5 discusses the main empirical implications and how they relate to the existing evidence. Section 6 concludes.

2. The model

We propose a simple growth model designed to study the agency problem between owners and managers in a world where managerial ability is not perfectly observable and contracts are incomplete. The model gives rise to a trade-off between selection and investment, with implications for the choice of contracts between the principal (the owner) and the agent (the manager).

2.1. Agents, preferences and technology

The economy is populated by overlapping generations of two-period lived agents. Similarly to Acemoglu et al. (2006), each generation consists of a mass \( L/2 \) of owners, who are endowed with ownership claims on new firms, and a mass \( L \) of managers, who have no wealth but are endowed with heterogeneous skills required to run firms.5 All agents are risk-neutral and discount the future at the rate \( \beta \in (0, 1) \). In every period, a mass \( L/2 \) of new firms – equal to the new cohort of owners – enters. Firms run for two periods and produce a single final good, which is taken as the numeraire. Therefore, at any period \( t \), there is a mass \( L \) of active firms (young and old) and total output is given by:

\[
Y_t = \int_0^L y_{jt} \, dj,
\]

where \( y_{jt} \) is production of firm \( j \) at time \( t \).

New firms start out with an initial level of productivity, which we call “knowledge capital” and we denote as \( k_{jt} \), randomly drawn from a distribution with positive support and mean proportional to the average knowledge capital of existing old firms. In particular, we assume that there are partial knowledge spillovers from old to new firms so that \( k_{jt} \) is drawn from a distribution with mean equal to \((1 - \rho)\) times the average knowledge capital of existing old firms. Thus, the aggregate capital of new firms is:

\[
\int_{j \in S_t} k_{jt} \, dj = (1 - \rho) \int_{j \in S_{t-1}} k_{jt} \, dj, \quad \text{with } 0 < \rho < 1.
\]

5 Thesmar and Thoenig (2000) study the trade-off between efficiency and adaptability in a model of organization choice and growth.

6 The fact that there is excess supply of managers implies that the outside option of idle managers is zero. Alternatively, we could have assumed that idle managers can be employed as “workers” in a final sector that uses labor and \( Y \) as inputs.
where $S_t$ denotes the set of new firms at time $t$. In other words, old knowledge used by a new firm depreciates at the rate $\rho$. The level of knowledge capital is the key state variable of the model, capturing the broad productive capacity of the economy, which will grow endogenously over time.

Besides the input $k_{jt}$ put by the owner, each firm requires one manager to be operated. Managers differ in their ability. In particular, managerial ability denoted as $\theta_j$, is assumed to be drawn from a Pareto distribution with support on $[\theta, \infty)$ and tail index $\alpha > 2$. Thus, the cumulative distribution function is $G(\theta) = 1 - (\theta/\theta_j)^{\alpha}$ and the corresponding population average, denoted by $\bar{\theta}$, is:

$$\bar{\theta} = \frac{\alpha}{\alpha - 1}.$$ 

While the distribution $G(\theta_j)$ is common knowledge, the realization of $\theta_j$ is initially unknown to all agents. Yet, the ability of a manager running a firm is persistent and will be fully learned after the first period of production.8

The manager of a young firm has access to an investment technology that converts units of current knowledge capital at time $t$ into higher knowledge capital at $t+1$. In particular,

$$k_{jt+1} = k_{jt} + f(i_{jt}),$$

where the function $f(\cdot)$ satisfies the regularity conditions: $f(0) = 0$, $f'(\cdot) > 0$, $f''(\cdot) < 0$, $f'(0) = \infty$ and $f'(\infty) = 0$.

First-period production of a firm born at time $t$ is

$$y_{jt} = \theta_{jt}(k_{jt} - i_{jt})$$

and second-period output is

$$y_{jt+1} = \theta_{jt+1}k_{jt+1}.$$ (2.3)

At the end of the second period, the firm exits. Note that $y_{jt}$ and $y_{jt+1}$ can be thought of as the cash flow generated by the firm during its life. This cash flow is shared between the manager and the firm’s owner, as specified by the contract analyzed below.

From now on, for notational convenience, we focus on symmetric equilibria where all new firms have the same amount of knowledge capital, $k_{jt} = k_t$, and we omit the $j$ index when this causes no confusion. In Section 3.4, we consider cross-sectional implications of our model when $k_{jt}$ is instead drawn from a non-degenerate distribution.9

### 2.2. Contractual environment

We now discuss the contractual frictions present in this economy. First, we assume that the market for managers is not competitive: due to search frictions, managers are assigned randomly. In particular, the owner can draw at most one manager each period and this draw can be taken either from the pool of young or old managers. Given that hiring an old manager to run a new firm is never optimal, the manager of a starting firm is randomly selected from the population of young.10 Once the match is formed, a potential surplus is created. Second, the contracts available to regulate the division of this surplus are incomplete in that they cannot be made contingent on third parties, such as a court. Third, we allow the parties to commit to verifiable actions and transfers. As a consequence, managerial contracts may specify the duration of the contract, i.e. the commitment to keep a young manager for one or two periods $A = \{1, 2\}$, a severance pay $s_t$ to the manager in case the match is broken and fixed wages to be paid in the first and second period, $\bar{w}_t$ and $\bar{w}_{t+1}$.11 Finally, we allow for transfers between the owner and the manager by assuming that agents can borrow or lend at the interest rate $1/\beta$. However, as it is standard in many models of inefficient management, we assume that ownership of $k_t$ cannot be traded.12

Once a match is formed, given that production requires both the manager and the knowledge capital of the owner, both parties have monopoly power in the determination of contractual elements. We therefore assume that all the contractual conditions defined by the tuple $(A, \bar{w}_t, s_t, \bar{w}_{t+1})$ are set through generalized Nash bargaining. We first characterize

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7 A tail index greater than two is required for the variance of the distribution to be finite. In this case, the variance is $\frac{\alpha^3}{(\alpha - 1)(\alpha - 2)}$. Note that a higher value of $\alpha$ corresponds to a lower variance.
8 The assumption that managers do not initially know their own ability simplifies the analysis by making all agents ex-ante identical and avoids the complications due to bargaining with asymmetric information.
9 To avoid corners, we assume that $k_0 > f^{-1}(1/\beta)$.
10 As it will be clear, old managers have no incentives to invest.
11 Fixed elements of the contract are perfectly enforceable. It can also be shown that they are equivalent to ex-ante transfers.
12 The existence of credit markets is not crucial, but it simplifies the analysis. The absence of a market for control rules out a “Coasian solution” whereby the manager buys the firm. This assumption is not difficult to justify, given that $k_t$ is likely to include ideas and other intangibles that are difficult to sell.
the equilibrium for a given bargaining power of the manager, equal to $\lambda \in (0, 1)$. In Section 4, we recognize that institutional variables and policies, such as measures of investor protection, may affect the power of managers and we study the incentives that a government may have to adopt them.

Given our assumptions on information, managerial ability is unknown to the parties when the contract is signed, but it is perfectly revealed once output materializes. This implies that the owner and the manager may have an interest to renegotiate over the division of the surplus and, if the contract allows it, the continuation of the match.\textsuperscript{13}

In the following sections, we analyze long- and short-term contracts separately. In particular, we derive the \textit{ex-post} payoffs in both periods, the optimal choices of investment and managerial retention (under short-term contracts) and the expected \textit{ex-ante} joint surpluses. We then compare the alternative contractual arrangements to the first-best benchmark that would arise in the absence of frictions. Finally, we study how the equilibrium choice of contracts changes with parameters and with the accumulation of knowledge capital.

\subsection*{2.3. Long-term contracts}

Consider a long-term contract defined by the tuple $\langle \Lambda, \bar{w}_t, \bar{w}_{t+1} \rangle$, where $\Lambda = 2$ is the time length of the relationship (two periods), and $\bar{w}_t$ and $\bar{w}_{t+1}$ are the fixed payments to the manager (to be solved later). Although the relationship lasts for two periods, at the end of each period the owner and the manager may have an interest to renegotiate over managerial compensation. If an agreement over the division of the surplus is not reached, current-period production is lost. To find the outcome of this \textit{ex-post} renegotiation, denoted as $(w_t, w_{t+1})$, we solve the bargaining problem by backward induction.

