Informality and long-run growth*

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Abstract

One of the most salient features of developing economies is the existence of a large informal sector. This paper uses quantitative theory to explore the implications of informality on wage inequality, human capital accumulation, child labor and long-run growth. Our model can generate transitory informality equilibria or informality-induced poverty traps. Its calibration reveals that the case for the poverty-trap hypothesis is strong: although informality serves to protect low-skilled workers from extreme poverty in the short-run, it prevents income convergence between developed and developing nations. Sudden elimination of informality would induce severe welfare losses for poor people on the transition path. Hence, we examine the effectiveness of different development policies to exit the poverty trap. Our numerical experiments show that subsidizing education to low-income families is the most cost-effective single policy option but for medium time horizons a combination of this subsidy with another which increases low-skilled formal firms workers wage turns out to be the most cost-efficient option.

Keywords: informality, development, education, child labor, inequality.

JEL codes: O11, O15, O17.

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1 Introduction

The informal economy is defined as the part of an economy that is not taxed, monitored by any form of government, or included in any gross national product. Although informality affects all types of nation, the nature and size of the informal economy differ across rich and poor countries.

In developed countries, the informal sector is characterized by unreported employment and sales. Informal activities are governed by the same production technology as in the formal sector and are simply hidden from the state for tax, social security or labor law purposes. Such a tax-based informality ranges from 10 to 15 percent of official GDP in high-income countries (Schneider, 2005).

Informality is of a different nature in developing countries (although tax evasion also plays a role). It is seen as the only way to earn a living for people who are outside the formal economy and not on anyone’s payroll. Most of them live and work in this sector not because it is their wish or choice, but because they have no chance to be hired by an employer from the formal sector for a decent wage except for a few hours or days, with no legal right to be hired again. Such a poverty-based informality is a way of life in poor countries. Many ubiquitous cottage microenterprises found on every street corner are not registered with authorities, and production is governed by different technologies intensive in low-skilled labor. It is difficult to measure it precisely, but informality is a much more severe phenomenon in developing countries. For example, Buehn et al. (2010) estimated the average size of the shadow economy as a percentage of “official” GDP and obtained an average size of 38.4 percent in Sub-Saharan Africa, 36.5 percent in Europe and Central Asia, and 13.5 percent in high-income OECD countries. Schneider’s data give a similar picture and show that informality represents two-thirds of official GDP in the most affected countries (Schneider, 2005).

This paper investigates the dynamic implications of informality, a relatively neglected aspect in the existing literature which mainly focuses on the tax-evasion motive and possible coordination failures in entrepreneurs’ decisions. As far as tax-based informality is concerned, a large literature has formalized firms’ and workers’ decisions to join the informal sector to avoid taxation or regulation from the government. Among others, Zenou (2008) exploits a search-matching model a la Mortensen-

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1 Measuring informality is a difficult task. People and firms who are engaged in illegal activities do not want to be known, or do not report their illegal activities. Measurement techniques can be grouped in direct and indirect methods; none of them are exempt from criticism. Indirect methods are more macroeconomic in essence: they look at the discrepancy between aggregate income and expenditure, electricity consumption versus economic activity, or monetary indicators (illegal activities use more cash). We also find authors who combine several indirect methods, as Schneider and coauthors. They use structural-equation estimation (MIMIC) that distinguishes between causes and indicators. The main causes are tax and social security contribution burdens, intensity of regulations, quality of public sector services, and state of the official economy. Among the indicators we find monetary indicators, labor market indicators (comparison between total labor force and formal employment), or the state of the economy. An example of a direct method is the use of micro surveys. But again, all of them are subject to different criticisms.
Pissarides to explain its emergence. An increasing empirical literature aims at assessing the effect of taxes on informality in middle-income countries. Using a survey of firms in Brazil, De Paula and Scheinkman (2010, 2011) emphasized the role of value added taxes in transmitting informality through chain effects: informality of a firm is correlated to the informality of firms from which it buys or sells. Inspired by the seminal work of Rosenstein-Rodan (1943), another strand of literature (see Murphy et al. 1989, or Krugman 1991) has demonstrated that predominance of poverty-based informality can be seen as a result of a coordination failure, impeding the process of industrialization and productivity growth. They develop models of multiple equilibria in which firms can choose to operate in the informal sector (characterized by low productivity and wages) or in the formal sector (characterized by high productivity and wages, and fixed equipment costs). Each firm has an incentive to move from informality to formality if the demand for the good produced is large enough. This happens when the economy-wide average income is high, i.e., when other firms industrialize and pay higher wages. Hence, a firm’s decision whether to industrialize or not depends on the expectation of what other firms will do.

