Consumer Obfuscation by a Multiproduct Firm∗

Vaiva Petrikaitė†

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Abstract

I show that a multiproduct firm has incentives to obfuscate its varieties by using search costs to screen out its customers by their individual preferences. By setting a different search cost per variety the seller prompts all consumers to search its varieties in a particular order. The consumers who draw high valuations for a particular product terminate their search earlier than the consumers who draw low valuations. Thus, the firm has incentives to raise the prices of earlier-searched varieties. The optimal search cost per variety is such that consumers inspect a variety only if the match values from previously-searched varieties have been very poor.

Keywords: Sequential search, obfuscation, horizontal differentiation, search costs, multiproduct firm

JEL classification: D21, D42, D83

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†University of Groningen and Institute for Economic Analysis (CSIC). E-mail: asvaiva@gmail.com
I Introduction

A firm’s ability to exploit information about individual consumer preferences is highly correlated with its profitability. If a firm knows how much a particular consumer values its product, and it can extract all consumer surplus by setting a personal price, then it earns the highest-possible profit. However, information about the valuation of a product by a particular consumer is rarely available; therefore, first-degree price discrimination is seldom feasible. Instead, firms employ other techniques to screen out their customers, such as varying the quality of their products (Deneckere and McAfee, 1996), splitting their products into base goods and their add-ons (Ellison, 2005), combining price and quantity bundles (McAfee et al., 1989) or applying two-part tariffs (Armstrong, 1999). These techniques bring less profit than first-degree price discrimination. However, profits are higher than when charging a uniform price.

In this paper, I propose a new mechanism through which a multiproduct monopolist can imperfectly screen out its customers according to their preferences, namely by obfuscating its products. In my model, like in Perloff and Salop (1985), consumers differ in their random valuations of the various products of the monopolist, the valuations are unobservable to the seller, and consumers want to buy the varieties that provide them with the highest utility. Next to setting its profit-maximising prices, the firm may obfuscate its products by setting a positive search cost per variety, which makes learning the utilities of products costly. The search cost is variety-specific, but it is identical for all consumers. If the firm obfuscates its varieties, then consumers search the varieties sequentially with perfect recall, like in Wolinsky (1986) and Anderson and Renault (1999). Because of different search costs, a consumer samples the products in an optimal order by starting with the less obfuscated varieties and terminating her search when she draws a sufficiently high match value for a product (Weitzman, 1979). Then, the mean valuation of a product by consumers who buy the product decreases with the search order. Therefore, the monopolist receives an incentive to increase the prices of earlier-searched varieties and to lower the prices of later-searched varieties. Thus, by obfuscating its products the firm extracts more consumer surplus and earns a higher profit.

I also analyse how a multiproduct firm selling two varieties obfuscates those products when they vary in quality or product design. To model vertical product differentiation, I assume that the distribution function of valuations of a higher-quality product dominates the distribution function of valuations of a lower-quality product in the sense of first-order stochastic dominance. I find that by obfuscating the lower-quality product, the firm decreases search intensity more than by obfuscating the higher-quality product. A lower search intensity leads to the better screening of consumers; thus, the seller earns a higher profit by obfuscating the lower-quality good.

To model differences in product design, I adopt the classification of varieties as either niche or mass products and the related methodology of Johnson and Myatt (2006) and Bar-Isaac et al. (2012). If a variety is a niche product, then the valuations of the product are very dispersed, and a monopolist earns more profit by charging a high price and serving a few consumers than by
charging a low price and selling many units. On the contrary, if a variety is a mass product, then the valuations of the product are accumulated around the mean, and a monopolist earns more by charging a low price and selling more units. I find that if both varieties are mass products, then the search intensity is the lowest, and the highest profit is obtained when the monopolist obfuscates the variety with more dispersed valuations. However, if both varieties are niche products, then it is optimal for the firm to obfuscate the variety with less dispersed valuations.

Finally, I study the incentives to obfuscate in the duopoly market in which every firm sells two horizontally differentiated varieties and a consumer pays an exogenous search cost to visit a seller. The firms choose their profit-maximising prices and may obfuscate one of their varieties by setting intra-store search costs. If the sellers obfuscate, then consumers pay two types of search costs: an exogenous search cost per firm, which is paid if a buyer visits a seller, and an intra-shop search cost, which must be paid if a consumer wants to observe the utility of an obfuscated variety in a shop. Consumers visit the firms sequentially without recall, whereas the sequential search of products inside a shop is with perfect recall. As a result, a firm sells to two groups of consumers: consumers who visit the seller in the first place and find the utility that is higher than the expected utility from visiting the other firm, and consumers who visit the firm after visiting the other firm. I show that a firm always chooses to obfuscate because it leads to a better extraction of consumer surplus. However, in order not to lose its customers, the seller sets its intra-store search cost sufficiently low such that a consumer observes the utilities of both of its products before visiting a competing firm.

Real-life examples and existing studies show that the practice of obfuscation is widely used. For instance, Apple, in its online store, obfuscates “Refurbished and Clearance” deals by placing a link at the bottom of the page of the online shop, as opposed to placing the regular items at the top. Similarly, by default Philips lists its more expensive products first in its online shop, and consumers must scroll down to find cheaper alternatives. Dréze et al. (1994) observe in their shelf reorganisation experiment that firm profits increased after raising the distance between different categories of bath tissues, which made product comparison more complicated. BBC Watchdog draws consumers’ attention to the fact that supermarkets place the cheaper options in less prominent positions. Derbyshire (2004) points out that supermarkets use a triangular product layout on a shelf by putting “the biggest, tallest products with the highest profit margin in the centre” where consumers notice them the fastest. Finally, it has been observed by Mannino (2012) that “[retailers] know shoppers want to easily find the size, price and item neatly displayed. So they purposely create the frustration of the poorly marked and poorly organized clearance area to tempt you toward the beautifully displayed and perfectly organized full-price merchandise.”

The fact that firms have incentives to obfuscate has been observed in the recent economics literature. Wilson (2010) and Ellison and Wolitzky (2012) study a homogeneous product market in which firms obfuscate by increasing search costs that consumers have to pay to visit the sellers. In

\[1\] For more details see http://www.bbc.co.uk/watchdog/consumer_advice/supermarket_psychology.shtml (accessed 2013 July)
the models, the degree of obfuscation intensity is either unobservable (Ellison and Wolitzky, 2012) or known (Wilson, 2010) to consumers *a priori*. Both Wilson (2010) and Ellison and Wolitzky (2012) find that obfuscation decreases competitive pressure and, therefore, firms earn more profits if they obfuscate. In the model of Wilson (2010), by obfuscating, a firm encourages its competitor to raise its price, which gives a competitive advantage to the obfuscating seller. In the paper of Ellison and Wolitzky (2012), an obfuscating firm raises the search costs both for itself and for other firms, which decreases competitive pressure in the market. Meanwhile, in my model the key role of obfuscation is to screen out consumers according to their valuations.

Ellison (2005) and Gabaix and Laibson (2006) analyse the markets in which firms sell composed products: base goods and their add-ons. In the model of Gabaix and Laibson (2006), a consumer buys both a base good and its add-on (shrouded attribute), a fraction of consumers are naive and underestimate the prices of shrouded attributes (add-ons), and firms may choose to inform the naive consumers by advertising the prices. Gabaix and Laibson (2006) find that the equilibrium where firms choose not to advertise the prices of shrouded attributes may be sustained. In the set-up of Ellison (2005), consumers can choose either to buy a base good only or to pay for both the base good and its add-on. The price of an add-on can be learned only after paying a costly visit to a firm. Ellison (2005) shows that there is no add-on pricing equilibrium where firms advertise both the prices of their base goods and their add-ons. The combination of a base good and its add-on can be interpreted as a higher-quality item. Therefore, in the models of Gabaix and Laibson (2006) and Ellison (2005) firms choose whether to obfuscate the prices of their higher-quality products and the prices of the lower-quality products (base goods) are observable. In my paper, the multiproduct monopolist may choose to obfuscate either its lower or higher-quality product.

By obfuscating some of its products, the monopolist controls the order in which products are sampled. In this sense, my model is related to the literature on ordered-search. Arbatskaya (2007) shows that when consumers shop for homogeneous products, equilibrium prices must fall as a consumer proceeds to search for lower prices. Zhou (2011) demonstrates that when consumers shop for differentiated products, equilibrium prices increase instead. Armstrong, Vickers, and Zhou (2009) explore the role of prominence in search markets and show that a prominent product is sold at a lower price relative to non-prominent products. Haan and Moraga-González (2011) and Armstrong and Zhou (2011) present alternative ways in which firms can become prominent. The main difference between these papers and mine is that I focus on the role of frictions among the products of a single firm. Zhou (2009) also observes that a multiproduct monopolist obtains higher profits if one of its varieties is prominent. My monopoly analysis extends this result by allowing the monopolist to choose a different obfuscation degree for every variety. Additionally, I analyse the incentives of a multiproduct firm to obfuscate when there are actual quality and product design differences between the varieties, and there is competition in the market.\(^2\)

\(^2\)The role of search costs in multiproduct settings has also been studied by Rhodes (2013), Shelegia (2011) and Zhou (2014). In these papers, too, search costs are not under the control of the firms.
The fact that a multiproduct firm prefers to make the comparison of its varieties for consumers more complicated has also been observed by Gu and Liu (2013). In their model, in which a retailer sells two horizontally differentiated varieties, consumers can obtain either a good or a bad match value of a product, and search costs differ across consumers. Differently from my set-up, in their model, the retailer sets the same price for both varieties. Gu and Liu (2013) show that the profit of a firm increases with obfuscation but their result stems not from the better extraction of consumer surplus, like in my paper, but from the effect of obfuscation upstream: the manufacturers lower their wholesale prices to get a prominent position.

This paper also relates to the work of Armstrong and Zhou (2013) with respect to the incentives to deter consumers from searching. In their paper, a single product monopolist applies buy-now discounts or requires deposits to discourage consumers from searching for alternatives. In this way, consumers are also screened out by their valuations. However, in contrast to their model, in this paper the multiproduct monopolist does not discriminate between consumers by charging different prices for the same variety.

Taking into consideration that the multiproduct seller introduces frictions for the consumers to compare the varieties, this paper relates to the literature on trade intermediaries (see e.g. Hagiu and Jullien, 2011; White, 2013; Burguet et al., 2014). In that literature, an intermediary controls only the degree of obfuscation and the prices are under the control of other firms, whereas in this paper the multiproduct monopolist sets both the profit-maximising prices and the search costs.

The decision of a firm to obfuscate its product leads to an outcome somewhat similar to the outcome of a firm’s decision to advertise information about its products due to information asymmetries in the market. Therefore, in this sense my paper is related to the work of Caminal (1996), Anderson and Renault (2006) and ?. In all three models, the decision to advertise affects consumers’ decisions on whether to pay a costly visit to a firm. Then the firm chooses the advertising option that leads to the highest-possible consumer traffic. The major difference between these papers and my paper is that those papers conclude that a single product advertising monopolist sells fewer units if fewer consumers search, whereas in my paper the multiproduct monopolist gains from consumers who decide not to search much: they buy the varieties that are more expensive, though less obfuscated.

The rest of the paper is organised as follows. The basic assumptions that are listed in section II are followed by an example with a two-product monopolist in section III. In section IV, I analyse the market where obfuscating monopolist sells several horizontally differentiated varieties. The incentives to obfuscate when there are differences in product quality and design are explored in section V. In section VI, I analyse whether a firm has incentives to obfuscate when it has a competitor. Finally, concluding remarks are in section VII. The proofs of propositions and other derivations can be found in the appendices of the paper.
II Market description

There is a mass of consumers that is normalised to one. A typical consumer has unit demand and wants to buy one of $2 \leq k < \infty$ horizontally differentiated varieties of a product. The net utility of consumer $l$ who buys variety $i$ is denoted by $u_{il}$, and it equals the difference between a valuation (match value), $\varepsilon_{il}$, and the price, $p_i$, of the product:

$$u_{il} = \varepsilon_{il} - p_i$$

The match value measures the valuation of variety $i$ by consumer $l$; it is consumer and product-specific and unobservable to a seller.\(^3\) However, it is common knowledge that match values are distributed independently and identically among consumers and varieties according to a uniform distribution in the interval $[0, 1]$.\(^4\)

All $k$ varieties are sold by a single seller. The firm has constant unit production costs. The costs are the same for all varieties, and they are normalised to zero. The monopolist chooses the set of profit-maximising prices and the layout of its varieties. The layout of the varieties determines the ease with which a consumer can observe a variety and learn its utility. The monopolist can make variety $i$

- freely observable, i.e., a consumer observes $u_i$ upon entering the shop, or
- obfuscated, i.e., a consumer must incur search cost $s_i$ in order to learn $\varepsilon_i$ and $p_i$.

Suppose that the monopolist has chosen to obfuscate its varieties. Then consumers search the products sequentially with costless recall by following the optimal stopping and search rules that have been given by Weitzman (1979), which I explain in the following paragraphs.

Suppose that $u_j$ is the highest utility that has been observed by a consumer, and the consumer considers searching variety $i$. If variety $i$ is the last-searched product, then the consumer is indifferent between continuing to search and terminating her search if the expected gain from the search equals the search cost. This is illustrated by equation (1).

$$\int_{\varepsilon_j - p_j + p_i}^{1} (\varepsilon_i - p_i - \varepsilon_j + p_j) d\varepsilon_i = s_i$$  \(1\)

I define $x_i \equiv \varepsilon_j - p_j + p_i$ such that (1) is satisfied. The difference $x_i - p_i$ is called the reservation utility of variety $i$. If $\varepsilon_j - p_j > x_i - p_i$, then the expected gain from the search is lower than the

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\(^3\)In order to simplify mathematical expressions, the consumer specific index is not used in further derivations unless it is necessary.