Consider the problem at $t + 1$. In case of agreement, the manager obtains $w_{t+1}$ and the owner $y_{t+1} - w_{t+1}$. In case of disagreement, the entire output is lost, but the owner is still bound to fulfill the payment to the manager specified in the contract, $w_{t+1}$. Thus, $\bar{w}_{t+1}$ works as an outside option for the manager. The bargained wage, $w_{t+1}$, satisfies:

$$w_{t+1} = \arg\max(w_{t+1} - \bar{w}_{t+1})^\lambda (y_{t+1} - w_{t+1} + \bar{w}_{t+1})^{1-\lambda}$$

where $\lambda$ is the bargaining power of the manager. The first-order condition implies:

$$w_{t+1} - \bar{w}_{t+1} = \lambda s_{t+1}$$

where $s_{t+1} = y_{t+1}$ is the total surplus from the agreement at $t + 1$. Thus:

$$w_{t+1} = \bar{w}_{t+1} + \lambda y_{t+1}, \quad (2.4)$$

that is, the manager obtains her outside option, plus a share of the surplus equal to her bargaining power, $\lambda$.

The problem at $t$ is identical. Following the same logic, yields:

$$w_t = \bar{w}_t + \lambda y_t. \quad (2.5)$$

With these \textit{ex-post} compensations at hand, we can solve for investment at time $t$, which is chosen by the manager so as to maximize the expected utility:

$$i_t = \arg\max[\mathbb{E} w_t + \beta \mathbb{E} w_{t+1}] = \arg\max\left\{\lambda \theta (k_t - i_t) + \beta \lambda \theta [k_t + f(i_t)] + \bar{w}_t + \beta \bar{w}_{t+1}\right\},$$

where $\mathbb{E}$ denotes the unconditional expectation at time $t$, and we substituted (2.4), (2.5), (2.2) and (2.3). Investment under long-term contracts, $i^L$, satisfies the following first-order condition:

$$\beta f'(i^L) = 1. \quad (2.6)$$

Note that $\bar{w}_t$ and $\bar{w}_{t+1}$ affect neither investment nor the joint surpluses. Yet, they affect the division of the surplus between the owner and the manager and are chosen through bargaining. In particular, upon forming the match and before signing a long-term contract, the parties must negotiate the initial contractual conditions. At this stage, in case of disagreement both parties can walk away so that the match is broken. In this event, the manager will remain unmatched forever and hence her outside option is zero.\textsuperscript{14} The firm owner, on the other hand, can start the firm in $t + 1$ with a new randomly drawn manager, thus his outside option is $\beta (1 - \lambda) \theta k_t$.

The expected joint surplus generated by a long-term contract at time $t$, denoted as $V^L(k_t)$, is:

$$V^L(k_t) = \mathbb{E} y_t + \beta \mathbb{E} y_{t+1} - \beta (1 - \lambda) \theta k_t = \theta (k_t - i^L) + \beta \theta [k_t + f(i^L)] - \beta (1 - \lambda) \theta k_t, \quad (2.7)$$

and the terms $(\bar{w}_t, \bar{w}_{t+1})$ are chosen so as to give a share $\lambda V^L(k_t)$ to the manager. As shown in Appendix A, this yields $\bar{w}_t + \beta \bar{w}_{t+1} = -\beta \lambda (1 - \lambda) \theta k_t$.

\textsuperscript{13} Note that bargaining takes place to split a surplus each time it is created, namely, when the contract is signed, and after production in each period.

\textsuperscript{14} Note that old managers are never rehired. If all contracts are long-term, then the only demand for managers comes from new firms who only hire from the pool of young. If some contracts are short-term, all firms still prefer to hire from the pool of young because the average ability of the pool of old is lower due to selection. This is always feasible because managers are twice as many as firms.
2.4. Short-term contracts

Consider now a short-term contract, defined by the tuple \((\Lambda, \bar{w}_t, s_t, \bar{w}_{t+1})\), where \(\Lambda = 1\) grants the option to replace the manager with a new random draw at \(t + 1\). \(w_t\) and \(\bar{w}_{t+1}\) are the fixed compensations and \(s_t\) a severance pay to the manager if the match is broken (to be solved later).

At time \(t + 1\), the bargaining problem between the owner and a manager (either a confirmed manager or a new random match) is identical to the case with long-term contracts. Thus:

\[
\bar{w}_{t+1} = \bar{w}_{t+1} + \lambda y_{t+1}.
\]

The problem at the end of period \(t\) is more complicated because the agents bargain both over the wage and the continuation of the match, knowing all the future payoffs (recall that at this stage ability is known). Note that, absent any contractual commitment, any agent is free to walk away so that the match is broken if no agreement is reached. In this case, the surplus from agreeing is higher than the value of current-period production, \(y_t\), whenever the parties have an incentive to continue the match at \(t + 1\). To see this, consider an agreement specifying a compensation of \(w_t\) and the confirmation of the manager for the second period. The surplus for the manager is then:

\[
S_t^{hi} = w_t + \beta w_{t+1} - \bar{w}_t - s_t.
\]

The value of agreeing now includes both \(w_t\) and the discounted expected payment at \(t + 1\), while the outside option is the fixed compensation \(\bar{w}_t\) (to be paid in any case) plus any severance pay \(s_t\) specified in the contract.\(^{15}\) Consider now the value for the owner. If the negotiation fails, \(y_t\) is lost and the payments \(\bar{w}_t + s_t\) are due. At \(t + 1\), the manager is replaced with a new random draw and the owner expects to obtain \((1 - \lambda)\theta k_{t+1}\), where \(\theta\) is the population average.\(^{16}\) In case of agreement, instead, the owner obtains \(y_t - \bar{w}_t\) in the first period and \((1 - \lambda)\theta k_{t+1} - \bar{w}_{t+1}\) in the second. Thus, his surplus is:

\[
S_t^{lo} = y_t - \bar{w}_t + \bar{w}_t + s_t + \beta \left[ (1 - \lambda)(\theta_t - \theta)k_{t+1} - \bar{w}_{t+1} \right].
\]

If instead the agents agree that the manager is to be replaced, then the payoffs at \(t + 1\) do not affect any bargaining position and the surpluses from the agreement are:

\[
S_t^{hi} = w_t - \bar{w}_t ,
\]

\[
S_t^{lo} = y_t - w_t + \bar{w}_t.
\]

In sum the joint surplus, after substituting \(w_{t+1} = \bar{w}_{t+1} + \lambda y_{t+1}\), is:

\[
S_t = \begin{cases} 
    y_t + \beta k_{t+1} [(\theta_t - \theta) + \theta \lambda] & \text{if the manager is confirmed,} \\
    y_t & \text{if the manager is replaced.}
\end{cases}
\]

Note that \(\beta k_{t+1} (\theta_t - \theta)\) is the surplus from keeping a manager of ability \(\theta_t\) (due to its effect on \(y_{t+1}\)) and \(\beta k_{t+1} \theta \lambda\) is the private surplus that the manager derives from being confirmed, which is proportional to her bargaining share. As before, the compensation \(w_t\) is chosen to maximize the Nash product \((S_t^{hi})^\lambda (S_t^{lo})^{1-\lambda}\) and must satisfy the first-order condition \(S_t^{lo} = \lambda S_t\). Substituting \(S_t^{lo}\) we obtain:

\[
w_t = \begin{cases} 
    \bar{w}_t + s_t - \beta w_{t+1} + \lambda S_t & \text{if the manager is confirmed,} \\
    \bar{w}_t + \lambda S_t & \text{if the manager is replaced.}
\end{cases}
\]

Note that the manager is willing to give up \(\beta w_{t+1}\) to keep her job.

Next, we need to find the condition for confirming a manager. Given that the Nash bargaining solution is efficient (i.e., it has to be on the Pareto frontier of \(S_t\)), the continuation of the match is decided so as to maximize \(S_t\). From (2.8), it follows immediately that the manager is retained for the second period if:

\[
\theta_t \geq (1 - \lambda)\theta.
\]

There are two important features of this retention rule. First, the manager is replaced when her ability, \(\theta_t\), is sufficiently below the population average, \(\theta\). The reason why a below-average manager may still be confirmed is that her private benefit from staying (the positive rents at \(t + 1\)) may be higher than the cost of keeping her for the owner. In that case, the manager is willing to compensate the owner for staying. The higher the bargaining power of the manager, the stronger her willingness to compensate the owner and the higher her probability to be reappointed. In other words, powerful managers are harder to replace.