Our angle of analysis is different. We want to explore the relationships between informality, wage inequality, human capital accumulation, child labor and long-run growth in a unified model. We build a two-sector model in which people choose to join or not to join the informal sector. We abstract from taxation and simply assume the existence of technological differences between sectors (as in Murphy et al. 1989, or Krugman 1991). Then we investigate the implications of poverty-based informality on welfare, inequality, growth, and effectiveness of development policies. Our philosophy is to use an abstract economic model which highlights the major economic mechanisms underlying the formation and persistence of the informal sector and development. Incentives to invest in children’s education and opportunities to get income from children will play a key role. We then confront data to theory, calibrate the parameters of our model and study its dynamic properties. Such a quantitative theory approach is now the dominant research paradigm used by economists incorporating rational expectations and dynamic choice into short-run macroeconomic and monetary economics models (King, 1995). However, very little has been done so far with this methodology in long-term macroeconomics and development economics.

We require our model to be compatible with five major stylized facts (SF1 to SF5) on poverty-based informality and development as illustrated on Figures 1(a) to 1(d).\footnote{\textsuperscript{2}In regression lines of Figures 1(a), 1(b), and 1(d), we exclude observations for socialist countries (in red) because informality in these countries seems to have different nature. Trends are steeper without these observations.}
Figure 1: Stylized facts on informality, education and development.
Data sources. Education: Barro and Lee (2010); Informality: Schneider (2005); GDP and child labor: World Development Indicators (2012); Returns to schooling: Hendricks (2004).
SF1. Informality decreases with development. Figure 1(a) shows the relation between the proportion of tertiary educated (individuals with some college education) and the ratio of output between the informal and formal sectors in year 2000. It shows a downward sloping relationship between informality and the economy-wide proportion of high-skilled workers. Our model will endogenize the size of the informal sector and will be consistent with this fact. The rationale is the following: low-skilled workers are mobile across sectors. When the proportion of high-skilled workers is low in the formal economy, the demand for low-skilled labor is also low and formal firms pay low wages to the less educated. Many workers then move to the informal sector where wages are more attractive. Informality thus serves to protect low-skilled workers against very low levels of income offered in the formal sector.

SF2. The informal sector exhibits lower productivity and uses low-skilled workers. This is a consensual hypothesis in informality models (Rosenstein-Rodan 1943, Murphy et al. 1989, Krugman 1991). It is supported by empirical studies. De Paula and Scheinkman (2011) showed that informal firms are managed by less able entrepreneurs, are smaller and exhibit low capital-labor ratios. They estimated that informal firms face at least 1.3 times the cost of capital of formal firms. Similarly, La Porta and Shleifer (2008) found evidence of a substantial difference between the registered and the unregistered firms in the skills of their managers, and suggested that this drives many other differences, including the quality of inputs and access to finance. Based on these facts, our model defines informality as a sector with lower productivity, low-skilled employment, and constant marginal productivity of labor. On the contrary, the formal sector combines high-skilled and less educated workers, exhibits decreasing marginal productivity, constant returns to scale, and higher total factor productivity.