\(^4\)The assumption about the uniform distribution of match values is not restrictive. It is shown in Appendix B by lemma 2 that the results of lemma 1 and proposition 1 hold for general distribution functions if $k = 2$. Simulation results with a power distribution with $k \geq 2$ lead to the same conclusion.
search cost. However, if \( \varepsilon_j - p_j < x_i - p_i \), then it is worthwhile for the consumer to search for the utility of variety \( i \).

At every search step there is a reservation utility that a consumer uses to make her decision on whether to search further. According to Weitzman (1979), if a consumer samples products in an optimal order, then the products’ reservation utilities are computed by using the myopic stopping rule (1).\(^5\) A consumer inspects products optimally if she ranks them according to their reservation utilities in a descending order and starts searching from the top of the list.

In order to ensure that there are consumers who search variety \( i \), I assume that for all \( i \) the search cost \( s_i \) is sufficiently small:

\[
s_i \leq \int_{p_i}^{1} (\varepsilon - p_i) d\varepsilon
\]

A consumer learns the search cost \( s_i \) only after she searches variety \( i - 1 \). However, the customer forms her expectations about \( s_i \), which equals \( s_i^* \), and the expectations are correct in equilibrium.\(^6\) The expected price of a not-yet-searched variety has a superscript *. A consumer computes the reservation utilities of the varieties by using their expected prices. Thus, the reservation utility of variety \( i \) is \( x_i (s_i^*) - p_i^* \).

### III Example with a two-product monopolist

In this section, I provide an example of a monopolist that sells two horizontally differentiated varieties, and show that the firm can increase its profit by obfuscating one of the products. Suppose that there is no obfuscation, and the monopolist considers two profit-maximising prices \( p_1 \) and \( p_2 \) for the first and the second variety respectively. Without loss of generality, I assume that \( p_2 < p_1 \).

A consumer arrives at the shop, observes two match values and prices, and either buys one of two products or leaves the shop without buying anything. The customer buys the first variety if its utility is the highest and non-negative. The probability of this event gives the demand for the first product that equals

\[
d_1 = \int_{p_1}^{1} (\varepsilon - p_1 + p_2) d\varepsilon
\]

Similarly, the consumer buys the second product if the utility of it is the highest and non-negative. Thus, the demand for the second product is

\[
d_2 = \int_{1-p_1+p_2}^{1} d\varepsilon + \int_{p_2}^{1-p_1+p_2} (\varepsilon - p_2 + p_1) d\varepsilon
\]

\(^5\) As if each variety were the last to be searched.

\(^6\) In case of costless recall, this assumption is qualitatively equivalent to the assumption that the vector of search costs is observable to consumers before they enter the shop.
The pay-off of the monopolist equals the total revenue from selling both products:

\[ \pi = p_1d_1 + p_2d_2 \]

After taking the first-order conditions with respect to both prices and solving the system of equations, I obtain that \( p_1 = p_2 = p^* = 1/\sqrt{3} \), and the profit of the monopolist equals \( \pi^* = 2/(3\sqrt{3}) \).

When the monopolist charges the same price for both of its products and does not obfuscate, the distribution of buyers between the two varieties is illustrated by Figure 1a. In the vertically striped area, \( \varepsilon_1 \) exceeds \( \varepsilon_2 \), and the consumers whose pairs of valuations of both goods are in this area buy variety 1. Similarly, the consumers whose pairs of valuations of both varieties are in the dotted area buy variety 2. The consumers for whom \( \max\{\varepsilon_1, \varepsilon_2\} < p^* \) do not buy anything (the white rectangle at the left-bottom corner).

Because the firm does not observe individual match values, the seller cannot set a higher price for consumers who buy the first variety and whose valuations are close to the upper limit of the square. Similarly, a consumer who buys the second variety pays the same price irrespective of her match value of the second product. The firm prefers to identify how a consumer values its products because then the firm could adjust its prices and earn more profit by extracting more consumer surplus. As a result, the seller obfuscates the second variety by introducing a positive search cost \( s \). If the monopolist obfuscates the second product, then a consumer observes the utility of the first variety as soon as she enters the shop. However, the customer must pay the search cost to learn the utility of the second product. Suppose that the monopolist obfuscates the second product but keeps
charging \( p^* \) for both varieties and it is common knowledge. Under these circumstances, consumers apply the optimal stopping rule and the customers who learn that \( \varepsilon_1 \geq x_2 \) optimally choose to terminate their search and buy the first variety. Other consumers continue searching, inspect the second variety, and buy the one that is the best for them, if any. As a result, compared to the situation without obfuscation, some of the consumers who used to buy the second variety switch to the first variety. These buyers are depicted in the grey area in Figure 1a (triangle ABC).

Because of obfuscation, the average \( \varepsilon_1 \) of consumers who buy the first variety increases, whereas the average \( \varepsilon_2 \) of consumers who buy the second variety decreases. These changes create the incentives to increase the price of the first variety and to decrease the price of the second variety.

To get the exact values of profit maximising \( p_1 \), \( p_2 \) and \( s \), I derive the demand functions for both products. Suppose that the monopolist considers two profit-maximising prices \( p_1 \) and \( p_2 \) and the search cost \( s \). Consumers expect that the price of the second variety equals \( p_2^* \) and they observe \( s \) and \( p_1 \) after entering the shop. The demand for the first product equals the sum of two probabilities: the probability that a consumer buys the first variety without searching for the utility of the second variety (\( \varepsilon_1 \geq x_2 - p_2^* + p_1 \)) and the probability that a consumer samples both products and buys the first product because its utility is higher and non-negative (\( \max \{ \varepsilon_2 - p_2, 0 \} \leq \varepsilon_1 - p_1 < x_2 - p_2^* \)).

As a result, the demand for the first product equals

\[
 d_1 = 1 - (x_2 - p_2^* + p_1) + \int_{p_1}^{x_2 - p_2^* + p_1} (\varepsilon - p_1 + p_2) \, d\varepsilon
\]

The consumer buys the second product if she pays the search cost (which implies that \( \varepsilon_1 < x_2 - p_2^* + p_1 \)) and finds that \( \max \{ \varepsilon_1 - p_1, 0 \} \leq \varepsilon_2 - p_2 \). The probability of this event constitutes the demand for the second product.

\[
 d_2 = (x_2 - p_2^* + p_1)(1 - x_2 + p_2^* - p_2) + \int_{p_2}^{x_2 - p_2^* + p_2} (\varepsilon - p_2 + p_1) \, d\varepsilon
\]

Again, the profit of the monopolist equals the total revenue from selling both products. However, in this case the seller chooses not only \( p_1 \) and \( p_2 \) but also \( x \) (or \( s \)). In equilibrium, the profit-maximising price \( p_2 \) equals \( p_2^* \). Then, after taking three first order conditions, setting \( p_2 = p_2^* \) and solving for both prices and \( s \) we obtain that \( p_2^* = 1/2 \), \( p_1^* = 5/8 \), and \( x = p_2^* \), which leads to \( s = 1/8 \). In this case the profit of the obfuscating monopolist equals \( \pi_0^* = 25/64 > \pi^* \).

Indeed, the price of the first product is higher and the price of the second variety is lower. Compared to the case with the symmetric price and obfuscation, some consumers switch from the first variety to the second variety. The new distribution of consumers across the varieties is depicted in Figure 1b. Again, the consumers whose pairs of valuations of both products are in the vertically striped area buy the first variety and the consumers whose pairs of valuations are in the dotted area buy the second product. By introducing a positive search cost for the second product, the monopolist encourages consumer self-selection according to their match values. The consumers who
draw high match values for the first product terminate their search and the consumers who draw low match values for the first product learn the utility of the second variety. Because of this imperfect self-selection, the monopolist can extract more consumer surplus and earn a higher profit.

IV Monopolist with $k$ varieties

In this section, I derive the set of profit-maximising prices and search cost when the monopolist sells more than two horizontally differentiated varieties. The seller simultaneously chooses the vector of search costs $\{s_1, s_2, \ldots, s_k\}$ (or equivalently $\{x_1, x_2, \ldots, x_k\}$) and the vector of prices $\{p_1, p_2, \ldots, p_k\}$, based upon consumer expectations and search behaviour. Irrespective of the chosen prices of the firm, consumers do not update their expectations about the prices of not-yet-searched varieties when they observe a deviation price. Because a consumer knows the search cost for a variety before searching it, the expected search costs do not enter the pay-off function of the monopolist. Therefore, I use $x_l(s_l)$ instead of $x_l(s^*_l)$ for the reservation utility of variety $l$ and omit $(s_l)$ while deriving the pay-off function of the firm. Without loss of generality I assume that $s_1 = 0$ and the rest of the search costs are set such that $x_l - p^*_l > x_{l+1} - p^*_{l+1}$ $\forall l \in [1, k].$\footnote{The search cost for the first variety can be positive. However, this cost has to be sufficiently small to ensure that $x_1 - p^*_1 = \max \{x_i - p^*_i\}_{i=1,\ldots,k}$. Moreover, the search cost for the first variety does not enter the pay-off function of the monopolist. Thus, it does not have any effect on the optimal prices and the obfuscation intensity of other varieties, and, for convenience, I set it equal to zero.}

Because of the order of reservation utilities, a consumer starts searching from the first variety. If the utility of the first variety is not sufficiently high, then she searches the second variety, and so on and so forth. For notational convenience, I assume that $\sum_{i=1}^l x_i = 0$ and $\prod_{i=1}^l x_i = 1$ if $j > l$ and $j - x_j - p_j^* = 0$ if $j > k$. Consider a consumer who has paid $s_i$ and learned $\varepsilon_i - p_i$. The consumer has searched variety $i$. Therefore, the utilities of all $i - 1$ varieties must have been less than $x_i - p_i^*$. Thus, if $\varepsilon_i - p_i \geq x_i - p_i^*$, then the consumer terminates her search and buys variety $i$. The joint probability of the event that the consumer searches variety $i$ and finds $\varepsilon_i - p_i \geq x_i - p_i^*$ equals

\[
\prod_{j=1}^{i-1} (x_i - p_i^* + p_j) (1 - x_i + p_i^* - p_i)
\]

Suppose that the consumer has searched variety $i$, and $x_{i+1} - p_{i+1}^* \leq \varepsilon_i - p_i < x_i - p_i^*$. The consumer compares all observed utilities with a new reservation utility $x_{i+1} - p_{i+1}^*$ when she considers whether to search further. The consumer has searched variety $i$ because $\max \{\varepsilon_h - p_h\}_{h={1,\ldots,i-1}} < x_i - p_i^*$. However, $x_i - p_i^* > x_{i+1} - p_{i+1}^*$. Therefore, the customer may decide not to search variety $i + 1$ and terminate her search by buying either variety $i$ or any other variety $j, j \in [1, i - 1]$. She buys variety $j$ if $\varepsilon_j - p_j > \max \{\varepsilon_h - p_h, x_{i+1} - p_{i+1}^*\}_{h={1,\ldots,i}} \setminus j$, and she buys variety $i$ if $\varepsilon_i - p_i > \max \{\varepsilon_h - p_h, x_{i+1} - p_{i+1}^*\}_{h={1,\ldots,i-1}}$. As a result, if $\varepsilon_i - p_i$ is in the above-specified interval, then
the consumer decides not to search variety $i+1$ and buys variety $i$ with probability

$$\Pr \left[ \max \{ \varepsilon_h - p_h, x_{i+1} - p_i^* \}_{h=\{1,\ldots,i-1\}} \leq \varepsilon_i - p_i < x_i - p_i^* \right] = \int_{x_{i+1} - p_i^* + p_i}^{x_i - p_i + p_i} \prod_{j=1}^{i-1} (\varepsilon - p_i + p_j) \, d\varepsilon = \int_{x_{i+1} - p_i^* + p_i}^{x_i - p_i + p_i} \prod_{j=1}^{i-1} (\varepsilon + p_j) \, d\varepsilon$$

Similarly, after searching variety $i+1$ the consumer may terminate her search and buy variety $i$ without searching variety $i+2$. This happens with probability

$$\Pr \left[ \max \{ \varepsilon_h - p_h, x_{i+2} - p_i^* \}_{h=\{1,\ldots,i-1,i+1\}} \leq \varepsilon_i - p_i < x_{i+1} - p_i^* \right] = \int_{x_{i+2} - p_i^* + p_i}^{x_{i+1} - p_i^* + p_i} \prod_{j=1, j \neq i}^{i+1} (\varepsilon - p_i + p_j) \, d\varepsilon = \int_{x_{i+2} - p_i^* + p_i}^{x_{i+1} - p_i^* + p_i} \prod_{j=1, j \neq i}^{i+1} (\varepsilon + p_j) \, d\varepsilon$$

Because of the order of reservations utilities, the consumer can terminate her search at any search step until she makes the decision to search for the utility of variety $k$. To put it differently, if the consumer has decided to search variety $i+1$, then after searching any other variety $j$, where $i < j < k$, there is a positive probability that the consumer will decide to stop searching and will buy variety $i$. This set of positive probabilities does not exist in the sequential search model of Wolinsky (1986). In the model of Wolinsky (1986), consumers pay the same search cost per firm and expect that all firms charge the same price. Therefore, the reservation utilities of all sellers are the same. If a consumer decides to search for the utility of variety $i+1$ after observing variety $i$, then she never considers terminating the search and buying variety $i$ until she searches for the utility of the last variety. Therefore, in the model of Wolinsky (1986), variety $i$ is bought either immediately after searching it or after sampling all varieties in the market.