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\(^{15}\) Recall that the outside option of an unmatched old manager is zero.

\(^{16}\) Note that the fixed wage of a new manager running an old firm has already been set to its equilibrium value of zero. The reason is that the owner is assumed to commit to this wage only when signing the new contract with the new manager. At that stage, the equilibrium fixed wage is indeed zero.
Second, all the contractual terms, \( \bar{w}_{t+1}, \bar{w}_t \) and \( s_t \), do not affect the condition for firing a manager, because they do not affect the total surplus, only its distribution. In particular, as in Lazear (1990), the severance pay does not affect the firing condition because it acts as a pure transfer: the manager is certain to get it either by being laid off, or as part of the negotiated compensation, \( w_t \).

We define the probability that a manager is retained as:

\[
p \equiv \Pr[\theta_t \geq (1 - \lambda)\theta].
\]

Using the properties of the ability distribution, the next lemma expresses this probability as a function of parameters.

**Lemma 1.** If \( \lambda < 1/\alpha \), then

\[
p = \left( \frac{\alpha - 1}{(1 - \lambda)\alpha} \right)^{\alpha},
\]

with \( \frac{\partial p}{\partial \alpha} > 0 \) and \( \frac{\partial p}{\partial \lambda} > 0 \). Moreover, \( 0.25 < p < 1 \). If \( \lambda \geq 1/\alpha \), then \( p = 1 \).

**Proof.** See Appendix A. \( \square \)

As already noted, a higher bargaining power, \( \lambda \), increases the probability that the manager is confirmed. Moreover, given that \( \alpha \) is an inverse measure of dispersion, the confirmation probability decreases with more heterogeneity in ability.\(^{17} \)

Notice further that whenever \( \lambda < 1/\alpha \), short-term contracts entail selection: the least ability managers are replaced with new draws, so that the ability distribution in continued matches is truncated below \( (1 - \lambda)\theta \). As a result, the average ability of confirmed managers is higher than the population average. We denote this difference, that is, the ex-ante expected ability premium of a confirmed manager, by \( \delta \):

\[
\delta \equiv \mathbb{E}[\theta_t \mid \theta_t \geq (1 - \lambda)\theta] - \theta.
\]

Using the properties of the Pareto distribution, the next lemma characterizes the determinants of the selection premium.

**Lemma 2.** If \( \lambda < 1/\alpha \), then

\[
\delta = \left( \frac{1 - \alpha \lambda}{\alpha - 1} \right)\theta > 0,
\]

with \( \frac{\partial \delta}{\partial \alpha} < 0 \) and \( \frac{\partial \delta}{\partial \lambda} < 0 \). If \( \lambda \geq 1/\alpha \), then \( \delta = 0 \).

**Proof.** See Appendix A. \( \square \)

Note that selection is more effective (high \( \delta \)) when ability is more dispersed (low \( \alpha \)). Intuitively, when talent is very concentrated, there is little to gain in confirming a manager, even when she is of above average ability. Selection is also stricter when the manager has a lower bargaining power (low \( \lambda \)) because, from (2.9), this implies a higher threshold for confirmation. If \( \lambda > 1/\alpha \), managers are so powerful that they can always compensate the owner for keeping them, even if they have the lowest ability. In this case, there is no selection. In the remaining of the paper, we focus on the more interesting case when \( \lambda < 1/\alpha \).

The selection premium is realized with probability \( p \). Thus, the ex-ante expected benefit of selection is \( p\delta \). The next corollary shows that the benefit of selection increases with more ability heterogeneity (lower \( \alpha \)) and decreases in \( \lambda \) (lower threshold for confirmation).

**Corollary 1.** The expected benefit of selection, \( p\delta \), is increasing in heterogeneity of ability and decreasing in \( \lambda \).

\[
\frac{\partial (p\delta)}{\partial \alpha} < 0; \quad \frac{\partial (p\delta)}{\partial \lambda} < 0.
\]

**Proof.** See Appendix A. \( \square \)

We are now in the position to study the incentives generated by this contract. In particular, the manager chooses \( i_t \) so as to maximize her expected compensation at time \( t \):

\[
i_t = \arg\max \left\{ \bar{w}_t + s_t + \lambda \theta (k_t - i_t) + \beta \bar{p} \lambda (\delta + \lambda \theta) [k_t + f(i_t)] \right\}
\]

\(^{17} \) More dispersion in ability reduces the probability of being confirmed because the Pareto distribution is right-skewed.
where \( p \) and \( \delta \) are given by (2.10) and (2.11) respectively. The associated first-order condition is:

\[
\beta f'(i^S)p(\delta + \lambda \theta) = \theta. \tag{2.12}
\]

The left-hand side of (2.12) is the expected marginal benefit of investment for the manager in terms of higher expected surplus at \( t + 1 \). This is equal to the discounted marginal product of investment multiplied by the probability that the manager will be retained. Notice that the manager recognizes that she will be confirmed when her ability is high enough, which explains the term \( \delta \), but also that she will appropriate only a share of her surplus from staying, which explains the term \( \lambda \theta \). The right-hand side is instead the expected marginal cost of foregone production today. Substituting (2.10) and (2.11), (2.12) becomes:

\[
\beta f'(i^S) = \left(1 - \frac{1 - \lambda}{\alpha - 1}\right) ^{\alpha - 1} \alpha^\alpha.
\]

The next lemma characterizes the determinants of \( i^S \).

**Lemma 3.** Investment under short-term contracts, \( i^S \), is an increasing function of patience, \( \beta \), of ability heterogeneity captured by the inverse of \( \alpha \), and of managerial bargaining power, \( \lambda \):

\[
\frac{\partial i^S}{\partial \beta} > 0; \quad \frac{\partial i^S}{\partial \alpha} < 0; \quad \frac{\partial i^S}{\partial \lambda} > 0.
\]

**Proof.** See Appendix A. \( \square \)

Not surprisingly, investment increases with patience, \( \beta \). More heterogeneity in ability (lower \( \alpha \)) increases investment since it raises the manager’s expected ability conditional on being confirmed in the second period (\( \delta \)) more than it reduces her probability of being kept (\( p \)). An increase in the bargaining power of the manager increases her probability of staying (\( p \)) more than it worsens selection (\( \delta \)), thereby inducing a higher investment.

The sum of the expected surpluses for the two parties generated by this contract at time \( t \), denoted as \( V^S(k_t) \), is:

\[
V^S(k_t) = \mathbb{E}y_t + \beta \mathbb{E}y_{t+1} \left[ 1 + \frac{p \delta}{\theta} - (1 - p) \lambda \right] - \beta(1 - \lambda) \theta k_t
\]

\[
= \theta(k_t - i^S) + \beta \left[ \theta + p \delta - (1 - p) \lambda \theta \right] [k_t + f(i^S)] - \beta(1 - \lambda) \theta k_t. \tag{2.13}
\]

Note that this surplus excludes the value of production appropriated by a new manager in case of replacement at \( t + 1 \). Finally, the optimal fixed compensations and the severance pay, \( \tilde{w}_t, s_t \) and \( \tilde{w}_{t+1} \), are chosen in the initial bargaining stage in such a way that the manager expects to get a share \( \lambda \) of the total surplus. As shown in Appendix A, this requires \( \tilde{w}_t + s_t = \lambda(1 - \lambda) \beta (\mathbb{E}y_{t+1} - \theta k_t) \), while \( \tilde{w}_{t+1} \) is immaterial.