SF3. Child labor increases with informality. One of the underlying aspects of informality is the existence of child labor. We can think of different forms of child labor, from shoeshine boys to children working in mining extraction. In general, children are not reported as part of the official labor force. Even if children produce in formal firms, they are not recorded as part of their formal workers by the state agencies. Figure 1(b), plots the percentage of male children who work, and the size of informality as percentage of GDP in 2000. The World Bank data considers child labor to be children involved in economic activity for at least one hour in the reference week of the survey. We can observe a direct relation between informality and child labor, if informality increases, child labor increases as well. The reasons for child labor to exist are related to wealth constrained households whose income is so low that they cannot send children to school. These families need their children to produce in order to increase their income instead of investing on education, which can increase their future wealth and help them to escape from poverty.

SF4. Skill premia are limited in poor countries, and no standard labor market model can account for such low skill premia. The relationship between the rate of return to one year of college (Hendricks 2004) and the proportion of college graduates
in the labor force (Barro and Lee 2010) is represented on Figure 1(c).³ Although returns to education decrease with human capital, highest rates do not exceed 20 percent per year of schooling. Standard labor market models predict much larger return rates in developing countries. The CES representation is common in labor markets studies (such as Katz and Murphy, 1992, Card and Lemieux, 2001) and in cross-country analysis of relative productivity (Caselli and Coleman, 2006). The range between 1.3 and 2 for the elasticity of substitution spans most labor market studies including Angrist (1995), Borjas and Katz (2007) and Katz and Murphy (1992). Assuming that college graduates have ten years of education more than the less educated and wages are equal to the marginal productivity of labor, the thin lines on Figure 1(c) represent the prediction of CES models with elasticities of substitution equal to one (Cobb-Douglas), 1.3 or 2.0. None of these models match the data? The average share of college graduates is around 3 percent in low-income countries. For such countries, the models predict a return to schooling comprised between 26 percent and 50 percent. The data provided in Hendricks (2004) show a maximal return to schooling of around 15 percent. We conclude that either the elasticities of substitution estimated for developed countries do not fit the production function of developing countries (an elasticity of 4.25 would be needed to match observations!), or the structure of the labor market differs across countries. We plead for the second hypothesis and see informality as a key factor limiting the skill premium and wage inequality in poor countries. Informality maintains a large skill ratio (i.e., ratio of college graduates to less educated workers) in the formal sector, thus keeping the return to schooling at a low level.

SF5. The elasticity of GDP per capita to human capital is close to unity and school enrolment are lower in poor countries. Although many studies point out that education has not generated as much growth as expected in developing countries, it is also reported that education is one of the necessary components to growth. As shown on Figure 1(d), the correlation between the proportion of college graduates in the labor force and GDP per capita is large, and the elasticity is close to unity. Despite scarcity in human capital and larger returns to schooling, contemporaneous school enrolment rates are lower in poor countries.

In this paper, we build a model compatible with these five stylized facts. The model may generate multiple equilibria or uniqueness, depending on the parameters’ values. In the absence of informality, the model predicts long-run convergence in income across nations. Informality may slow down this convergence process or be the source of a poverty trap. The reason is that informality keeps skill premia at a relatively low level, reduces incentive to invest in education, and is conducive to child labor. Using the stylized facts above and other consensual parameters from the literature, we calibrate our model and study its quantitative properties. This allows us discriminating between the poverty-trap and slow-convergence hypotheses. It comes

³We use the most recent year of information of Mincerian returns in each country from Hendricks (2004).
out that the case for the poverty-trap hypothesis is strong: although informality serves to protect low-skilled workers in the short-run, it prevents income convergence across countries. On this basis, we assess the effectiveness of different policy options. Sudden elimination of informality would induce large welfare losses for several generations of poor people on the transition path. We thus compare different Pigouvian policies (subsidizing education to all families, or to low-income families, subsidizing high-skilled formal employment, or low-skilled formal employment) assuming that subsidies are financed by development assistance. Two criteria are used to evaluate these policies: cost-effectiveness and the length of the transition required to exit the poverty trap. Among the four subsidies considered, education subsidies to low-income families dominates the others in terms of cost efficiency. Moreover, only wage subsidies for low-skill jobs in the formal sector play a distinct and complementary role in the transition to the high-income equilibrium. Whereas the education and the high-skilled formal employment subsidies speed up the accumulation of human capital, the low-skill wage subsidy reduces the threshold at which the informal sector disappears. Therefore, targeted education subsidies are the cheapest single policy but for medium time horizons a combination of the two policies turns out to be the most cost-efficient choice.