Suppose that the consumer has searched variety $k$. Then, as well as in the sequential search model of Wolinsky (1986), she can decide to buy variety $i < k$. This happens if the utility of variety $i$ is the highest and is non-negative. This event happens with probability

$$\Pr \left[ \max \{ \varepsilon_h - p_h, 0 \}_{h=\{1,\ldots,k\}\setminus i} \leq \varepsilon_i - p_i < x_k - p_k^* \right] = \int_{p_i}^{x_k - p_k^* + p_i} \prod_{j=1, j \neq i}^{k} (\varepsilon - p_i + p_j) \, d\varepsilon = \int_{p_i}^{x_k - p_k^* + p_i} \prod_{j=1, j \neq i}^{k} (\varepsilon + p_j) \, d\varepsilon$$

The total demand for variety $i$ ($d_i$) is the sum of all of the probabilities that the consumer buys variety $i$,

$$d_i = \prod_{j=1}^{i-1} (x_i - p_i^* + p_j) (1 - x_i + p_i^* - p_i) + \sum_{l=i}^{k} \left( \int_{x_{i+1} - p_{i+1}^* + p_i}^{x_l - p_l^* + p_i} \prod_{j=1, j \neq i}^{l} (\varepsilon + p_j) \, d\varepsilon \right)$$
and the pay-off function of the monopolist equals

\[ \pi = \sum_{i=1}^{k} p_i d_i. \]

The firm chooses the optimal set of search costs and prices simultaneously. Furthermore, due to consistent consumer beliefs, the optimal choice of \( s \) and \( p \) coincides with \( s^* \) and \( p^* \). Then the system of the first-order conditions of the monopolist for all \( i \in [1, k] \) is

\[
\prod_{j=1}^{i-1} (x_i - p_i^* + p_j^*) (1 - x_i - p_i^*) + \sum_{l=i}^{k} \left( \int_{x_{i+1} - p_{i+1}^*}^{x_i - p_i^*} \prod_{j=1, j \neq i}^{l} (\varepsilon + p_j^*) d\varepsilon \right) + \\
\sum_{h=i+1}^{k} p_h^* \left( \prod_{j=1, j \neq i}^{h-1} (x_h - p_h^* + p_j^*) (1 - x_h) + \sum_{l=h}^{k} \left( \int_{x_{i+1} - p_{i+1}^*}^{x_i - p_i^*} \prod_{j=1, j \neq i}^{l} (\varepsilon + p_j^*) d\varepsilon \right) \right) = 0 \quad (2) \\
\sum_{h=1}^{i-1} (p_h^* - p_i^*) \prod_{j=1, j \neq h}^{i-1} (x_i - p_i^* + p_j^*) (x_i - 1) = 0 \quad (3)
\]

**Lemma 1.** Suppose that there is an exogenously given vector of search costs and a set of prices \( \{p_1^*, \ldots, p_k^*\} \) that satisfy (2) and \( x_i - p_i^* > x_{i+1} - p_{i+1}^* \), \( \forall i \in [1, k] \). Then \( p_i^* > p_{i+1}^* \), \( \forall i \in [1, k] \).

The price ranking of lemma 1 is different from the equilibrium price ranking when every variety is sold by a different seller. Zhou (2011) shows that if the varieties are sold by distinct firms, and consumers search the varieties in a specified order, then the equilibrium price increases with the search order, i.e. \( p_i^* < p_{i+1}^* \), \( \forall i \in [1, k] \). This is the opposite of the result in lemma 1. This difference in results implies that the incentives to raise or lower the prices depend on whether the varieties are sold by a multiproduct monopolist or by distinct competing sellers. In order to explain it, I focus on a market with two varieties. If the varieties are sold by two distinct sellers, then, following the terminology of Armstrong et al. (2009), the seller that variety is searched the first is said to be a prominent firm and the other firm is called a non-prominent firm.

Consider the case when both varieties are sold by a single multiproduct seller. In section III, it was shown that because of obfuscation, the mean valuation of the first variety by consumers who buy the product, is higher, which creates the incentives to raise the first price. Now I look at the incentives to lower the price of the second product. Suppose that the second price is lowered. Then the buyers of the second variety pay a lower price than before obfuscation, and this has a negative effect on the revenue of the multiproduct firm. However, after the second price is lowered, the average \( \varepsilon_1 \) of consumers who buy the first variety increases. This creates additional incentives to
increase the first price. The positive effect on profits that comes from the increase in \( p_1^* \) outweighs the negative effect on profits that comes from the decrease in \( p_2^* \). As a result, with obfuscation the multiproduct firm lowers the price of the second variety and raises the price of the first variety.

If both varieties are sold by distinct sellers, then the externality of \( p_i \) on the average \( \varepsilon_j \), where \( i \neq j \), of consumers who buy variety \( j \) is not internalised. Thus, the incentives for the firms to raise and lower their prices differ from those of the multiproduct monopolist. Suppose that the price of the first variety increases. As a result, the prominent firm gains additional revenue from the consumers who continue buying the first variety. However, this gain is outweighed by the loss of revenue that happens because some consumers switch to the non-prominent seller. Thus, the prominent firm does not have incentives to increase its price. On the contrary, the prominent firm prefers to decrease its price. If the price of the first variety decreases, then the prominent firm stops many consumers from visiting the second firm. As a result, the seller attracts many new customers and earns less from selling to its old customers. The additional revenue from the new customers is higher than the loss of revenue from the old customers. Therefore, the prominent firm has strong incentives to lower its price.

Now consider the case when the non-prominent firm decreases its price. The seller earns less from the consumers who continue buying its product and additionally earns from the consumers who switch from the prominent firm or enter the market. Unfortunately, the additional revenue from the new customers does not compensate the loss of revenue from the old customers. Therefore, the non-prominent firm does not have incentives to decrease its price. On the contrary, the non-prominent firm has incentives to raise its price. After \( p_2^* \) increases, the firm earns more from the consumers who continue buying the second product and loses some revenue from the consumers who either switch to the first variety or leave the market without buying anything. The additional revenue from the remaining customers is higher than the loss of revenue from the leaving customers. Therefore, the non-prominent firm prefers to raise its price.

I now move on to the next result.

**Proposition 1.** In equilibrium, the monopolist sets the search costs for variety \( i \) such that \( x_i = p_i^* \), \( \forall i \in [2, k] \) and the set of equilibrium prices is computed by the recursive expression (4), where \( p_k^* = \frac{1}{2} \), and the profit of the firm equals \( \pi^* = (p_1^*)^2 \).

\[
p_i^* = \frac{1}{2} \left( 1 + (p_{i+1}^*)^2 \right)
\]

Learning the utility of every subsequent variety is more and more costly. If the prices of all \( k \) varieties were the same, then the ex ante expected utility would decrease with the search order. Therefore, obfuscation introduces a product differentiation aspect that is similar to vertical product differentiation. This product differentiation exists only ex ante, i.e., before a consumer searches a given variety, thereby helping to screen out consumers according to their valuations. Because
of the increasing search costs, the mean valuation of every product by the consumers who go on searching decreases with the search order. Therefore, every subsequent price is less than a preceding price. In this respect, my results resemble the findings of the models with an inter-temporal price-discriminating monopolist (see e.g. Stokey, 1979, 1981; Bulow, 1982). However, differently from the inter-temporal price discriminating monopolist, the multiproduct obfuscating monopolist faces no commitment problem. This is because in my model, consumers can decide to buy one of the already-observed varieties after every search step, whereas consumers cannot go back in time in inter-temporal price discrimination models. Because the multiproduct monopolist takes into account that consumers can exercise earlier-observed options when it sets the vector of profit-maximising prices, the firm has no incentives to revise its optimal price schedule after any consumer’s search step.

When the firm sets $s_l$ such that $x_l = p_l^*$, then the reservation utility of every variety equals zero. As a result, the only consumers who search variety $l$ are those who would leave the market after searching $l-1$ varieties and not buying anything. Therefore, at every search step the firm acts as a monopolist in a different market segment, and the optimal prices of previously searched varieties have no effect on the optimal prices that are chosen for the varieties that are searched later. Additionally, the monopolist would never increase the search cost $s_l$ as much as $x_l < p_l^*$, because it would imply a negative reservation utility of variety $l$. Then no consumer would ever sample and buy the variety. Taking into account that the profit of the monopolist increases with the number of varieties, the firm has no incentives to remove a product from the market by setting its reservation utility below zero.

![Graph showing search costs and prices of all $k$ varieties, $k = 20$](image)

**Figure 2**: Search costs and prices of all $k$ varieties, $k = 20$

In Figure 2, I depict the equilibrium sets of prices and search costs when the monopolist sells 20 varieties. Because of obfuscation, the first ten prices are higher than the equilibrium price $p^*$ that would be charged in the absence of obfuscation. Therefore, the surplus of the consumers who buy one of the first 10 varieties decreases not only because of the search costs, but also because of higher prices. According to simulation results, the share of consumers who buy one of the first 10 varieties is 59%. The consumers who buy one of the last 10 varieties pay lower prices than in the
absence of obfuscation. However, their surplus is cannibalised by the search costs. Therefore, the total consumer surplus decreases if the monopolist obfuscates.\footnote{In case of obfuscation, consumer surplus equals $CS = \int_{p_1^*}^{1} (\varepsilon - p_1^*) \, d\varepsilon$.}

The profit of the firm increases when the seller obfuscates. However, the increase is not monotonic in the number of varieties. Suppose that the monopolist does not obfuscate and the number of the varieties increases. Then, because of a wider choice, the mean valuation of a product by consumers who buy the variety increases, which leads to a higher symmetric equilibrium price and a higher profit for the seller. If the seller obfuscates, then its profit also becomes higher when $k$ increases. This happens because a higher number of varieties leads to a more homogeneous pool of valuations by consumers who buy a product; thus the firm can adjust its prices better to extract more consumer surplus. If the number of varieties is small, then the profit of the obfuscating monopolist increases more with $k$ than the profit of the non-obfuscating monopolist, and the gain from obfuscation increases with $k$. However, if $k$ is greater than or equal to 18, then the profit of the non-obfuscating monopolist increases more with $k$ than the profit of the obfuscating monopolist, and the gain from obfuscation decreases with the number of varieties.

\begin{figure}[h]
\centering
\begin{subfigure}{.5\textwidth}
  \centering
  \includegraphics[width=\textwidth]{figure3a.png}
  \caption{gains from obfuscation}
\end{subfigure} \hfill
\begin{subfigure}{.5\textwidth}
  \centering
  \includegraphics[width=\textwidth]{figure3b.png}
  \caption{welfare}
\end{subfigure}
\caption{The change of profits and total welfare ($\Delta W = \Delta CS + \Delta \pi$) when the monopolist obfuscates}
\end{figure}

\section{Ex ante differentiated varieties}

So far I have studied the case when the match values of all of the varieties were distributed identically. In that case, all of the products were ex ante identical and the firm was indifferent with respect to which variety to obfuscate less and which variety to obfuscate more. In this section, I analyse the case when the distributions of match values differ across the varieties. Thus, the products are no longer identical ex ante, and the seller must choose not only the set of prices and search costs, but also which variety to obfuscate more. For analytical tractability, I analyse the case when the monopolist sells only two varieties. As before, a consumer observes the utility of the first
variety directly upon entering the shop, whereas the utility of the second variety must be searched at a positive search cost.

In order to model ex ante product differentiation, I use the framework and terminology of Johnson and Myatt (2006) and Bar-Isaac et al. (2012). I assume that the match values for variety \( i \), \( i = \{1, 2\} \) are distributed in the interval \([0, \bar{\varepsilon}]\) according to a differentiable distribution function \( F_i \) with a positive density \( f_i \) and an increasing hazard rate. Additionally, \( F_i \) is parametrised by \( \beta_i \).

The distribution function is continuous in \( \beta_i \), and an increase in \( \beta_i \) turns the distribution function clockwise around a particular point \( \varepsilon_i^* \), which can be either equal to zero or strictly positive. If \( \varepsilon_i^* = 0 \), then, after a rotation, the new distribution function dominates the previous one in the sense of first-order stochastic dominance. This type of rotation shifts consumers to the high values of \( \varepsilon_i \). I interpret this as an increase in the quality of product \( i \). If \( \varepsilon_i^* > 0 \), then, after an increase in \( \beta_i \), the tails of the density function \( f_i \) become thicker. As a result, both the share of consumers who like the variety very much and the share of consumers who like the variety very little increase. If the tails of the density function \( f_i \) are sufficiently thick, then, according to Johnson and Myatt (2006), it is optimal for a monopolist to sell fewer units of variety \( i \) by setting a high price. In that case, the match value of the marginal consumer who buys variety \( i \) is above the mean of \( \varepsilon_i \), and the variety is said to be a niche product. If a sufficiently large mass of consumers is close to the mean of \( \varepsilon_i \), then Johnson and Myatt (2006) show that it is optimal for a monopolist to sell many units by setting a low price. In that case, the match value of the marginal consumer who buys variety \( i \) is below the mean of \( \varepsilon_i \), and the variety is said to be a mass product. If \( \varepsilon_i^* > 0 \), then I adopt the assumption of Johnson and Myatt (2006) from proposition 1 that the rotation point does not move to the right when \( \beta_i \) increases.

Additionally, I put the following restrictions on the change of \( f_i \) with respect to \( \beta_i \):

- if \( \partial F_i / \partial \beta_i < 0 \), then the density function \( f_i \) either decreases with \( \beta_i \) or increases very slowly \((\partial F_i (\varepsilon) / \partial \beta_i + \varepsilon \partial f_i (\varepsilon) / \partial \beta_i \leq 0)\);

- if \( \partial F_i / \partial \beta_i > 0 \), then \( f_i \) either increases or decreases with \( \beta_i \) sufficiently slowly \((\partial F_i / \partial \beta_i + \varepsilon \partial f_i / \partial \beta_i \geq 0)\).

Finally, I assume that the density function is also not too decreasing with \( \varepsilon \) such that \( f_i (\varepsilon) + y f_i' (\varepsilon) > 0, y \leq \varepsilon \).