**2.5. The first best**

Before comparing the relative performance of short- and long-term contract, it is useful to characterize the first-best solution that would be attained if investment and output were verifiable (hence contractible), and there were no frictions in the market for managers. Under these conditions, investment \( i^{FB} \) and selection would be chosen so as to maximize the expected present discounted value of the firm, \( W(k_t) \). We refer to this allocation as the “first best” because it corresponds to the natural benchmark of a frictionless economy.\(^{18}\)

In this case, managers would be replaced if their ability turns out to be lower than the average and investment solves\(^{19}\):

\[
\max_{i_t} W(k_t) = \theta(k_t - i_t) + \beta(\theta + p^{FB} \delta^{FB}) \left[ k_t + f(i_t) \right],
\]

where \( \delta^{FB} = \frac{\theta}{\alpha - 1} \) and \( p^{FB} = (\frac{\alpha - 1}{\alpha})^\alpha \). The first-order condition is:

\[
\beta f'(i^{FB}) = \frac{\theta}{\theta + p^{FB} \delta^{FB}} < 1. \tag{2.14}
\]

Comparing, (2.6), (2.12) and (2.14), it is immediate to see that \( i^S < i^S < i^{FB} \). Thus, contract incompleteness implies underinvestment relative to the first-best equilibrium. The reason why investment is inefficiently low under long-term contracts is

\(^{18}\) Note that this allocation corresponds to the one maximizing the expected welfare of the newborn generation. We defer normative considerations to Section 4.

\(^{19}\) Given the existence of a competitive market, the managerial compensation is driven down to zero.
that they exclude the beneficial effect of selection on the return to investment (the value of $k_{t+1}$ is proportional to expected ability at $t+1$). Under short-term contracts, instead, investment is inefficiently low for two reasons. First, the manager does not appropriate the full return from investing due to tenure uncertainty ($p < 1$) and ex-post bargaining ($\lambda < 1$). This inefficiency, captured by the term $p\lambda < 1$ in (2.12), is due to a combination of myopia and a classical hold-up problem. Second, as long as managers have private incentives to keep their position ($\lambda > 0$), there is too little selection ($p\delta < pFB\delta$). In turn, a lower expected ability in case of confirmation reduces the value of investment.

Finally, note that underinvestment is relatively more severe with short-term contracts: given our assumptions, the beneficial effect of selection on $i_t$ is not enough to compensate the appropriability problem. It is also interesting to note that the result $i^S < i^L$ holds even when managers are so powerful that they are never replaced (i.e., $\lambda\alpha \rightarrow 1$, implying $p \rightarrow 1$). In this case, managers know that they will be confirmed. Yet, they refrain from investing as much as with short-term contracts because investing more at the beginning of $t$ weakens their bargaining position at the end of the period.20

3. Equilibrium contracts and economic development

We can now compare the relative performance of short- and long-term contracts and study how the choice of contractual arrangements changes along the process of economic development.

3.1. Equilibrium choice of contracts

The type of contract chosen in equilibrium is the one that maximizes the expected joint surplus at the beginning of time $t$. More precisely, short-term contracts will prevail whenever $V^S(k_t) > V^L(k_t)$. Qualitatively, the major difference between the two types of contracts is that long-term contracts maximize investment, but sacrifice managerial selection. On the contrary, short-term contracts allow to replace bad managers, but the benefit of selection comes at two costs. First, incentives to invest are lower. Second, there is an appropriability problem: short-term contracts generate lower surpluses to the contrary, short-term contracts allow to replace bad managers, but the benefit of selection comes at two costs. First, the effect of selection can potentially be so large as to induce higher investment than with long-term contracts. Yet, the result $i^S < i^L$ is always positive, for an intuitive reason: selection is chosen precisely to maximize the joint surplus.

To this end, it is convenient to define the relative performance (for the initial pair owner–manager) of short-term contract as:

$$\Delta V^{S-L}(k_t) \equiv \frac{V^S(k_t) - V^L(k_t)}{\theta}. \quad (3.1)$$

Short-term contracts will prevail if and only if $\Delta V^{S-L}(k_t) > 0$. Replacing (2.7) and (2.13) into (3.1) and rearranging, we find that $\Delta V^{S-L}(k_t) > 0$ when the following condition holds:

$$[\beta f(i^L) - i^L] - [\beta f(i^S) - i^S] < [k_t + f(i^S)]\beta[p\delta/\theta - (1 - p)\lambda]. \quad (3.2)$$

The left-hand side of (3.2) is equal to the additional surplus from investment generated by long-term contracts. This term is always positive because $i^L$ is chosen precisely to maximize $[\beta f(i) - i]$. The right-hand side is the net benefit of selection: higher expected ability at $t + 1$ (the term $p\delta/\theta$ minus the expected share appropriated by a newly drawn manager (the term $1 - p)\lambda$, times the stock of capital, $k_{t+1}$. As shown in Appendix A, the net benefit of selection (the right-hand side) is always positive, for an intuitive reason: selection is chosen precisely to maximize the joint surplus.

Condition (3.2) holds, i.e., short-term contracts are chosen, when selection is relatively more important than investment. In fact, it holds trivially when there is no investment (e.g., $f(i) = 0$). More interestingly, since investment does not depend on $k_t$, it is immediate to see that capital accumulation makes short-term contracts more attractive and that $\Delta V^{S-L}(k_t) > 0$ if $k_t$ is sufficiently high. The level $k^*$ above which short-term contracts are preferred can be obtained solving $\Delta V^{S-L}(k_t) = 0$:

$$k^* = \left[\frac{\beta p\delta}{\theta} - \beta(1 - p)\lambda\right]^{-1}\left[\left[\beta f(i^L) - i^L\right] - \left[\beta f(i^S) - i^S\right] - f(i^S)\right]. \quad (3.3)$$

Note that $k^*$ can potentially be negative, so that, depending on parameters, an economy may start out with short-term contracts. Yet, $k^* \rightarrow \infty$ as $\alpha\lambda \rightarrow 1$.

The reason why $\Delta V^{S-L}(k_t)$ increases in $k_t$ is that ability becomes relatively more important as the economy grows because it is complementary to the level of technological sophistication captured by knowledge capital. Intuitively, the higher the productive capacity of the economy, the higher the value of selecting talent to operate it. Key to this property is the assumption that managerial ability has a multiplicative effect on technology, as in the majority of models designed to study the effect of managerial quality (e.g., Rosen, 1981, and Gabaix and Landier, 2008). It is also consistent with the

20 The result that $i^S < i^L$ partly depends on properties of the Pareto distribution for ability. Under alternative distributional assumptions, the beneficial effect of selection can potentially be so large as to induce higher investment than with long-term contracts. Yet, the result $i^S < i^L$ will still hold whenever $\delta$ is below some critical value (i.e., when ability is not very dispersed or selection is subject to mistakes). See Bonfiglioli and Gancia (2011) for an example.
large literature on capital-skill complementarity (e.g., Krusell et al., 2000) and skill-biased technical change (e.g., Nelson and Phelps, 1966; Caselli, 1999; Violante, 2002, and Acemoglu et al., 2012).

Moreover, greater heterogeneity in managerial ability, as captured by a lower $\alpha$, makes short-term contacts relatively more efficient because it increases the selection premium $\delta$, which raises $V^S(k_t)$ both directly and indirectly through the rise in $i^S$. This suggests that more homogeneous societies are more likely to choose long-term contracts. In turn, this result may help explain why relatively rigid production relationships may be common even in some advanced countries where workers are less heterogeneous (an example could be lifetime employment policies in Japan).

Finally, the bargaining power of managers, $\lambda$, does not affect the value of long-term contracts, but has ambiguous effects on the value of short-term contracts. A higher $\lambda$ increases the probability that a manager is retained: this raises investment, but it also lowers the net benefit of selection. Given that the benefit of selection is proportional to $k_t$, the negative effect must prevail for high levels of knowledge capital. In other words and consistently with the previous findings, a lower bargaining power of managers makes contracts more flexible (lower $p$) and this is beneficial provided that $k_t$ is high enough.

All these results are summarized in the following proposition.

**Proposition 1.** $\Delta V^{S-L}(k_t)$ is increasing in knowledge capital and heterogeneity; it is decreasing in managerial power for high enough levels of knowledge:

$$\frac{\partial \Delta V^{S-L}(k_t)}{\partial k_t} > 0; \quad \frac{\partial \Delta V^{S-L}(k_t)}{\partial \alpha} < 0,$$

$$\frac{\partial \Delta V^{S-L}(k_t)}{\partial \lambda} \leq 0 \iff k_t \geq \hat{k}.$$

Moreover, $\Delta V^{S-L}(k_t) > 0$ for $k_t > k^*$ where $k^*$ is given by (3.3).

**Proof.** See Appendix A. \(\square\)

3.2. Dynamics and the transition to short-term contracts

So far, we have characterized the equilibrium for a given starting level of knowledge capital, $k_t$, of a new firm at time $t$. We now describe the dynamic evolution of the economy. In each period $t$, aggregate knowledge is equal to the total stock of knowledge capital of new and old firms:

$$K_t = \int_{j \in S_t} k_{jt} \, dj + \int_{j \in S_{t-1}} \left[ k_{jt-1} + f(i_{jt-1}) \right] \, dj = \left( 1 - \frac{\rho}{2} \right) \left[ K_{t-1} + f(i_{t-1}) \right], \quad (3.4)$$

where $S_t$ denotes the set of new firms at time $t$ and the second line makes use of (2.1). For given $i_{t-1} = i \in [i^S, i^L]$, (3.4) is a difference equation converging asymptotically to the steady state:

$$K_{ss} = \left( \frac{2 - \rho}{\rho} \right) f(i). \quad (3.5)$$

In this steady state, the average starting capital of a new firm is:

$$k_{ss} = \left( \frac{1 - \rho}{\rho} \right) f(i).$$

Note that different types of contracts generate different steady states: $k^S_{ss} < k^L_{ss}$ since $i^S < i^L$. Together with Proposition 1, this implies that whether a country will converge to an equilibrium with short-term contracts will depend on whether $k^S_{ss}$ is above or below $k^*$. As formalized in Proposition 2, there are three cases. If $k^* < k^S_{ss}$, then the economy may start with long-term contracts, but will eventually switch to short-term contracts. If instead $k^* > k^S_{ss}$, then the economy will reach its steady state before switching to short-term contracts. If the region $k^L_{ss} < k^* < k^S_{ss}$, instead, the economy will fluctuate between short- and long-term contracts around $k^*$.

**Proposition 2.** If $k^* < k^S_{ss}$ then the economy converges to a steady state with short-term contracts. If $k^* > k^L_{ss}$ the economy never switches to short-term contracts. If $k^S_{ss} < k^* < k^L_{ss}$, then the economy will fluctuate between short- and long-term contracts in a neighborhood of $k^*$.

**Proof.** With $k^* < k^S_{ss}$, an economy starting with $k_0 < k^*$ will have long-term contracts until it reaches $k_t > k^*$ (due to discreteness), then it will switch to short-term contracts and keep them until it converges asymptotically to $k^S_{ss}$; an economy starting with $k_0 > k^*$ will stay forever with short-term contracts and converge asymptotically to $k^L_{ss}$. With $k^* > k^L_{ss}$; an economy starting with $k_0 < k^*$ will have long-term contracts forever and converge asymptotically to $k^L_{ss}$; an economy starting
with $k_0 \geq k^*$ will stay with short-term contracts until $k_t < k^*$, then it will switch to long-term contracts and keep them until it converges asymptotically to $k^*_{ss}$. With $k^*_{ss} < k^* < k^*_L$, an economy starting with $k_0 < k^*$ will have long-term contracts until $k_t > k^*$, then it will switch to short-term contracts. Given that $k^*_{ss} < k^*$, $k_t$ will fall until $k_t < k^*$, then the economy will shift back to long-term contract and start growing again, and so on in a cycle.

Our model thus predicts that countries starting from a low level of capital may go through an initial phase where long-term production relationships and low managerial turnover prevail. If $k_t$ reaches a critical threshold, however, ability becomes more important and the economy will endogenously switch to flexible short-term contracts. Contractual relationships may thus evolve with economic development as suggested by Kuznets (1966, 1973), Gerschenkron (1962) and North (1994). Yet, countries converging to different steady states, for example because of differences in patience or in the accumulation technology, may end up with persistent differences in contractual arrangements.

3.3. Long-run growth

Finally, note that the model can be extended to make it consistent with (exogenous) long-run growth. For example, we may assume that overall productivity also depends on the evolution of an exogenous world technology frontier, $A_t$, that grows exogenously at the rate $g$. If we modify the production technology to $y_t = A_t \theta(k_t - i_t)$ and we assume that $A_t$ is persistent throughout the life of a firm, then we can obtain the following law of motion for aggregate output:

$$Y_t = A_t \int_{j \in S_t} \theta_{jt}(k_{jt} - i_{jt}) \, dj + A_{t-1} \int_{j \in S_{t-1}} \theta_{jt}[k_{jt-1} + f(i_{jt-1})] \, dj.$$

Moreover, $A_t$ will not affect the ratio $V^S(k_t)/V^L(k_t)$ and therefore the transition to short-term contracts.

Once knowledge capital has converged to its steady-state level $k_{ss}$, the economy will be in a balanced growth path where aggregate output grows at the rate $g$:

$$Y_t = \frac{A_t}{2} \left[ \theta(k_{ss} - i) + \frac{\theta + p \delta}{1 + g} \left( k_{ss} + f(i) \right) \right].$$

Notice that in this model the evolution of contracts is a feature of the transition and not of balanced growth. This is intuitive, given that a key reason why short-term contracts are better at higher level of $k_t$ is the presence of diminishing returns to knowledge accumulation, so that ability becomes relatively more important than accumulation. Yet, diminishing returns to accumulation are a common feature of models designed to explain conditional convergence, i.e., the fact that *ceteris paribus* poor countries tend to grow faster than rich countries, and seem particularly appropriate to study economies at different levels of development.

3.4. Firm heterogeneity and contract choice

So far we have emphasized the implications of the model for cross-country comparisons. By adding heterogeneity across firms and sectors, the model can shed light on cross-industry comparisons as well. To this end, we first relax the assumption that all new firms start with the same level of knowledge capital. In particular, suppose that $k_{jt}$ is observable and it is drawn from a non-degenerate distribution, $Z(k)$, with mean equal to $(1 - \rho)$ times the average knowledge capital of existing old firms.

Given that investment does not depend on the level of $k_{jt}$, we still have $i_{jt} = i$ so that the law of motion of the average knowledge capital in the economy is unaffected. Moreover, condition (3.2) and Proposition 1 also hold, meaning that the threshold level of knowledge capital, $k^*$, above which short-term contracts become optimal is the same as before. The difference is that in each period there will be a fraction $Z(k^*)$ of new firms who prefer to choose long-term contracts. Yet, as long as the mean of $Z(k)$ grows over time with knowledge accumulation, the fraction of firms below $k^*$ will fall. Thus, the main new prediction is a smooth transition along which flexible contracts are first chosen by the most productive firms and then gradually adopted by the others.

Following the same logic, we can also assume that firms are grouped into different sectors, indexed by $i$, each characterized by possibly different investment technologies, types of relevant skills and managerial power. In this case, investment will be sector-specific, but it is straightforward to see that the general properties of the model will still hold. Introducing this additional dimensions of heterogeneity allows us to obtain both cross-firm and cross-industry predictions. In particular, the modified model suggests that rigid contractual relationships should tend to prevail among small and less productive firms, in more traditional sectors where skills matter less or where the relevant skills are more homogeneous. Whenever short-term contracts are chosen, selection will also be tougher in sectors and firms where the managers have lower bargaining power.
4. Appropriate contracts and institutions

We now study some normative implications of the model. Consistently with Section 2.5, we adopt as a welfare criterion the maximization of the value of new firms. As already mentioned, this corresponds to maximizing the expected welfare of the newborn generation.\footnote{In doing so, we abstract from intergenerational externalities due to knowledge spillovers from old to new firms, because they are not the main focus of the paper. Moreover, in an OLG setup, the effect of these externalities on social welfare would depend on arbitrary weights assigned to future generations.} First, we compare the equilibrium choice of contracts derived in Section 3.1 with one that maximizes the expected value of a new firm and ask whether the switch to short-term contracts occurs too early or too late. Next, we ask whether policies of investor protection aimed at limiting the bargaining power of managers may improve the allocation. In particular, we characterize the level of $\lambda$ that maximizes the expected value of new firms and study how it varies with economic development.