Consider a consumer who arrives at the firm, observes the first variety, and decides whether to search for the utility of the second variety at search cost \( s_2 \). If \( \varepsilon_1 - p_1 \geq x_2 - p_2^* \), then she terminates her search and buys the first variety. Otherwise she searches the second variety and buys one of the varieties, or she leaves the market without buying anything. If the consumer observes both utilities, then she buys the first variety if \( \max \{0, \varepsilon_2 - p_2 \leq \varepsilon_1 - p_1 < x_2 - p_2^*\} \) and buys the second variety.

---

9This assumption is made to assure the concavity of the pay-off with respect to both prices at the equilibrium point.
if \( \varepsilon_2 - p_2 \geq \max \{ \varepsilon_1 - p_1, 0 \} \). By following the reasoning in section IV, the pay-off function of the firm is

\[
\pi = p_1 \left[ 1 - F_1 (x_2 - p_2^* + p_1) + \int_{p_1}^{x_2-p_2^*+p_1} F_2 (\varepsilon - p_1 + p_2) dF_1 (\varepsilon) \right] \\
+ p_2 \left[ F_1 (x_2 - p_2^* + p_1) (1 - F_2 (x_2 - p_2^* + p_2)) + \int_{p_2}^{x_2-p_2^*+p_2} F_1 (\varepsilon - p_2 + p_1) dF_2 (\varepsilon) \right]
\]

After taking the derivatives of the pay-off with respect to both prices and \( x_2 \) and setting the deviation prices equal to their expected values, the system of three first-order conditions is expressed by (5)-(7):

\[
1 - F_1 (x_2 - p_2^* + p_1^*) + \int_{p_1^*}^{x_2-p_2^*+p_1^*} F_2 (\varepsilon - p_1^* + p_2^*) dF_1 (\varepsilon) + (p_2^* - p_1^*) f_1 (x_2 - p_2^* + p_1^*) (1 - F_2 (x_2)) + (p_2^* - p_1^*) \int_{p_1^*}^{x_2-p_2^*+p_1^*} f_2 (\varepsilon - p_1^* + p_2^*) dF_1 (\varepsilon) - p_1^* F_2 (p_2^*) f_1 (p_1^*) = 0
\]

(5)

\[
F_1 (x_2 - p_2^* + p_1^*) (1 - F_2 (x_2)) + \int_{p_2^*}^{x_2} F_1 (\varepsilon - p_2^* + p_1^*) dF_2 (\varepsilon) - (p_2^* - p_1^*) \int_{p_2^*}^{x_2} f_1 (\varepsilon - p_2^* + p_1^*) dF_2 (\varepsilon) - p_2^* F_1 (p_1^*) f_2 (p_2^*) = 0
\]

(6)

\[
(p_2^* - p_1^*) f_1 (x_2 - p_2^* + p_1^*) (1 - F_2 (x_2)) = 0
\]

(7)

Suppose that \( \beta_1 = \beta_2 = \beta \) and \( F_1 = F_2 = F \). Then it is shown by lemma 2 in Appendix B that the result of lemma 1 and proposition 1 can be extended to the case with general distribution functions when the firm sells two varieties, i.e., \( p_1^* > p_2^* \) and \( x_2 = p_2^* \). The subsequent analysis shows that the same result holds if \( \beta_1 \neq \beta_2 \) and the monopolist chooses optimally which variety to obfuscate.

### A Vertical product differentiation

Before determining which of two varieties the monopolist chooses to obfuscate when they are both horizontally and vertically differentiated, it is worth examining how both profit-maximizing prices and the search cost change when the quality of one variety increases. During this analysis I keep the order of the search fixed: a consumer always starts searching from the first variety.

Suppose that the quality of the first product increases. Then \( \beta_1 \) increases to \( \beta' \), and \( \beta_2 \) remains
equal to $\beta$. As a result, $F_1$ dominates $F_2$ in the sense of first-order stochastic dominance. Then the frequency of consumers who draw high match values for the first variety increases and the frequency of consumers who draw very low valuations of the first variety decreases. In terms of Figure 1a, there are more consumers whose pairs of valuations of both varieties are close to the upper-horizontal side of the square and fewer consumers whose pairs of valuations of both varieties are close to the lower-horizontal side of the square. Thus, the mean valuation of the first product by consumers who buy the first variety is higher. Therefore, more consumers terminate their search without searching the second variety, which gives the firm additional incentives to set a high price for the first product. Meanwhile, the mean valuation of the second product by consumers who buy the second variety can be either higher or lower. This depends on how high the threshold value $x_2$ is. The threshold determines how many consumers are captured by the triangle $ABC$ in Figure 1a. The greater the mass of valuations in the triangle, the lower the mean valuation of the second product by consumers who buy the second variety. If, after increasing the quality of the first product, the mean of $\varepsilon_2$ by consumers who buy the second variety decreases, then the firm has additional incentives to set a low value of $p^*_2$. In that case, the inequality $p^*_1 > p^*_2$ is preserved, and the seller sets the maximum search cost for the second variety. If, after increasing the quality of the first product, the mean of $\varepsilon_2$ by consumers who buy the second variety increases, then the firm has incentives to set a higher $p^*_2$. However, the incentives to set a higher price for the first variety are stronger. Thus, the price of the first variety remains higher than the price of the second variety, and the monopolist sets the maximum search cost for the second variety.

Now I consider the case when the quality of the second variety increases. Then $\beta_2$ increases to $\beta'$ and $\beta_1$ remains equal to $\beta$, which causes $F_2$ to dominate $F_1$ in the sense of first-order stochastic dominance. As a result, the frequency of consumers who draw high valuations of the second variety increases, and the frequency of consumers who draw low valuations of the second variety decreases. Regarding Figure 1a, there are more consumers whose pairs of valuations of both varieties are close to the right-vertical side of the square and fewer consumers whose pairs of valuations of both varieties are close to the left-vertical side of the square. Consequently, the mean $\varepsilon_2$ by consumers who buy the second variety is higher. Therefore, the firm has incentives to raise the price of the second product. The mean of $\varepsilon_1$ by consumers who buy the first variety also increases. This happens because the increase in the frequency of consumers whose pairs of valuations of both varieties are in the triangle $ABC$ is higher than the decrease in the frequency of consumers whose pairs of valuations are close to the left-vertical side of the square. As a result, the firm has incentives to increase $p^*_1$ after the quality of the second product increases. Nevertheless, the incentives to increase $p^*_1$ are weaker than the incentives to increase $p^*_2$. Therefore, if $\beta'$ is close to $\beta$, then the inequality $p^*_1 > p^*_2$ remains, and the monopolist sets the maximum search cost for its higher quality product. However, if $\beta'$ is sufficiently far from $\beta$, then it may happen that the price of the first variety is lower than the price

\footnote{In Appendix B, this result is proven analytically for a power function $F_1(\varepsilon_i) = \varepsilon_i^{\beta_i}, \beta_i \geq 1$. However, numerical simulation results show that it holds for other distribution functions as well.}
of the second variety. Then by using (7) I obtain that for the given search order the monopolist does not obfuscate at all.

The monopolist earns more profit if consumer search intensity is lower, because then the firm can better screen out its customers and extract more consumer surplus from the buyers of the first variety. If the quality of the first variety is higher, then consumers search less because the probability to draw a high match value of the second variety is small. On the contrary, if the second product is of a higher quality, then search intensity is higher. Therefore, the screening of consumer preferences is worse and profits are lower. Thus, the monopolist always obfuscates its lower-quality product.

**Proposition 2.** Suppose that the multiproduct monopolist sells two vertically differentiated varieties. Then, in equilibrium, the firm always obfuscates the lower-quality product.

It has been observed by Deneckere and McAfee (1996) that it is profitable for a single product monopolist to introduce a lower-quality product in a market. By using proposition 2, I also find that if the multiproduct monopolist could manipulate the distribution functions of match values, then the firm would increase the quality of the first variety and would decrease the quality of the obfuscated variety (corollary of proposition 2). However, the incentives of the multiproduct monopolist to decrease the quality of the second variety are different from the incentives of the single product monopolist to sell a damaged good in the model of Deneckere and McAfee (1996). By introducing a damaged good the single product monopolist serves an additional market segment and, therefore, earns more profit. Moreover, all consumers benefit from the damaged product because the monopolist decreases the price of a higher quality product, and the consumers who buy the damaged good are better off than buying nothing. (Theorem 1 in Deneckere and McAfee, 1996). Meanwhile, the multiproduct monopolist decreases the quality of the obfuscated variety, mainly in order to extract more consumer surplus from the buyers of the first variety. Furthermore, the total consumer surplus may either increase or decrease. Even if both prices decrease after decreasing the quality of the second variety, the total consumer surplus may decrease. This is because the consumers who continue to buy the second variety get lower match values and pay higher search costs, and the consumers who switch to the first variety get lower match values.

**Corollary (of proposition 2).** Suppose that the monopolist sells two horizontally differentiated varieties by following the optimal obfuscation policy of proposition 1 and the firm can change the quality of the varieties. Then the monopolist increases the quality of the first variety and decreases the quality of the second variety.

**B Mass and niche products**

Whether the monopolist obfuscates the variety with more or less dispersed match values depends on whether both varieties are niche or mass products. Again, before determining which variety it
is optimal to obfuscate, I study how the rotation of distribution functions affects profit-maximising prices and the search cost, given that consumers always begin searching from the first variety.

I start with the case where both varieties are niche products. Then, the tails of $f_1$ and $f_2$ are already sufficiently thick. In terms of Figure 1a, the frequency of consumers whose pairs of valuations of both varieties are close to the sides of the square is higher than the frequency in the centre of the square. Because both varieties are niche products, both $p_1^*$ and $p_2^*$ that maximise the profit of the monopolist are to the right of the rotation point. Suppose that $\beta_1$ increases to $\beta'$. Then the tails of $f_1$ become thicker. Thus, regarding Figure 1a, the frequency of consumers whose pairs of valuations of both varieties are close to the upper and lower horizontal sides of the square increases, and the frequency of consumers in the middle of the square decreases. Therefore, the mean $\varepsilon_1$ by consumers who buy the first variety increases, and the firm wants to set a higher $p_1^*$. As a result, if both varieties are niche products, then making the first variety more niche leads to the same result as increasing the quality of the first variety: $p_1^* > p_2^*$ and the monopolist sets $x_2 = p_2^*$. Similarly, if $\beta_2$ increases to $\beta'$ while $\beta_1 = \beta$, then the effect on the profit-maximising prices and the search cost is similar to the effect of increasing the quality of the second variety. In summary, if both varieties are niche products and one variety is more niche than the other, then the monopolist treats the variety that is more niche as a higher quality product and obfuscates the variety that is less niche (proposition 3a).

Now I consider the case when both varieties are mass products. Then the tails of $f_1$ and $f_2$ are very thin and a large mass of consumers draw the match values that are close to the mean. In terms of Figure 1a, the frequency of pairs of valuations of both varieties is very high in the centre of the square and very low near the sides. Because both varieties are mass products, both prices that maximise the profit of the monopolist are to the left of the rotation point. Suppose that $\beta_1$ increases to $\beta'$. Then $F_1$ rotates clockwise and the tails of $f_1$ become thicker. As a result, consumers become more evenly spread over the distribution of the match values of the first variety. Regarding Figure 1a, the frequency of consumers whose pairs of valuations of both varieties are in the centre decreases and the frequency increases near the upper and lower horizontal sides of the square. Then, after the increase in $\beta_1$, more consumers terminate their search without inspecting the second variety, and the firm gets incentives to raise the price of the first variety. However, these incentives are outweighed by the following factor: after the rotation more consumers tend to leave the market without buying anything because the lower tail of $f_1$ is thicker. Because the frequency of consumers near the mean of $\varepsilon_1$ decreases, fewer consumers who search the second variety buy the first variety. As a result, the share of consumers whose valuations of the first variety are very low become very important, and the firm decreases the price $p_1^*$.

After the rotation of $F_1$ the mean $\varepsilon_2$ by consumers who buy the second variety also decreases, which pushes the price of the second variety down. This happens because, after the increase in $\beta_1$, $\beta_2$ increases to $\beta'$ while $\beta_1 = \beta$, then the effect on the profit-maximising prices and the search cost is similar to the effect of increasing the quality of the second variety. In summary, if both varieties are niche products and one variety is more niche than the other, then the monopolist treats the variety that is more niche as a higher quality product and obfuscates the variety that is less niche (proposition 3a).

\footnote{The distribution function $F_1$ decreases in the neighbourhood of $p_1^*$ as $p_2^*$, which is similar to when the quality of the first variety increases.}
the triangle $ABC$ in Figure 1a captures more consumers who draw high valuations of the second variety. However, the negative effect of $\beta_1$ on the mean of $\varepsilon_2$ by consumers who buy the second variety is weaker. As a result, if both varieties are mass products and the first variety becomes less mass, then this affects both profit-maximising prices and the search cost as a decrease in quality of the first variety. In order to prove it, it is sufficient to observe that after $\beta_1$ increases to $\beta'$, the distribution function $F_1$ increases in the neighbourhood of $p_1^* \text{ and } p_2^*$. Likewise, if $\beta_2$ increases, then the effect on the profit-maximising prices and the search cost is similar to the effect when the quality of the second variety decreases. Thus, if the second variety becomes less mass, then the seller sets optimal prices and search costs as if the quality of the first variety were higher. I conclude that the monopolist prefers to obfuscate the variety which is less mass (proposition 3b).

**Proposition 3.** Suppose that the multiproduct monopolist sells two varieties that differ in design.

a) if both varieties are niche products, then the monopolist obfuscates the variety that is less niche;

b) if both varieties are mass products, then the monopolist obfuscates the variety that is less mass.