4.1. Optimal choice of contracts

We take as given the outcomes generated by different contracts (from Sections 2.3 and 2.4) and characterize the level of knowledge capital $k^{**}$ above which the expected value of a new firm (rather than the surplus for the owner-manager match) is higher with short-term contracts. To do so, recall that the expected value of a new firm is:

$$W(k_t) = \theta(k_t - i_t) + \beta(\theta + p)\delta[k_t + f(i_t)].$$

To find the expected value with long-term contracts, $W^L$, we simply substitute $i^L$ from (2.6), $p = 1$ and $\delta = 0$ into $W$. Likewise, we obtain the expected value with short-term contracts, $W^S$, by substituting $i^S$, $p$ and $\delta$ from (2.12), (2.10) and (2.11), respectively. Short-term contracts maximize the value of a new firm (i.e., $W^S > W^L$) when the following condition is satisfied:

$$\left[\beta f(i^S) - i^S\right] < \left[k_t + f(i_t)\right]\frac{\delta}{\beta}.$$ \hspace{1cm} (4.1)

Comparing (4.1) to (3.2), we see that the left-hand side, i.e., the cost of underinvestment with short-term contracts, is the same in both conditions. The right-hand side, i.e. the benefit from selection, is instead higher in (4.1) than in (3.2). The reason is the appropriability problem: the expected joint surplus for the manager and owner at time $t$, $V^S$, is lower than $W^S$ because it does not include the share that may go to a new manager in case of replacement. This implies that, for $W$ to be maximized, the transition to short-term contracts should occur at a lower level of capital. This can be seen by finding the value $k^{**}$ such that $W^S = W^L$ (i.e., solving (4.1) as an equality) and comparing it to (3.3):

$$k^{**} = \theta \left[\beta f(i^L) - i^L\right] - \left[\beta f(i^S) - i^S\right] - f(i^S) < k^*.$$ \hspace{1cm} (4.2)

In terms of policy implications, this result suggests that a government aiming at maximizing the value of new firms may want to impose more flexible labor market policies as soon as the economy reaches a sufficient level of development ($k_t \geq k^{**}$).

4.2. Optimal investor protection

Next, we characterize the level of managerial power, $\lambda \in (0, 1/\alpha)$, that maximizes the expected value of new firms with short-term contracts.\footnote{Of course, $W^L$ does not depend on $\lambda$.} Recall that a higher $\lambda$ induces lower managerial turnover. This means worse selection, but also more investment. Thus, $\lambda$ poses a trade-off. To characterize the optimal $\lambda$ for the firm, we maximize $W^S$ subject to (2.10), (2.11) and (2.12). Moreover, $\lambda$ is constrained to be within the admissible range $(0, 1/\alpha)$.

The first-order condition for $\lambda$ is:

$$\left[\beta f'(i^S)(\theta + p) - \beta\right]\frac{\partial i^S}{\partial \lambda} = \lambda \beta (\delta p)\delta k_t + f(i^S)).$$ \hspace{1cm} (4.3)

The right-hand side is the marginal benefit of increasing $\lambda$, through the rise in investment that it generates ($\partial i^S/\partial \lambda > 0$). The right-hand side is the marginal cost of worse selection ($\partial (\delta p)/\partial \lambda < 0$), which is proportional to the capital operated by the manager. After substituting (2.10), (2.11) and (2.12), the first-order condition for $\lambda$ can be expressed as:

$$\frac{(\alpha - 1)(1 - \rho \lambda)}{p} \frac{\partial i^S}{\partial \lambda} = \lambda \alpha p \beta [k_t + f(i^S)].$$ \hspace{1cm} (4.4)

Note that the right-hand side (the marginal cost) is increasing in $\lambda$ (recall that both $p$ and $i^S$ are increasing in $\lambda$). Under mild conditions, the left-hand side (the marginal benefit) is decreasing in $\lambda$ and there is a unique interior solution, $\lambda^*$ (see Appendix A for a formal proof).
How does $\lambda^*$ change with economic development? Given that the marginal cost of $\lambda$ is increasing in knowledge capital, $\lambda^*$ is necessarily decreasing in $k_t$. This result is consistent with the literature on incomplete contracts showing that ex-ante efficiency requires a higher bargaining power to be allocated to the party that makes the most important task (e.g., Hart, 1995). In the context of the present model, we can think of investment as a task performed by managers and selection as a task performed (partly) by firm owners. When knowledge capital is high, selection is more important than investment. In this case, owners should be granted more protection so as to make contracts more flexible. Thus, the model suggests that institutions granting more protection to investors and owners (low $\lambda$) are not very useful (may even be detrimental) during the initial stages of development, while they become more important at later stages.

We summarize all these policy prescriptions in Proposition 3.

**Proposition 3.** For $k_t < k^{*\ast}$, where $k^{*\ast}$ is defined in (4.2), the expected value of new firms is maximized with long-term labor contracts and no corrective policy intervention is required. For $k_t \geq k^{*\ast}$, the optimal policy is to impose short-term labor contracts and simultaneously set measures of investor protection so as to have $\lambda = \lambda^*$ where $\lambda^*$ is defined in (4.3). For $k_t > k^{*\ast}$, investors should be granted stronger protection (i.e., lower $\lambda^*$) as $k_t$ grows.

**Proof.** See Appendix A. □

5. **Empirical predictions and the evidence**

We now discuss some of the empirical implications of the model and compare them with the existing evidence. A key premise of our theory is that short-term contracts maximize selection at the expenses of investment. This basic trade-off is consistent with the findings reported in Aghion et al. (2013) that increased institutional ownership is positively correlated with innovation and negatively correlated with the incidence of performance-driven replacement of managers in a panel of U.S. firms.

Next, the model predicts that the intensity of selection, i.e., the likelihood that bad performance leads to managerial turnover (1 − $p$), should be higher in larger firms (high $k_t$) and in firms and countries subject to stronger investor protection and better corporate governance (lower $\lambda$). These implications are broadly consistent with both cross-firm and cross-country studies on the determinants of managerial turnover. Among these, Zhou (2000) provides evidence that large Canadian firms are more likely to terminate their CEO after bad performance than small firms. Using data for approximately 38,000 firms from 59 countries, Le and Miller (2008) show that firms from weak investor protection regimes that are cross-listed on a major U.S. Stock Exchange, subject to severe disclosure requirements, are more likely to terminate poorly performing CEOs. The same is not true for firms that cross-list in the London Stock Exchange, which has less severe requirements. Using a sample of 21,483 firm-year observations in 33 countries during the period 1997 through 2001, De Fond and Hung (2004) show that CEO termination is more performance sensitive in countries with better corporate governance.

Interestingly, a recent paper by Bloom and Van Reenen (2010) provides evidence on how managerial practices differ across the world. Their unique dataset is obtained from a survey conducted on about 6000 firms from 16 developed and emerging economies and can be used to correlate the value assigned by firms to selection with country-level indicators of investor protection. To measure the value of selection, we take the average score of the answers to three questions on the extent to which bad performance or failing the objectives leads to the removal of managers and on the determinants of promotions.24 A higher score means that bad-performing managers are more likely to be removed and that promotion is based on performance. As expected, the correlation between the average score and the indicator of shareholders protection by Doing Business (see La Porta et al., 2008) is positive and high (0.684 with a $p$-value of 0.002).

Finally, the most important prediction of the model is that the effect of short-term contracts and selection on economic performance depends on the level of development: it may be negative in countries far from their steady state, but may turn positive in more advanced economies. Given that investor protection affects the extent of selection, its impact should therefore depend on the level of development in the same way. Testing these predictions poses an obvious difficulty: contracts and policies are endogenous and, as our model suggests, they may adjust optimally along the development path. Thus, the mere correlation between measures of contract flexibility, investor protection, and growth is not informative of any causal effect.

Fortunately, however, an extensive empirical literature has identified plausibly exogenous sources of cross–country variation in institutional factors driven by colonial history. In particular, La Porta et al. (2008) document that today's legal rules protecting investors vary systematically with legal traditions that were introduced through conquest and colonization. Thus, legal origin can be used as an instrument to identify the causal effect of investor protection on economic growth.

In a seminal paper, Acemoglu and Johnson (2005) show that legal origins have no impact on economic development after controlling for property rights institutions. Our paper can help to shed light on this finding. The reason is that in our theory the effect of investor protection is expected to depend on the level of development of a given country: although the

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23 Unfortunately, we cannot establish the effect of $\alpha$ on $\lambda^*$ analytically. Yet, numerical simulations suggest that more heterogeneity in ability (low $\alpha$) implies a lower $\lambda^*$.