**Proof.** Similar to the proof of proposition 2.

If a single product monopolist could choose the distribution of its match values in the model of Johnson and Myatt (2006), then the firm would take either very dispersed or very concentrated match values (proposition 1 in Johnson and Myatt, 2006). In other words, the firm would prefer its product to be either a very niche or a purely mass product. This happens because if the product is niche, then by making it more niche the monopolist makes the demand less elastic. Therefore, the monopolist can increase its price without losing many customers. Similarly, if the product is mass and becomes more mass, then the demand becomes more elastic and the firm can attract relatively many consumers by slightly lowering its price. The incentives of the multiproduct obfuscating monopolist are similar. By using the result of proposition 3, I obtain that if both varieties are mass products, then the firm has incentives to rotate the distribution function of the first variety anticlockwise and to rotate the distribution function of the second variety clockwise. After the rotation more consumers terminate their search at the first variety (these consumers are mainly in the triangle $ABC$ in Figure 1a), and many consumers who search the second variety still buy the first variety. Therefore, by making the second variety less mass, the firm not only increases the price of the first variety, but also increases the demand for the first variety, which results in an increase in profits. If both varieties are niche then the monopolist prefers to rotate the distribution function of the first variety clockwise and to rotate the distribution function of the second variety anticlockwise. Then consumers are less sensitive to the price of the first variety. Thus, the monopolist earns more profit by increasing the price of the first product.

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12The same result has been obtained by Bar-Isaac et al. (2012) in proposition 1 in the analysis of a competitive market with sequential consumer search.
Corollary (of proposition 3). Suppose that the monopolist sells two varieties, the varieties are identical ex ante and horizontally differentiated ex post, the firm follows the optimal obfuscation policy of proposition 1 and can change the design of the products. Accordingly, if

- the varieties are mass products, then the firm makes the first variety more mass and the second variety less mass as long as both varieties remain mass products and $p^*_2 < p^*_1$.
- the varieties are niche products, then the firm makes the first variety more niche and the second variety less niche as long as both products are niche.

VI Competition

In the previous sections, I have analysed the incentives of a multiproduct monopolist to obfuscate its varieties. In this section, I show that incentives to obfuscate do not vanish if there is competition in the market. To show this, I study a duopoly model where each seller sells two horizontally differentiated varieties. The match values of products are identically and independently distributed across consumers and products in the interval $[0, 1]$ according to a uniform distribution. The constant unit production cost is identical for both sellers and it is normalised to zero. Consumers do not observe the features of the product without visiting the sellers. When a consumer wants to visit a shop, she pays a positive search cost $s_f$. Additionally, the consumer visits the firms without recall, i.e. if the buyer is at firm $i$ and pays $s_f$ to visit firm $j \neq i$, then she never returns to firm $i$.

Similarly to the case in section IV, the firms choose their profit-maximising prices and whether to obfuscate one of their varieties and by how much. If a firm obfuscates one of its varieties, then a consumer must pay an intra-store search cost $s_p$ to learn the utility of the product. I assume that $s_p$ is sufficiently smaller than $s_f$. This assumption ensures that if a consumer samples the first variety of firm $i$, then she does not leave for firm $j$ without observing both varieties of firm $i$. Inside a shop, consumers search for the utilities of the products with perfect recall. A consumer observes an intra-store search cost only after entering a shop.

Suppose that a consumer is at firm $i$, the highest observed utility there is $u_i$, and the buyer considers visiting firm $j \neq i$. The consumer knows that she will not return to firm $i$. Therefore she bases her decision on a further search by using the expected utility that she can get at firm $j$.

Specifically, a consumer is indifferent between buying at firm $i$ or visiting firm $j$ if equation (8) is satisfied.

$$
\begin{align*}
    u_i &= \int_{x^*_2 - p^*_2}^{1-p^*_1} u_{1j} du_{1j} + (x^*_2 - p^*_2 + p^*_1) \int_{x^*_2 - p^*_2}^{1-p^*_1} u_{2j} du_{2j} \\
    &+ \int_0^{x^*_2 - p^*_2} u^2_{2j} du_{2j} + \int_0^{x^*_2 - p^*_2} u^2_{1j} du_{1j} - s_{p_j} (x^*_2 - p^*_2 + p^*_1) - s_f
\end{align*}
$$

In equation (8), $u_{1j}$ stands for the utility of the first variety of firm $j$, $u_{2j}$ is the utility of the second variety of firm $j$, and $x^*_2 - p^*_2$ is the expected reservation utility of the second variety of firm
I label the RHS of (8) \( \mu_j^* \). Then a consumer goes to firm \( j \) if \( u_i \) is less than \( \mu_j^* \) and terminates her search at firm \( i \) if the inequality is reversed. An important thing that has to be observed here is that \( \mu_j^* \) does not depend on the price and the intra-store search cost of firm \( i \), i.e. the threshold value is based on consumer expectations about the prices and the intra-store search cost of firm \( j \) only.

Suppose that \( \mu_1^* > \mu_2^* > 0 \). Then all consumers start searching from the first firm. As a result, the second seller acts as a monopolist in section IV, i.e., it obfuscates its second variety by setting its reservation utility equal to zero. However, the first firm obfuscates less because an outside option for its customers gives utility \( \mu_2^* \), which is more than zero. Therefore, the expected utility from the first firm is higher than the expected utility from the second firm, and there can be an asymmetric equilibrium in the market where one seller obfuscates less than the other and consumers start their shopping from the less obfuscating seller.

Now I analyse the case when consumers expect \( \mu_1^* = \mu_2^* = \mu^* > 0 \). I show that, irrespective of the choice of firm \( j \), firm \( i \neq j \) always obfuscates. When the expected utilities are identical, consumers sample the sellers randomly: half of the consumers start searching from firm 1 and half of the consumers start searching from firm 2. Suppose that firm \( i \) considers deviation prices \( \{p^1_i, p^2_i\} \neq \{p^1_{1i}, p^2_{1i}\} \) and search cost \( s_{pi} \) per its second variety. Then the pay-off of the seller equals

\[
\pi_i = p_{1i} (d_{11i} + d_{12i}) + p_{2i} (d_{21i} + d_{22i})
\]

where \( d_{11i} \) and \( d_{12i} \) stand for the demands for products 1 and 2 when consumers start searching from firm \( i \), and \( d_{21i} \) and \( d_{22i} \) are the demands for products 1 and 2 when consumers arrive from firm \( j \). The demands from the first group of consumers are very similar to the ones derived in section V. The only difference is that the utility of the outside option for these consumers equals \( \mu^* \) instead of zero. Therefore, the expressions of the demands are

\[
d_{11i} = \frac{1}{2} \left( 1 - (x_{2i} - p^*_{2i} + p_{1i}) + \int_{\mu^*}^{x_{2i} - p^*_{2i}} (\varepsilon + p_{2i}) \, d\varepsilon \right)
\]

\[
d_{12i} = \frac{1}{2} \left( (x_{2i} - p^*_{2i} + p_{1i}) (1 - (x_{2i} - p^*_{2i} + p_{2i})) + \int_{\mu^*}^{x_{2i} - p^*_{2i}} (\varepsilon + p_{1i}) \, d\varepsilon \right)
\]

The firm gets demand from the second group of consumers if the highest observed utility at firm \( j \) is bellow \( \mu^* \). Therefore, the demands from the second group of consumers are

\[
d_{21i} = \frac{1}{2} \left( \mu^* + p^*_{2j} \right) \left( \mu^* + p^*_{1j} \right) \left( 1 - (x_{2i} - p^*_{2i} + p_{1i}) + \int_{0}^{x_{2i} - p^*_{2i}} (\varepsilon + p_{2i}) \, d\varepsilon \right)
\]

\[\text{\textsuperscript{13}}\text{No consumer would visit a firm at which the expected utility is negative.}\]
\[d_{2i}^2 = \frac{1}{2} (\mu^* + p_{2i}^*) (\mu^* + p_{1i}^*) \left( (x_{2i} - p_{2i}^* + p_{1i}) (1 - (x_{2i} - p_{2i}^* + p_{2i})) + \int_{0}^{x_{2i} - p_{2i}^*} (\varepsilon + p_{1i}) d\varepsilon \right)\]

Suppose that seller \(i\) sets positive \(s_{pi}\) such that \(x_{2i} - p_{2i}^* > \mu^*\). Then the firm sells to the same number of consumers as in the absence of obfuscation. However, the seller earns more profit by obfuscating as it screens out its customers’ preferences better. Therefore, the firm has incentives to increase its intra-store search cost as long as it sells to the same number of customers. As a result, it is more profitable for the seller to set \(x_{2i} = \mu_j^* + p_{2i}^*\) than not to obfuscate at all.

Suppose that the firm increases its intra-store search cost further. Then \(d_{1i}^1 = 1 - \mu^* + p_{1i}^*\) and \(d_{2i}^1 = 0\) because the reservation utility of the second variety is below \(\mu^*\). If the seller does so, the firm sets \(x_2 = p_{2i}^*\) and screens out the preferences of the consumers who arrive from firm \(j\) the best. Regardless of the better screening of consumers who arrive from firm \(j\), the loss of \(d_{2i}^1\) has a stronger negative effect on the profit of firm \(i\) than the additionally extracted consumer surplus from the consumers who arrive from firm \(j\). Therefore, in equilibrium, firm \(i\) sets \(s_{pi}\) such that \(x_{2i} = \mu^* + p_{2i}^*\).

**Proposition 4.** Suppose that there are two firms and each sells two horizontally differentiated varieties, and consumers visit the sellers without recall by paying positive search cost \(s_f\) per firm and search with perfect recall within a firm. Then there can be

- an asymmetric equilibrium where both firms obfuscate one of their varieties, one seller sets the reservation utility of its obfuscated variety equal to the expected utility from searching its competitor, the second firm obfuscates more by setting the reservation utility of its obfuscated variety equal to zero, and consumers start searching from the first firm.
- an equilibrium where the expected utilities from searching both sellers are the same, consumers sample the sellers randomly, both firms obfuscate one of their varieties, and each firm sets the reservation utility of its obfuscated variety equal to the expected utility from searching a competitor.

**VII Concluding remarks**

A firm can extract more consumer surplus and earn higher profits if it learns how much a particular consumer values its product. In this paper, I show that a multiproduct seller has incentives to obfuscate its varieties to screen out its customers. By doing this, the firm sacrifices income from its obfuscated varieties because fewer consumers buy them and pay a lower price. However, by discouraging consumers from searching, the firm directs consumers who draw high valuations to buy less obfuscated but more expensive varieties, which results in a higher profit. If there are differences in product design or quality then the firm obfuscates more the varieties for which a consumer is less likely to find a good match. By doing so, the monopolist makes consumer incentives to search the weakest, and this results in the highest profit.
The incentives to obfuscate are present both when a firm is a monopolist and has a competitor. However, the degree of obfuscation in equilibrium depends on a market structure. If a firm is a monopolist, then it chooses the highest-possible degree of obfuscation by setting its intra-store search costs such that the reservation utilities of its products are equal to zero. However, if there is more than one seller in the market, then an obfuscating firm takes into account the expected utility that a consumer may get while visiting a competing firm when it sets intra-store search costs. As a result, if there are several multiproduct sellers in the market, then the degree of obfuscation is lower than in the monopoly.

Although obfuscation increases the profit of a firm, worse matching of products with consumer preferences, higher prices, and positive search costs lead to a lower total welfare. In this sense, obfuscation practices should draw the attention of regulatory bodies.
References


Appendix A

Proof of lemma 1. I prove this lemma by using the following argument: if for all \( l \leq i \) it is true \( p_i^* > p_{i+1}^* \), then \( p_i^* > p_{i+1}^* \).

Suppose that for all \( l \leq i \) we have \( p_i^* > p_{i+1}^* \) but \( p_i^* < p_{i+1}^* \). I denote the LHS of (2) by \( H_i \) and show that it decreases with \( p_i^* \).

\[
\frac{\partial H_i}{\partial p_i^*} = - (1 - x_i - p_i^*) \sum_{l=1}^{i-1} \prod_{j=1, j \neq l}^{i-1} (x_i - p_i^* + p_j^*) - 2 \prod_{j=1}^{i-1} (x_i - p_i^* + p_j^*) \frac{d\varepsilon}{d\varepsilon} \\
- \sum_{h=1}^{i-1} p_h^* \prod_{j=1, j \neq h}^{i-1} (x_i - p_h^* + p_j^*) \\
= -2 \prod_{j=1}^{i-1} (x_i - p_i^* + p_j^*) - \sum_{h=1}^{i-1} (1 - (x_i - p_h^* + p_i^*)) \prod_{j=1, j \neq h}^{i-1} (x_i - p_i^* + p_j^*) < 0
\]

Now I define \( G_i \equiv H_i \prod_{j=1}^{i-1} (x_i - p_i^* + p_j^*)^{-1} = 0 \) and take the derivative of \( G_i \) with respect to \( p_i^* \).

\[
\frac{\partial G_i}{\partial p_i^*} = \frac{2 \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{i, -1\}}^{l-1} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} - \frac{\sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{i, -1\}}^{l-1} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} \\
+ \frac{\sum_{h=1}^{i-1} p_h^* \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{h, i, -1\}}^{l} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} - \frac{\sum_{h=1}^{i-1} p_h^* \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{h, i, -1\}}^{l} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} \\
+ \frac{\sum_{h=1}^{i-1} p_h^* \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{h, i, -1\}}^{l} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} - \frac{\sum_{h=1}^{i-1} p_h^* \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{h, i, -1\}}^{l} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} \\
+ \frac{\sum_{h=1}^{i-1} p_h^* \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{h, i, -1\}}^{l} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} - \frac{\sum_{h=1}^{i-1} p_h^* \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{h, i, -1\}}^{l} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} \\
= \frac{\sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{i, -1\}}^{l-1} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} - \frac{\sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{i, -1\}}^{l-1} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} \\
+ \frac{\sum_{h=1}^{i-1} p_h^* \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{h, i, -1\}}^{l} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} - \frac{\sum_{h=1}^{i-1} p_h^* \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{h, i, -1\}}^{l} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} \\
+ \frac{\sum_{h=1}^{i-1} p_h^* \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{h, i, -1\}}^{l} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} - \frac{\sum_{h=1}^{i-1} p_h^* \sum_{l=i}^{k} \int_{x_{i+1} - p_i^*}^{x_{i+1} - p_i^*} \prod_{j=1, j \neq \{h, i, -1\}}^{l} (\varepsilon + p_j^*) \frac{d\varepsilon}{d\varepsilon}}{\prod_{j=1}^{i-2} (x_i - p_i^* + p_j^*) (x_i - p_i^* + p_{i-1}^*)} > 0
\]
Therefore, if \( p^*_i < p^*_{i+1} \), then

\[
H_{i|p^*_i=p^*_{i+1}} < 0 \quad \text{and} \quad G_{i+1|p^*_i=p^*_{i+1}} > 0
\]

More particularly, it must be that

\[
H_{i|p^*_i=p^*_{i+1}} = \prod_{j=1}^{i-1} (x_i - p^*_i + p^*_j) (1 - x_i - p^*_i) + \int_{x_{i+1} - p^*_i}^{x_i} \prod_{j=1}^{i-1} (\varepsilon + p^*_j) \, d\varepsilon
\]

\[
+ \sum_{h=1}^{i-1} p^*_h \int_{x_{i+1} - p^*_i + 1}^{x_i - p^*_i} \prod_{j=1, j \neq \{i, h\}}^l (\varepsilon + p^*_j) \, d\varepsilon
\]

\[
+ p^*_i \left( \prod_{j=1}^{i-1} (x_{i+1} - p^*_i + p^*_j) (1 - x_{i+1}) + \sum_{l=i+1}^k \left( \int_{x_{i+1} - p^*_i + 1}^{x_i - p^*_i} \prod_{j=1, j \neq \{i, i+1\}}^l (\varepsilon + p^*_j) \, d\varepsilon \right) \right)
\]

\[
+ g_i < 0
\]

and

\[
\prod_{j=1}^{i-1} (x_{i+1} - p^*_i + p^*_j) x_{i+1} \left( G_{i+1|p^*_i=p^*_{i+1}} \right) = \prod_{j=1}^{i-1} (x_{i+1} - p^*_i + p^*_j) x_{i+1} (1 - x_{i+1} - p^*_i)
\]

\[
+ p^*_i \sum_{l=i+1}^k \int_{x_{i+1} - p^*_i + 1}^{x_i - p^*_i} \prod_{j=1, j \neq \{i, i+1\}}^l (\varepsilon + p^*_j) \, d\varepsilon + g_i > 0
\]

where

\[
g_i = \sum_{l=i+1}^k \left( \int_{x_{i+1} - p^*_i + 1}^{x_i - p^*_i} \prod_{j=1, j \neq i}^l (\varepsilon + p^*_j) \, d\varepsilon \right) + \sum_{h=1}^{i-1} p^*_h \sum_{l=i+1}^k \int_{x_{i+1} - p^*_i + 1}^{x_i - p^*_i} \prod_{j=1, j \neq \{i, h\}}^l (\varepsilon + p^*_j) \, d\varepsilon
\]

\[
+ \sum_{h=i+2}^k p^*_h \left( \prod_{j=1, j \neq i}^{h-1} (x_h - p^*_h + p^*_j) (1 - x_h) + \sum_{l=h}^k \left( \int_{x_{i+1} - p^*_i + 1}^{x_i - p^*_i} \prod_{j=1, j \neq \{i, h\}}^l (\varepsilon + p^*_j) \, d\varepsilon \right) \right) > 0
\]
Now I take difference between $H_i|_{p_l^* = p_{l+1}}$ and $\prod_{j=1}^{i-1} (x_{i+1} - p_l^* + p_j^*) x_{i+1} \left( G_{i+1} |_{p_l^* = p_{l+1}} \right)$.

$$H_i|_{p_l^* = p_{l+1}} - \prod_{j=1}^{i-1} (x_{i+1} - p_l^* + p_j^*) x_{i+1} \left( G_{i+1} |_{p_l^* = p_{l+1}} \right) =$$

$$\prod_{j=1}^{i-1} (x_i - p_l^* + p_j^*) (1 - x_i - p_{l+1}^*) + \int_{x_{i+1} - p_{l+1}^*}^{x_l - p_{l-1}^*} \prod_{j=1}^{i-1} (\varepsilon + p_j^*) d\varepsilon +$$

$$\sum_{h=1}^{i-1} p_h^* \left( \prod_{j=1}^{i-1} (x_{i+1} - p_{l+1}^* + p_j^*) (1 - x_{i+1}) + \sum_{l=i+1}^{k} \left( \int_{x_{i+1} - p_{l+1}^*}^{x_l - p_{l-1}^*} \prod_{j=1}^{l} (\varepsilon + p_j^*) d\varepsilon \right) \right) +$$

$$\sum_{h=1}^{i-1} p_h^* \int_{x_{i+1} - p_{l+1}^*}^{x_l - p_{l-1}^*} \prod_{j=1}^{l} (\varepsilon + p_j^*) d\varepsilon - \prod_{j=1}^{i-1} (x_{i+1} - p_{l+1}^* + p_j^*) x_{i+1} (1 - x_{i+1} - p_l^*) -$$

$$p_l^* \sum_{l=i+1}^{k} \int_{x_{i+1} - p_{l+1}^*}^{x_l - p_{l-1}^*} \prod_{j=1}^{l} (\varepsilon + p_j^*) d\varepsilon$$

The difference increases with $x_i$ because its derivative with respect to $x_i$ is positive:

$$\frac{\partial}{\partial x_i} \left( H_i|_{p_l^* = p_{l+1}} - \prod_{j=1}^{i-1} (x_{i+1} - p_{l+1}^* + p_j^*) x_{i+1} \left( G_{i+1} |_{p_l^* = p_{l+1}} \right) \right) =$$

$$\sum_{l=1}^{i-1} \prod_{j=1, j \neq l}^{i-1} (x_i - p_{l+1}^* + p_j^*) (1 - x_i - p_{l+1}^* + p_l^*) > 0$$

Thus, the difference is larger than after setting $x_i = x_{i+1}$. After this replacement the difference becomes

$$H_i|_{p_l^* = p_{l+1}} - \prod_{j=1}^{i-1} (x_{i+1} - p_{l+1}^* + p_j^*) x_{i+1} \left( G_{i+1} |_{p_l^* = p_{l+1}} \right) >$$

$$H_i|_{p_l^* = p_{l+1}} - \prod_{j=1}^{i-1} (x_{i+1} - p_{l+1}^* + p_j^*) x_{i+1} \left( G_{i+1} |_{p_l^* = p_{l+1}} \right) \bigg|_{x_i = x_{i+1}} =$$

$$\prod_{j=1}^{i-1} (x_{i+1} - p_{l+1}^* + p_j^*) (1 - x_{i+1})^2 > 0$$

The last result establishes the contradiction and, therefore, $p_l^* > p_{l+1}^* \forall i \in [1, k]$.

**Proof of proposition 1.** By using lemma 1 one gets that the LHS of (3) is always negative. Thus, the monopolist will set $x_i$ the smallest possible, which is $p_l^*$. By setting all $x_l = p_l^* \forall l \in [2, k]$ on the LHS of (2) I obtain

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The profit of the monopolist in equilibrium is

\[ p_i^* = \frac{1}{2} \left( 1 + \sum_{k=i+1}^{h-1} p_h^* \prod_{j=i+1}^{h-1} p_j^* \right) \]

\[ = \frac{1}{2} \left( 1 + p_{i+1}^* (1 - p_{i+1}^*) + p_{i+1}^* \left( \sum_{h=i+2}^{k} p_h^* \prod_{j=i+2}^{h-1} p_j^* \right) \right) \]

\[ = \frac{1}{2} \left( 1 + p_{i+1}^* (1 - p_{i+1}^*) + p_{i+1}^* (2p_{i+1}^* - 1) \right) = \frac{1}{2} \left( 1 + (p_{i+1}^*)^2 \right) \]

The profit of the monopolist in equilibrium is

\[ \pi^* = \sum_{i=1}^{k} \prod_{j=1}^{i-1} p_j^* p_i^* (1 - p_i^*) = p_i^* (1 - p_i^*) + p_i^* \left( \sum_{i=2}^{k} \prod_{j=2}^{i-1} p_j^* (1 - p_i^*) \right) \]

\[ = p_i^* (1 - p_i^*) + p_i^* (2p_i^* - 1) = (p_i^*)^2 \]

**Local concavity**

All diagonal elements of the Hessian matrix \( H \) at the point where \( x_i = p_i^* \) and \( p_i = p_i^* \) are negative:

\[ H_{ii} = \frac{\partial^2 \pi}{\partial p_i^2} \bigg|_{p_i=p_i^*, x_i=p_i^*} = -2 \prod_{j=1}^{i-1} p_j^* < 0 \]

The elements that are to the left of the diagonal equal zero when \( x_i = p_i^* \) and \( p_i = p_i^*, \forall i: \)

\[ H_{ii} = \frac{\partial^2 \pi}{\partial p_i \partial p_l} \bigg|_{p_i=p_i^*, x_i=p_i^*, p_l=p_l^*} = \prod_{j=1, j\neq l}^{i-1} p_j^* (1 - 2p_i^*) + \sum_{j=i+1}^{k} p_j^* (1 - p_j^*) \prod_{h=1, h\neq \{i,l\}}^{j-1} p_h^* = 0 \]

where \( l \) is the index of a column and \( l < i \).

The elements that are to the right of the diagonal also equal zero when \( x_i = p_i^* \) and \( p_i = p_i^*, \forall i: \)

\[ H_{ii} = \frac{\partial^2 \pi}{\partial p_i \partial p_l} \bigg|_{p_i=p_i^*, x_i=p_i^*, p_l=p_l^*} = \prod_{j=1, j\neq i}^{i-1} p_j^* (1 - 2p_i^*) + \sum_{j=i+1}^{k} p_j^* (1 - p_j^*) \prod_{h=1, h\neq \{i,l\}}^{j-1} p_h^* = 0 \]

where \( l > i \). Hence, the determinant of the \( i^{th} \) principal minor of the Hessian matrix is

\[ |A_i| = \prod_{j=1}^{i} H_{jj} \]

It is negative when \( i \) is odd and positive when \( i \) is even. Hence the pay-off function is locally concave in prices.  

\[ \square \]
Proof of proposition 2. Suppose that only $\beta_1$ increases and $\beta_2 = \beta$ remains the same. If the inequality $p_1^* > p_2^*$ is preserved for any $x_2$, then the monopolist sets $x_2 = p_2^*$. If after an increase in $\beta_1$ the monopolist sets $x_2 = p_2^*$, then I show that the inequality $p_1^* > p_2^*$ remains. For that I take the derivatives of the first order conditions (5) and (6) with respect to both prices and $\beta$.

To obtain the second line, I have used $\lim_{x_2 \to p_2^*} H_1^a (p_1^*, p_2^*) = 0$, which has allowed to replace $p_2 (1 - F_2 (p_2^*)) = (F_1 (p_1^*) - 1) / f_1 (p_1^*) + p_1^*$.

$$
\lim_{x_2 \to p_2^*} \frac{\partial H_1^a}{\partial p_1^*} = -2 f_1 (p_1^*) - p_1^* f_1' (p_1^*) + p_2^* f_1' (p_1^*) (1 - F_2 (p_2^*))
$$

$$
= -2 f_1 (p_1^*) - f_1' (p_1^*) \frac{1 - F (p_1^*)}{f (p_1^*)} < 0
$$

To obtain the second line, I have used $\lim_{x_2 \to p_2^*} H_1^a (p_1^*, p_2^*) = 0$, which has allowed to replace $p_2 (1 - F_2 (p_2^*)) = (F_1 (p_1^*) - 1) / f_1 (p_1^*) + p_1^*$.

$$
\lim_{x_2 \to p_2^*} \frac{\partial H_1^a}{\partial p_1^*} = 2 f_1 (p_1^*) (1 - F_2 (p_2^*)) + (p_1^* - p_2^*) f_1' (p_1^*) (1 - F_2 (p_2^*))
$$

$$
- p_2^* f_2 (p_2^*) f_1 (p_1^*)
$$

$$
= 2 f_1 (p_1^*) (1 - F_2 (p_2^*)) + (p_1^* - p_2^*) f_1' (p_1^*) (1 - F_2 (p_2^*)) - (1 - F_2 (p_2^*)) f_1 (p_1^*)
$$

$$
= (1 - F_2 (p_2^*)) \left( f_1 (p_1^*) + (p_1^* - p_2^*) f_1' (p_1^*) \right) > 0
$$

where the first equality has been obtained by using the first order condition (6).

$$
\lim_{x_2 \to p_2^*} \frac{\partial H_1^a}{\partial \beta_1} = \frac{\partial F_1 (p_1^*)}{\partial \beta_1} + p_2^* \frac{\partial f_1 (p_1^*)}{\partial \beta_1} (1 - F_2 (p_2^*)) - p_1^* \frac{\partial f_1 (p_1^*)}{\partial \beta_1} > 0
$$

$$
\lim_{x_2 \to p_2^*} \frac{\partial G_1^a}{\partial p_1^*} = f_1 (p_1^*) (1 - F_2 (p_2^*)) - p_2^* f_1 (p_1^*) f_2 (p_2^*) = 0
$$

$$
\lim_{x_2 \to p_2^*} \frac{\partial G_2^a}{\partial p_2^*} = -f_1 (p_1^*) (1 - F_2 (p_2^*)) - F_1 (p_1^*) \left( 2 f_2 (p_2^*) + p_2^* f_2' \right) + (p_2^* - p_1^*) f_1 (p_1^*) f_2 (p_2^*)
$$

$$
= -F_1 (p_1^*) \left( 2 f_2 (p_2^*) + p_2^* f_2' \right) - p_1^* f_1 (p_1^*) f_2 (p_2^*) < 0
$$

where the equality has been obtained by replacing $1 - F_2 (p_2^*) = p_2^* f_1 (p_2^*)$.