24 In the dataset, these questions are called talent3, perf5 and talent4. We took a simple average.
average impact may well be insignificant, the effect should become positive and grow bigger in more developed countries. This is indeed the finding reported in Aguirre (2012). Investor protection, once appropriately instrumented for, has a positive impact on GDP growth only in advanced economies, while it has none in developing countries.  

6. Conclusions

In this paper, we have built a simple growth model where economic success requires both incentives to undertake investments that pay out in the future and managerial selection. Investment is relatively more important at early stages of development, when productive capacity is low. It is then optimal to choose long-term contracts that maximize the incentives to invest, even at the cost of no managerial selection. As knowledge capital grows, however, ability becomes more important and the economy endogenously switches to short-term contracts that maximize managerial talent, even at the cost of some underinvestment. Measures of investor protection, by reducing the power of managers, increase the extent of selection of short-term contracts. For this reason, they become more beneficial as a country approaches its steady state.

The results in this paper have been obtained with the help of a stylized model that abstracts from several potentially interesting issues. First, following the incomplete contract literature, we have excluded contracts contingent on production due to a lack of verifiability, but we have abstracted from commitment problems. That is, we have assumed that legal enforcement is imperfect, but sophisticated enough to make the choice of one or two period contracts binding. This seems a reasonable compromise. Yet, it is worth noticing that, in the absence of commitment, rigid contracts would be harder to implement due to a time-consistency problem. Even if owners would like to promise reappointment ex-ante, they may want to deviate once ability is learned. Still, the effect of managerial power on contract flexibility would remain and so the basic trade-off between investment and selection.

Second, in the interest of clarity, we have assumed that information is symmetric and that ability is fully revealed after one period. In Bonfiglioli and Gancia (2011), we study the implications of relaxing both assumptions in a slightly different model. Adding noise to the learning process affects the quality of selection and therefore reduces the benefit of short-term contracts. Asymmetric information may have similar effects: if owners can only observe current economic performance and not investment, managers will have an incentive to give up some long-term investment in favor of activities with an immediate payoff in an effort to manipulate the perception of their ability and hence increase the probability of being retained. In both cases, the result is that more information frictions slow down the convergence to short-term contracts.

Third, endogenizing the ability distribution may open the door to multiple equilibria and development traps. The reason is that with long-term contracts ability is less important so that managers may have a lower incentive to invest in activities, such as education, that could increase talent. At the same time, this may lead to a more compressed ability distribution that in turn justifies the adoption of long-term contracts. This may help explain why some countries appear to be trapped in a no-selection, low-human capital equilibrium.

Fourth, we believe that the basic trade-off between investment and selection is likely to be present in several different contexts that are worth studying. For example, in Bonfiglioli and Gancia (forthcoming) we explore the implications of this trade-off in a political-economy model where an incumbent politician subject to electoral uncertainty must choose how to deviate once ability is learned. Still, the effect of managerial power on contract flexibility would remain and so the basic trade-off between investment and selection.

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Appendix A

A.1. Determination of \((\bar{w}_t, s_t, \bar{w}_{t+1})\)

The constant terms of the contract, \(\bar{w}_t\), \(s_t\) and \(\bar{w}_{t+1}\), are determined through Nash bargaining at the beginning of period \(t\). At this stage, the outside option of the manager is zero, while the owner can run the firm in \(t + 1\) with a new manager and thus faces an outside option of \(\beta(1 - \lambda)\theta k_t\).  

\[ A.2 \quad A(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ B.2 \quad B(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ C.2 \quad C(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ D.2 \quad D(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ E.2 \quad E(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ F.2 \quad F(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ G.2 \quad G(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ H.2 \quad H(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ I.2 \quad I(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ J.2 \quad J(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ K.2 \quad K(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ L.2 \quad L(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ M.2 \quad M(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ N.2 \quad N(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ O.2 \quad O(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ P.2 \quad P(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ Q.2 \quad Q(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ R.2 \quad R(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ S.2 \quad S(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ T.2 \quad T(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ U.2 \quad U(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ V.2 \quad V(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ W.2 \quad W(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ X.2 \quad X(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ Y.2 \quad Y(\bar{w}_t, s_t, \bar{w}_{t+1}) \]

\[ Z.2 \quad Z(\bar{w}_t, s_t, \bar{w}_{t+1}) \]
Under long-term contracts, the expected surpluses for the manager and the owner are, respectively:

\[ E S^m_t = \bar{w}_t + \lambda \bar{E}y_t + \beta \bar{w}_{t+1} + \beta \lambda \bar{E}y_{t+1} , \]

\[ E S^o_t = (1 - \lambda) \bar{E}y_t - \bar{w}_t + \beta (1 - \lambda) \bar{E}y_{t+1} - \beta \bar{w}_{t+1} - \beta (1 - \lambda) \theta k_t , \]

where \( \bar{E} \) denotes unconditional expectations at time \( t \). The optimal \( \bar{w}_t \) and \( \bar{w}_{t+1} \) maximizing \( (E S^m_t )^\lambda (E S^o_t )^{1-\lambda} \) satisfy \( \lambda (E S^o_t ) = (1 - \lambda) (E S^m_t ) \), which implies:

\[ \bar{w}_t + \beta \bar{w}_{t+1} = -\beta \lambda (1 - \lambda) \theta k_t . \]

Under short-term contracts, expected surpluses are:

\[ E S^m_t = \lambda \bar{E}y_t + \bar{w}_t + s_t + \beta p \lambda \left( \frac{\delta}{\theta} + \lambda \right) \bar{E}y_{t+1} , \]

\[ E S^o_t = (1 - \lambda) \bar{E}y_t - \bar{w}_t - s_t + \beta (1 - \lambda) \left( 1 + p \lambda + p \frac{\delta}{\theta} \right) \bar{E}y_{t+1} - \beta (1 - \lambda) \theta k_t . \]

The optimal \( \bar{w}_t \) and \( s_t \), maximizing \( (E S^m_t )^\lambda (E S^o_t )^{1-\lambda} \) satisfy \( \lambda (E S^o_t ) = (1 - \lambda) (E S^m_t ) \), which implies:

\[ \bar{w}_t + s_t = \lambda (1 - \lambda) \beta (\bar{E}y_{t+1} - \theta k_t ) . \]

A.2. Proof of Lemma 1

Deriving (2.10) with respect to \( \lambda \) yields:

\[ \frac{\partial p}{\partial \lambda} = \frac{\alpha p}{1 - \lambda} > 0 . \]

If \( \lambda > 1/\alpha \), then \( (1 - \lambda) \theta = \frac{(1 - \lambda) \theta}{\alpha - 1} < \theta \). Thus, the minimum ability to be confirmed is lower than the lowest ability, hence \( p = 1 \).

Deriving (2.10) with respect to \( \lambda \) yields:

\[ \frac{\partial p}{\partial \alpha} = p \left( \frac{1}{\alpha - 1} + \frac{1}{\alpha} \ln p \right) . \]

Note that \( \ln p \) is always negative and increasing in \( \lambda \). For the minimum value of \( \lambda \to 0 \), the expression in parenthesis becomes

\[ \frac{1}{\alpha - 1} + \ln \left( \frac{\alpha - 1}{\alpha} \right) . \]

which is positive since it is decreasing in \( \alpha \) (the derivative being \(-\alpha^{-1}(\alpha - 1)^{-2}\)) with a minimum of zero for \( \alpha \to \infty \). Therefore,

\[ \frac{\partial p}{\partial \alpha} > 0 . \]

Moreover, with \( 0 < \lambda < 1/\alpha \), the fact that \( p \) is increasing in \( \lambda \) and \( \alpha \) implies that the lower bound of \( p \) corresponds to \( \lambda \to 0 \) and \( \alpha \to 2 \). Thus, \( p > 0.25 \).

A.3. Proof of Lemma 2

Substituting \( \theta = \frac{\alpha}{\alpha - 1} \) into (2.11) yields:

\[ \delta = \frac{1 - \alpha \lambda}{\alpha - 1} \frac{\alpha}{\alpha - 1} - \theta > 0 \]

because \( \lambda < 1/\alpha \). The derivative of \( \delta \) w.r.t. \( \alpha \) is

\[ \frac{\partial \delta}{\partial \alpha} = \theta \frac{2 \lambda \alpha - 1 - \alpha}{(\alpha - 1)^3} . \]

Since \( \lambda < 1/\alpha \):

\[ \frac{\partial \delta}{\partial \alpha} < \theta \frac{1 - \alpha}{(\alpha - 1)^3} = -\frac{1}{(\alpha - 1)^2} \theta < 0 . \]

The derivative of \( \delta \) w.r.t. \( \lambda \) is:

\[ \frac{\partial \delta}{\partial \lambda} = -\left( \frac{\alpha}{\alpha - 1} \right)^2 \theta < 0 . \]
A.4. Proof of Corollary 1

Using (2.10) and (2.11) and \( \theta \) yields:

\[
p\delta = \left( \frac{\alpha - 1}{(1 - \lambda)\alpha} \right)^\alpha \frac{(1 - \alpha \lambda)\alpha}{(\alpha - 1)^2} \theta.
\]

Then:

\[
\frac{dp\delta}{d\alpha} = -\theta p \left(\alpha - 2\alpha\lambda + 1 - (1 - \alpha\lambda)(\alpha - 1)\ln p \right) \frac{1}{(\alpha - 1)^3} < 0
\]

since \( \lambda < 1/\alpha \), \( p < 1 \), and \( \alpha > 2 \).

Next:

\[
\frac{\partial(p\delta)}{\partial\lambda} = \delta \frac{\partial p}{\partial\lambda} + p \frac{\partial \delta}{\partial\lambda} = -\lambda \frac{1}{1 - \lambda} \alpha p \theta < 0.
\]

A.5. Proof of Lemma 3

Recall the first-order condition for investment:

\[
\beta f'(i^S) = \frac{(\alpha - 1)}{p(1 - \lambda)}.
\]

We compute the derivatives of investment w.r.t. to the parameter \( x \) as follows:

\[
\frac{\partial i^S}{\partial x} = -\frac{\partial}{\partial x} \left[ \frac{(\alpha - 1)}{p(1 - \lambda)} - \beta f'(i^S) \right] \frac{\partial}{\partial i^S} \left[ \frac{(\alpha - 1)}{p(1 - \lambda)} - \beta f'(i^S) \right],
\]

where

\[
\frac{\partial}{\partial i^S} \left[ \frac{(\alpha - 1)}{p(1 - \lambda)} - \beta f'(i^S) \right] = -\beta f''(i^S) > 0.
\]

To prove that \( i^S \) is decreasing in \( \alpha \) we compute:

\[
\frac{\partial}{\partial\alpha} \left[ \frac{(\alpha - 1)}{p(1 - \lambda)} - \beta f'(i^S) \right] = -\frac{1}{\alpha} (\ln p) \frac{\alpha - 1}{p(1 - \lambda)} > 0,
\]

which implies \( \frac{\partial i^S}{\partial\alpha} < 0 \).

To show that \( i^S \) is increasing in \( \lambda \) we derive:

\[
\frac{\partial}{\partial\lambda} \left[ \frac{(\alpha - 1)}{p(1 - \lambda)} - \beta f'(i^S) \right] = -\beta f'(i^S) \frac{\alpha - 1}{1 - \lambda} < 0,
\]

which implies \( \frac{\partial i^S}{\partial\lambda} > 0 \).

To prove \( i^S \) increasing in \( \beta \) we compute:

\[
\frac{\partial}{\partial\beta} \left[ \frac{(\alpha - 1)}{p(1 - \lambda)} - \beta f'(i^S) \right] = -f'(i^S) < 0,
\]

which implies \( \frac{\partial i^S}{\partial\beta} > 0 \).

A.6. Proof of Proposition 2

Replace (2.7) and (2.13) into (3.1) to obtain:

\[
\Delta V^{S-L}(k) = \left[ p \left( \frac{\delta}{\theta} + \lambda \right) - \lambda \right] \beta \left[ k_t + f(i^S) \right] - \left[ \beta f(i^S) - i^S \right] - \left[ \beta f(i^S) - i^S \right].
\]

Next, compute:

\[
\frac{\partial \Delta V^{S-L}(k_t)}{\partial k_t} = \left[ p \left( \frac{\delta}{\theta} + \lambda \right) - \lambda \right] \beta = \left[ \frac{(\alpha - 1)^{\alpha - 1}}{(1 - \lambda)^{\alpha - 1} \alpha^\alpha - \lambda} \right] \beta,
\]

where we used (2.11) and (2.10). This derivative is positive iff:
\[
\frac{1}{\alpha} \left(1 - \frac{1}{\alpha} \right)^{\alpha-1} > \lambda (1 - \lambda)^{\alpha-1},
\]

(A.1)

which is always true because \(1/\alpha > \lambda\).

To prove that \(\Delta V^{S-L}(k_t)\) is decreasing in \(\alpha\), it is useful to substitute \(p(\lambda + \lambda)\) from (2.12):

\[
\Delta V^{S-L}(k_t) = \left[ \frac{1}{\beta f'(i^S)} - \lambda \right] \beta [k_t + f(i^S)] - \left\{ \beta f(i^L) - i^L \right\} - \left\{ \beta f(i^S) - i^S \right\},
\]

which shows that \(\Delta V^{S-L}(k_t)\) is increasing in \(i^S\), and therefore decreasing in \(\alpha\).

The derivative of \(\Delta V^{S-L}(k_t)\) w.r.t. \(\lambda\) is

\[
\frac{\partial \Delta V^{S-L}(k_t)}{\partial \lambda} = -\frac{\alpha - 1}{\beta} \frac{\partial i^S}{\partial \lambda} - (1 - p)\beta [k_t + f(i^S)].
\]

A.7. Proof of Proposition 3

From the first-order condition for \(\lambda^*\) (4.4):

\[
\frac{\partial W}{\partial \lambda} = \frac{(\alpha - 1)(1 - p)\lambda}{\beta} \frac{\partial i}{\partial \lambda} - \lambda \alpha p \beta [k + f(i)].
\]

We now verify the second-order condition. Recall that \(\frac{\partial^2 W}{\partial \lambda^2} < 0\), \(\frac{\partial^2 W}{\partial i^2} > 0\). Thus, a sufficient condition for \(\frac{\partial^2 W}{\partial \lambda^2} < 0\) is \(\frac{\partial^2 W}{\partial i^2} > 0\). In turn, it can be shown that a sufficient condition for \(\frac{\partial^2 W}{\partial i^2} < 0\) is \(f''(i) > 0\), which is satisfied, for example, for \(f(i) = i^p\), \(\forall \gamma \in (0, 1)\). This insures uniqueness of \(\lambda^*\). To prove existence, note that:

\[
\lim_{\delta \to 0} \frac{\partial W}{\partial \lambda} = \frac{\partial i}{\partial \lambda} \left( \alpha - 1 \right) \left( \frac{\alpha}{\alpha - 1} \right)^{\alpha} > 0.
\]

Thus, \(\lambda = 0\) is never optimal. Likewise, \(\lambda = 1/\alpha\) is never optimal because in that case long-term contracts are preferred. Whenever short-term contracts are preferred, for (4.1) to hold we need:

\[
\lim_{\lambda \to 1/\alpha} \beta [k_t + f(i^S)] = \lim_{\lambda \to 1/\alpha} \left\{ \beta f(i^L) - i^L \right\} - \left\{ \beta f(i^S) - i^S \right\} \frac{\theta}{p\delta} = \infty,
\]

which implies:

\[
\lim_{\lambda \to 1/\alpha} \frac{\partial W}{\partial \lambda} = \frac{(\alpha - 1)^2}{\alpha} \frac{\partial i}{\partial \lambda} - \beta [k_t + f(i)] < 0.
\]

The optimal \(\lambda\) varies with knowledge \(k_t\) as follows:

\[
\frac{\partial \lambda^*}{\partial k_t} = -\frac{\partial^2 W}{\partial \lambda^* \partial k_t} \frac{\partial^2 W}{\partial (\lambda^*)^2}.
\]

We showed above a sufficient condition for \(\frac{\partial^2 W}{\partial (\lambda^*)^2}\) to be negative. Then, we compute:

\[
\frac{\partial^2 W}{\partial \lambda^* \partial k_t} = -\lambda \alpha \theta p \beta < 0,
\]

which implies \(\frac{\partial \lambda^*}{\partial k_t} < 0\).

References