$$
\lim_{x_2 \to p_2^*} \frac{\partial G_2^a}{\partial \beta_1} = \frac{\partial F_1 (p_1^*)}{\partial \beta_1} (1 - F_2 (p_2^*) - p_2^* f_2 (p_2^*)) = 0
$$

Then, by applying Cramer’s rule, I define the signs of the derivatives of $p_1^*$ and $p_2^*$ with respect
\[
\lim_{x_2 \to p^*_2} \frac{\partial H_1^a}{\partial p_1^*} \lim_{x_2 \to p^*_2} \frac{\partial G_2^a}{\partial p_2^*} - \lim_{x_2 \to p^*_2} \frac{\partial G_2^a}{\partial p_1^*} \lim_{x_2 \to p^*_2} \frac{\partial H_1^a}{\partial p_2^*} > 0
\]

Therefore,
\[
\operatorname{sgn}\left[\lim_{x_2 \to p^*_2} \frac{\partial p_1^*}{\partial \beta_1}\right] = \operatorname{sgn}\left[- \lim_{x_2 \to p^*_2} \frac{\partial H_1^a}{\partial \beta_1} \lim_{x_2 \to p^*_2} \frac{\partial G_2^a}{\partial p_2^*} + \lim_{x_2 \to p^*_2} \frac{\partial G_2^a}{\partial \beta_1} \lim_{x_2 \to p^*_2} \frac{\partial H_1^a}{\partial p_2^*}\right] = 1
\]
\[
\operatorname{sgn}\left[\lim_{x_2 \to p^*_2} \frac{\partial p_2^*}{\partial \beta_1}\right] = \operatorname{sgn}\left[- \lim_{x_2 \to p^*_2} \frac{\partial G_2^a}{\partial \beta_1} \lim_{x_2 \to p^*_2} \frac{\partial H_1^a}{\partial p_2^*} + \lim_{x_2 \to p^*_2} \frac{\partial H_1^a}{\partial \beta_1} \lim_{x_2 \to p^*_2} \frac{\partial G_2^a}{\partial p_1^*}\right] = 0
\]

Therefore, if \(\beta_1\) increases and \(x_2 = p^*_2\) then \(p^*_1 > p^*_2\).

From equation (7) one can see that the monopolist obfuscates the second variety by setting \(x_2 = p^*_2\) if \(p^*_1 > p^*_2\), and \(x_2 = 1\) if \(p^*_2 > p^*_1\). Therefore, after the change in both prices which happens after \(\beta_2\) increases and \(\beta_1 = \beta\) does not change, the monopolist either fully obfuscates the second variety or does not obfuscate at all.

Suppose that \(p^*_2 < p^*_1\) and the monopolist fully obfuscates the second variety \((x_2 = p^*_2)\). Then the equilibrium prices and profits are
\[
p^*_2 = \frac{1 - F_2(p^*_2)}{f_2(p^*_2)} \quad p^*_1 = \frac{1 - F_1(p^*_1)}{f_1(p^*_1)} + p^*_2 (1 - F_2(p^*_2))
\]

Then by applying the envelope theorem I get
\[
\frac{\partial \pi^*}{\partial \beta_1} = \frac{\partial F_1(p^*_1)}{\partial \beta_1} (p^*_2 (1 - F_2(p^*_2)) - p^*_1) > 0
\]

and
\[
\frac{\partial \pi^*}{\partial \beta_2} = F_1(p^*_1) p^*_2 \frac{\partial F_2(p^*_2)}{\partial \beta_2} < 0
\]

The profit of the firm increases with the quality of the first product and decreases with the quality of the second product.

Suppose that the second variety is a higher-quality product but because of the search order \(p^*_1 > p^*_2\) and \(x_2 = p^*_2\). If the seller swapped the products, then the price of the higher-quality product would be higher than the price of the lower-quality product and the seller would set the maximum obfuscation degree. Additionally, as the profit of the obfuscating monopolist increases with the quality of the first variety and decreases with the quality of the second variety, the multiproduct monopolist would earn more after the swap.

Now suppose that the second variety is of a higher quality but the quality difference is so large that the second price is higher than the first. Then the firm does not obfuscate. However, the seller would definitely earn more by obfuscating the lower-quality product than not obfuscating at all.
a result, the if the monopolist sells a lower and a higher-quality products then it always obfuscates a lower-quality product. □

Appendix B

Lemma 2. Suppose that $\beta_1 = \beta_2 = \beta$ and there is a pair of prices that satisfies (5)-(6) for any $x_2$. Then $p_1^* > p_2^*$ and $x_2 = p_2^*$.

Proof. From equation (5) I define an implicit solution $p_1^* = \nu_1 (p_2^*)$. Similarly, from equation (6) I define $p_1^* = \nu_2 (p_2^*)$. I label the LHS of (5) as $H_1^a$ and the LHS of (6) as $G_2^a$. Now I prove two claims that are used to prove the result of the lemma.

Claim 1. $\nu_1 (0) > 0$ and $\nu_2 (0) < 0$.

Proof.

\[
H_1^a (p_1^*, 0) = 1 - F (x_2 + p_1^*) + \int_{p_1^*}^{x_2+p_1^*} F (\varepsilon - p_1^*) dF (\varepsilon) - p_1^* f (x_2 + p_1^*) (1 - F (x_2)) \\
- p_1^* \int_{p_1^*}^{x_2+p_1^*} f (\varepsilon - p_1^*) dF (\varepsilon)
\]

$H_1^a (p_1^*, 0)$ decreases with $p_1^*$ because its derivative with respect to $p_1^*$ is negative:

\[
\frac{\partial H_1^a (0, p_1^*)}{\partial p_1^*} = -2 f (x_2 + p_1^*) (1 - F (x_2)) - 2 \int_{p_1^*}^{x_2+p_1^*} f (\varepsilon - p_1^*) dF (\varepsilon) - p_1^* f (x_2 + p_1^*) (1 - F (x_2)) \\
- p_1^* \int_{p_1^*}^{x_2+p_1^*} f (\varepsilon - p_1^*) f' (\varepsilon) d\varepsilon < 0
\]

Additionally,

\[
H_1^a (0, 0) = 1 - F (x_2) + \int_0^{x_2} F (\varepsilon) dF (\varepsilon) > 0
\]

Therefore, I conclude that $\nu_1 (0) > 0$.

Now I take equation (6).

\[
G_2^a (p_1^*, 0) = F (x_2 + p_1^*) (1 - F (x_2)) + \int_0^{x_2} F (\varepsilon + p_1^*) dF (\varepsilon) \\
+ p_1^* \int_0^{x_2} f (\varepsilon + p_1^*) dF (\varepsilon)
\]
\( G_2^a(p_1^*, 0) \) increases with \( p_1^* \) because its derivative with respect to \( p_1^* \) is positive:

\[
\frac{\partial G_2^a(p_1^*, 0)}{\partial p_1^*} = f(x_2 + p_1^*) (1 - F(x_2)) + 2 \int_0^{x_2} f(\varepsilon + p_1^*) dF(\varepsilon) + p_1^* \int_0^{x_2} f'(\varepsilon + p_1^*) dF(\varepsilon)
\]

Additionally,

\[
G_2^a(0, 0) = F(x_2) (1 - F(x_2)) + \int_0^{x_2} F(\varepsilon) dF(\varepsilon) > 0
\]

Therefore, \( \nu_2(0) < 0 \)

Claim 2. Both \( \nu_1(p_2^*) \) and \( \nu_2(p_2^*) \) cross 45-degrees line in the plane \( p_2^* \times p_1^* \in [0, x_2] \times [0, \bar{\varepsilon}] \) once and \( \nu_1(p_2^*) \) does it later than \( \nu_2(p_2^*) \).

Proof. At the point where \( \nu_1(p_2^*) \) crosses 45-degree line \( p_1^* = p_2^* = p_a \); and the point where \( \nu_2(p_2^*) \) crosses 45-degree line \( p_1^* = p_2^* = p_b \). I start with showing that \( p_a \) and \( p_b \) are unique. For that it is sufficient to show that \( H_1^a(p_a) \) decreases with \( p_a \), \( G_2^a \) decreases with \( p_b \), both functions are negative when \( p_b = x_2 \) and \( p_a = \bar{\varepsilon} \) and positive when \( p_a = p_b = 0 \)

\[
H_1^a(p_a) = 1 - F(x_2) + \int_{p_a}^{x_2} F(\varepsilon) dF(\varepsilon) - p_a F(p_a) f(p_a)
\]

(9)

\[
G_2^a(p_b) = F(x_2) (1 - F(x_2)) + \int_{p_b}^{x_2} F(\varepsilon) dF(\varepsilon) - p_b F(p_b) f(p_b)
\]

(10)

\[
\frac{\partial H_1^a(p_a)}{\partial p_a} = -2F(p_a)f(p_a) - p_a f^2(p_a) - p_a F(p_a) f'(p_a) < 0
\]

\[
\frac{\partial G_2^a}{\partial p_b} = -2F(p_b)f(p_b) - p_b f^2(p_b) - p_b F(p_b) f'(p_b) < 0
\]

Also note that

\[
H_1^a(p_a)|_{p_a=0} = 1 - F(x_2) + \int_0^{x_2} F(\varepsilon) dF(\varepsilon) > 0
\]

\[
G_2^a(p_b)|_{p_b=0} = F(x_2) (1 - F(x_2)) + \int_0^{x_2} F(\varepsilon) dF(\varepsilon) > 0
\]
\[ G_2^a (p_b) \bigg|_{p_b = x_2} = F (x_2) (1 - F (x_2) - x_2 f (x_2)) < 0 \]

\[ H_1^a (p_a) \bigg|_{p_a = x} = \int_x^{x_2} F_2 (\varepsilon) dF_1 (\varepsilon) - \varepsilon f_1 (\varepsilon) \leq 0 \]

Now I use a contradiction to prove that \( p_a > p_b \). Suppose it is on the contrary: \( p_a < p_b \) and both prices are defined by (9) and (10).

After taking the derivative of \( G^a_2 (p_b) \) with respect to \( x_2 \), I obtain that the derivative is positive:

\[ \frac{\partial G^a_2 (p_b)}{\partial x_2} = f (x_2) (1 - F (x_2)) > 0 \]

Thus, if \( G^a_2 (p_b) = 0 \), then

\[ \int_{p_b}^x F (\varepsilon) dF (\varepsilon) - p_b F (p_b) f (p_b) = \frac{1}{2} (1 - F^2 (p_b) - 2p_b F (p_b) f (p_b)) \geq 0 \] \( (11) \)

It has been shown that \( \partial H^a_1 / \partial p_a < 0 \). As a result, if \( p_b > p_a \) then \( H^a_1 (p_b) \) must be negative. By using \( G^a_2 (p_b) = 0 \), which allows to replace \(-F (x_2) + \frac{1}{2} F (x_2)^2 \) by \(-\frac{1}{2} F (p_b)^2 - p_b F (p_b) f (p_b) \), I get

\[ H^a_1 (p_b) = 1 - F^2 (p_b) - 2 p_b F (p_b) f (p_b) \]

By using (11), I get that \( H^a_1 (p_b) \geq 0 \), which establishes the contradiction and \( p_a > p_b \). \( \square \)

Then, by using claims 1 and 2, I conclude that if \( \beta_1 = \beta_2 = \beta \), then \( \nu_1 (p_2) \) and \( \nu_2 (p_2) \) cross in the area where \( p_1 > p_2 \) and \( p_1^* > p_2^* \). The fact that \( x_2 = p_2^* \) follows from (7). \( \square \)

**Local concavity.**

When \( x_2 = p_2^* \), then by using the first order conditions at the equilibrium point I obtain that the diagonal elements of the Hessian matrix \( \mathcal{H} \) at the equilibrium are

\[ \mathcal{H}_{11} = -2 f (p_1^*) - p_1^* f^\prime (p_1^*) + p_2 f^\prime (p_1^*) (1 - F (p_2^*)) \]
\[ = -2 f (p_1^*) - p_1^* f^\prime (p_1^*) + f^\prime (p_1^*) \left( -\frac{1 - F (p_1^*)}{f (p_1^*)} + p_1^* \right) \]
\[ = -2 f_1^2 (p_1^*) - f_1 (p_1^*) (1 - F_1 (p_1^*)) \frac{1}{f_1 (p_1^*)} < 0 \]

\[ \mathcal{H}_{12} = \mathcal{H}_{21} = f (p_1^*) (1 - F (p_2^*) - p_2^* f (p_2^*)) = 0 \]
\[ \mathcal{H}_{22} = -2F(p_1^*)f(p_2^*) - p_2^*F(p_1^*)f'(p_2^*) < 0 \]

The diagonal elements of the Hessian matrix are negative and \(|\mathcal{H}| > 0\). Thus, the pay-off function is locally concave at equilibrium.

**The changes of** \(p_1^*\) **and** \(p_2^*\) **when** \(\beta_2\) **increases**, \(x_2 = p_2^*\) **and** \(F_i(\varepsilon) = \varepsilon^{\beta_i}\)**

\[
\text{sgn} \left[ -\lim_{x_2 \to p_2^*} \frac{\partial H_1^a}{\partial \beta_2} \lim_{x_2 \to p_2^*} \frac{\partial G_2^a}{\partial p_2^*} + \lim_{x_2 \to p_2^*} \frac{\partial G_2^a}{\partial \beta_2} \left( \lim_{x_2 \to p_2^*} \frac{\partial H_1^a}{\partial p_2^*} + \lim_{x_2 \to p_2^*} \frac{\partial H_1^a}{\partial p_1^*} \right) \right] =
\]

\[
\lim_{x_2 \to p_2^*} \frac{\partial H_1^a}{\partial \beta_2} = -(p_2^* - p_1^*)f_1(p_1^*) \frac{\partial F_2(p_2^*)}{\partial \beta_2} - p_1^* \frac{\partial F_2(p_2^*)}{\partial \beta_2} f_1(p_1^*)
\]

\[
= -\frac{\partial F_2(p_2^*)}{\partial \beta_2} p_2^* f_1(p_1^*) = -\beta (p_1^*)^{\beta - 1} (p_2^*)^{\beta_2 + 1} \ln p_2^*
\]

\[
\lim_{x_2 \to p_2^*} \frac{\partial G_2^a}{\partial \beta_2} = -F_1(p_1^*) \frac{\partial F_2(p_2^*)}{\partial \beta_2} - p_2^* F_1(p_1^*) \frac{\partial f_2(p_2^*)}{\partial \beta_2}
\]

\[
= -(p_1^*)^\beta (p_2^*)^{\beta_2} (1 + \beta_2) \ln p_2^*
\]

\[
\lim_{x_2 \to p_2^*} \frac{\partial G_2^a}{\partial p_2^*} = -F_1(p_1^*) \left( 2f_2(p_2^*) + p_2^* f_2(p_2^*) \right) - p_1^* f_1(p_1^*) f_2(p_2^*)
\]

\[
= -\beta_2 (p_1^*)^\beta (p_2^*)^{\beta_2 - 1} (1 + \beta_2 - \beta)
\]

\[
\lim_{x_2 \to p_2^*} \frac{\partial H_1^a}{\partial p_2^*} = (1 - F_2(p_2^*)) \left( f_1(p_1^*) + (p_1^* - p_2^*) f'_1(p_1^*) \right)
\]

\[
= p_2^* f(p_2^*) \left( f_1(p_1^*) + (p_1^* - p_2^*) f'_1(p_1^*) \right)
\]

\[
= \beta_2 (p_2^*)^{\beta_2 - 1} \beta (p_1^*)^{\beta - 2} (p_1^* + (\beta - 1) (p_1^* - p_2^*))
\]
\[
\lim_{x_2 \to p_2} \frac{\partial H_1^a}{\partial p_1^*} = -2 f_1 (p_1^*) - p_1^* f_1' (p_1^*) + p_2^* f_1' (p_1^*) (1 - F_2 (p_2^*)) \\
= -2 f_1 (p_1^*) - f_1' (p_1^*) (p_1^* + f (p_2^*) p_2^*) \\
= -\beta (p_1^*)^{\beta-2} \left( 2p_1^* + (\beta - 1) \left( p_1^* + \beta_2 (p_2^*)^{\beta_2} \right) \right)
\]

\[
- \lim_{x_2 \to p_2} \frac{\partial H_1^a}{\partial \beta_2} \lim_{x_2 \to p_2} \frac{\partial G_2^a}{\partial p_2^*} + \lim_{x_2 \to p_2} \frac{\partial G_2^a}{\partial \beta_2} \left( \lim_{x_2 \to p_2} \frac{\partial H_1^a}{\partial p_2^*} + \lim_{x_2 \to p_2} \frac{\partial H_1^a}{\partial \beta_1^*} \right) = \\
(p_1^*)^{2\beta-1} \beta (p_2^*)^{\beta_2-1} \ln p_2^* \left[ (\beta + 1) (\beta_2 + 1) + \beta_2 (p_2^*)^{\beta_2-1} (\beta \beta_2 + \beta + p_2^* (\beta - \beta_2 - 1)) \right] \tag{12}
\]

The expression in the brackets of (12) increases with \( \beta \) because its derivative with respect to \( \beta \) is positive:

\[
\beta_2 + 1 + \beta_2 (p_2^*)^{\beta_2-1} (\beta_2 + 1 + p_2^* ) > 0
\]

Therefore, the expression in the brackets of (12) attains its smallest value when \( \beta = 1 \). After setting \( \beta = 1 \), the expression simplifies to \( 2 (\beta_2 + 1) + \beta_2^2 (p_2^*)^{\beta_2-1} (1 - p_2^*) > 0 \). As a result, (12) is negative, which implies that \( p_1^* \) increases with \( \beta_2 \) slower than \( p_2^* \).

**Proof of proposition 4**

The first part of the proposition has been proved in section VI. Thus, here I prove the second part of the proposition. It has been discussed in section VI that a firm can earn more by setting \( x_2i = \mu^* + p_{2i}^* \) than any lower \( s_{pi} \). Therefore, it remains to show that setting \( x_2i = \mu^* + p_{2i}^* \) is more profitable that setting \( x_2i = p_{2i}^* \).

Suppose that \( x_2i \) is set such that \( x_2i - p_{2i}^* \geq \mu^* \). Then the first-order conditions with respect to both prices are

\[
1 - x_2i + p_{2i}^* - 2p_{1i}^* + \int_{\mu^*}^{x_2i - p_{2i}^*} (\varepsilon + p_{2i}^*) d\varepsilon + p_{2i}^* (1 - p_{2i}^* - \mu^*) + \\
(\mu^* + p_{2i}^* \left( \mu^* + p_{1i}^* \right) \left( 1 - x_2i - 2p_{1i}^* + \int_{0}^{x_2i - p_{2i}^*} (\varepsilon + p_{2i}^*) d\varepsilon + p_{2i}^* (2 - p_{2i}^*) \right) = 0 \tag{13}
\]

\[
p_{1i}^* (x_2i - p_{2i}^* - \mu^*) + (x_2i - p_{2i}^* + p_{1i}^*) (1 - x_2i - p_{2i}^*) + \int_{\mu^*}^{x_2i - p_{2i}^*} (\varepsilon + p_{1i}^*) d\varepsilon + \\
(\mu^* + p_{2i}^* \left( \mu^* + p_{1i}^* \right) \left( p_{1i}^* (x_2i - p_{2i}^*) + (x_2i - p_{2i}^* + p_{1i}^*) (1 - x_2i - p_{2i}^*) + \int_{0}^{x_2i - p_{2i}^*} (\varepsilon + p_{1i}^*) d\varepsilon \right) = 0 \tag{14}
\]
If the firm sets \( x_{2i} = \mu^* + p_{2i}^* \). Then the pair of profit-maximising prices satisfy (15) and (16).

\[
1 - \mu^* - 2p_{1i}^* + p_{2i}^* (1 - p_{2i}^* - \mu^*) + \\
(\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) \left( 1 - \mu^* - 2p_{1i}^* + p_{2i}^* + \frac{1}{2} (\mu^*)^2 + \mu^* p_{2i}^* - (p_{2i}^*)^2 \right) = 0 \quad (15)
\]

\[
(\mu^* + p_{1i}) (1 - \mu^* - 2p_{2i}^*) + \\
(\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) \left( p_{1i}^* \mu^* - 2(\mu^* p_{2i}^* + \mu^* + p_{1i} (1 - 2p_{2i}^*) - \frac{1}{2} (\mu^*)^2 \right) = 0 \quad (16)
\]

Now I look at the case when firm \( i \) behaves as a monopolist: \( x_{2i} = p_{2i}^* \). In order to distinguish between the two different search costs, if the seller sets \( x_{2i} = p_{2i} \) then its profit-maximising prices are denoted by \( \tilde{p}_{1i} \) and \( \tilde{p}_{2i} \), and the profit of the firm is \( \tilde{\pi} \). If the firm sets \( x_{2i} = \mu^* + p_{2i}^* \) then \( p_{1i} = p_{1i}^* \) and \( p_{2i} = p_{2i}^* \) are its profit maximizing prices, and the seller earns \( \pi^* \). If \( x_{2i} = \tilde{p}_{2i} \) then the demand \( d_{2i}^* \) equals zero and \( d_{1i} = 1 - \mu^* - \tilde{p}_{1i} \). Whether \( d_{1i} \) is positive depends on the profit-maximising price \( \tilde{p}_{1i} \).

Suppose that \( \tilde{p}_{1i} \leq 1 - \mu^* \). Then after taking the first-order conditions of the pay-off and setting \( x_{2i} = \tilde{p}_{2i} \), I obtain that \( \tilde{p}_{2i} = 1/2 \) and the price \( \tilde{p}_{1i} \) satisfies equation (17).

\[
1 - \mu^* - 2\tilde{p}_{1i} + (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) \left( \frac{5}{4} - 2\tilde{p}_{1i} \right) = 0 \quad (17)
\]

The pay-off \( \tilde{\pi} \) equals

\[
\tilde{\pi} = \frac{1}{2} \tilde{p}_{1i} (1 - \mu^* - \tilde{p}_{1i}) + \frac{1}{2} \tilde{p}_{1i} (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) (1 - \tilde{p}_{1i}) + \\
+ \frac{1}{8} (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) \tilde{p}_{1i} \quad (18)
\]

If \( \tilde{p}_{1i} > 1 - \mu^* \) and \( x_{2i} = \tilde{p}_{2i} \) then both \( d_{1i} \) and \( d_{2i} \) equal zero and the pay-off of the firm is similar to the pay-off of the monopolist because the only consumers who buy at the firm are the ones who arrive from firm \( j \neq i \). Hence, \( \tilde{p}_{2i} = 1/2, \tilde{p}_{1i} = 5/8 \) and

\[
\tilde{\pi} = (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) \frac{25}{128} \quad (19)
\]

Now I compare the profit \( \pi^* \) and \( \tilde{\pi} \). Firstly, I take the case when \( \tilde{p}_{1i} < 1 - \mu^* \). I show that by setting \( x_{2i} = p_{2i}^* + \mu^*, p_{1i} = \tilde{p}_{1i} \) and \( p_{2i} = \tilde{p}_{2i} \) the firm earns \( \tilde{\pi}^* \) that is more than \( \tilde{\pi} \). The price \( \tilde{p}_{1i} \) is given by equation (17) and it is higher than \( 1/2 \). By using the expressions \( d_{1i}^*, d_{2i}^*, d_{1i}^2 \) and \( d_{2i}^2 \) and plugging appropriate values for deviation prices and \( x_{2i}, \) I get the pay-off which equals
\[
\tilde{\pi}^* = \frac{1}{2} \tilde{p}_{1i} (1 - \mu^* - \tilde{p}_{1i}) + \frac{1}{2} \tilde{p}_{1i} (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) (1 - \tilde{p}_{1i}) \\
+ \frac{1}{8} (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) \tilde{p}_{1i} \\
- \frac{1}{4} \tilde{p}_{1i} \mu^* (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) (1 - \mu^*) + \frac{1}{4} (\mu^* + \tilde{p}_{1i}) \left( \frac{1}{2} - \mu^* \right) \\
+ \frac{1}{8} \mu^* (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) (1 - \mu^*)
\]

I define function \( l(\mu^*) \) where

\[
l(\mu^*) = \frac{1}{4} (\mu^* + \tilde{p}_{1i}) \left( \frac{1}{2} - \mu^* \right) - \frac{1}{4} \tilde{p}_{1i} \mu^* \delta^* (1 - \mu^*) + \frac{1}{8} \mu^* \delta^* (1 - \mu^*)
\]

where \( 0 < \delta^* < 1 \). The second derivative of \( l(\mu^*) \) with respect to \( \mu^* \) is negative:

\[
\frac{\partial^2 l}{\partial (\mu^*)^2} = -\frac{1}{2} + \frac{1}{2} \tilde{p}_{1i} \delta^* - \frac{1}{4} \delta^* < 0
\]

Therefore, if \( l(\mu^*) \) is positive for the highest and the lowest value of \( \mu^* \) then it is always positive.

\[
l(0) = \frac{1}{8} \tilde{p}_{1i} > 0
\]

\[
4l (1 - \tilde{p}_{1i}) = \tilde{p}_{1i} - \frac{1}{2} - \tilde{p}_{1i}^2 (1 - \tilde{p}_{1i}) \delta^* + \frac{1}{2} \tilde{p}_{1i} (1 - \tilde{p}_{1i}) \delta^* \\
= \tilde{p}_{1i} - \frac{1}{2} \tilde{p}_{1i} (1 - \tilde{p}_{1i}) \delta^* \left( \frac{1}{2} - \tilde{p}^* \right) \\
= \left( \tilde{p}_{1i} - \frac{1}{2} \right) (1 - \tilde{p}_{1i} (1 - \tilde{p}_{1i}) \delta^*) > 0
\]

Therefore, if \( \tilde{p}_{1i} < 1 - \mu^* \) and the maximum \( s_p \) is such that \( x_{2i} = p_{2i}^* + \mu^* \) then \( \tilde{\pi} < \tilde{\pi}^* < \pi^* \).

Next, I take the case when \( \tilde{p}_{1i} > 1 - \mu^* \). The profit \( \pi^* \) is higher than the profit that the firm would earn if it chose deviate by setting zero intra-store search costs. This is because by setting \( x_2 = \mu^* + p_{2i}^* \) the firm sells to the same number of consumers and screens their preferences.

If firm \( i \) deviates to zero intra-store search costs, then it sets the same price for both varieties that equals \( 1/\sqrt{3} \) and its profit equals

\[
\pi_0 = \frac{1}{\sqrt{3}} (1 + (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*)) \int_{1/\sqrt{3}}^{1} \varepsilon d\varepsilon = \frac{1 + (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*)}{3\sqrt{3}}
\]
I take the difference, between $\pi_0$ and $\tilde{\pi}$.

$$\pi_0 - \tilde{\pi} = \frac{1 + (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*)}{3\sqrt{3}} - (\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*) \frac{25}{128}$$

The difference $1/(3\sqrt{3})-25/128$ is negative. Therefore, $\pi_0 - \tilde{\pi}$ decreases with $(\mu^* + p_{2j}^*) (\mu^* + p_{1j}^*)$. Then

$$\pi_0 - \tilde{\pi} \geq \frac{1 + 1}{3\sqrt{3}} - \frac{25}{128} > 0$$

As a result, the seller will never obfuscate as much as the reservation utility of the obfuscated variety was equal to zero. Therefore, the firm will obfuscate by setting the search cost per its second variety such that $x_{2i} = \mu^* + p_{2i}^*$.