The remainder of this paper is organized as follows. Section 2 describes the model. The implications of informality are examined in Section 3. In Section 4, we calibrate the model and study its quantitative properties. Section 5 concludes.

2 Model

We develop a two-period overlapping generations model in infinite discrete time with children and working-age adults. In every period, a single homogeneous good can be produced in two different sectors, the formal and informal sectors (labeled $f$ and $i$). Formal firms employ high- and low-skilled workers whereas the informal sector only employs low-skilled workers. In each period there is an endogenous number of adults of each type who choose how much to consume and how much to invest in the education of their children. All decisions are made in the adult period of life, i.e., children do not get to decide anything. Below, we describe the technology, preferences, the dynamics, and define the competitive equilibrium path of our economy.

2.1 Production

A single good can be produced in two sectors. The formal sector employs high- and low-skilled labor and an informal one only uses low-skilled labor. Let $h_t$ be the proportion of high-skilled adults at time $t$, and $N_t$ the total labor force of adults. We denote by $H_t = h_tN_t$ and $L_t = (1 - h_t)N_t$ the size of high- and low-skilled labor forces, respectively. Low-skilled workers are assumed to be perfectly mobile across
sectors, whereas high-skilled workers have no incentive to join the informal sector.\(^4\)

Output \(Y_t\) is the sum of output \(Y_{f,t}\) produced in the formal sector and output \(Y_{i,t}\) produced in the informal one. Output produced in each sector is

\[
\begin{align*}
Y_{f,t} &= A_t H_t^\alpha L_{f,t}^{1-\alpha}, \\
Y_{i,t} &= B L_{i,t},
\end{align*}
\]

where \(\alpha\) is the share of output produced in the formal sector by high-skilled workers, \(A_t\) is a time-varying scale factor representing the state of technology, \(H_t\) is the quantity of high-skilled workers employed in the formal sector, \(L_{f,t}\) and \(L_{i,t}\) are the quantities of low-skilled workers employed in formal and informal sectors, respectively, and \(B\) is a scale factor associated to the technology in the informal sector, which is assumed to be constant. For simplicity purposes, we write \(B = \tilde{\gamma} A_0\), where \(\tilde{\gamma}\) is a parameter that allows us to write \(B\) in terms of the scale factor \(A_0\).

Firms choose inputs by maximizing profits

\[
Y_{f,t} - w_{h,t} H_t - w_{l,t} L_{f,t}
\]

and

\[
Y_{i,t} - w_{l,t} L_{i,t},
\]

subject to \(Y_{i,t} \geq 0\).\(^5\) Under perfect competition, firms in formal and informal sectors choose employment levels by equalizing the marginal productivity of high- and low-skilled workers with their wage rates \(w_{h,t}\) and \(w_{l,t}\). In the formal sector, these conditions are

\[
\begin{align*}
w_{h,t} &= A_t \alpha \left( \frac{L_{f,t}}{H_t} \right)^{1-\alpha}, \\
w_{l,t} &= A_t (1 - \alpha) \left( \frac{L_{f,t}}{H_t} \right)^{-\alpha}.
\end{align*}
\]

The output and employment decisions in the informal sector can be described by the complementary slackness conditions

\[
\frac{w_{l,t}}{\tilde{\gamma} A_0} \geq 1, \quad Y_{i,t} \geq 0, \quad \text{and} \quad \left( \frac{w_{l,t}}{\tilde{\gamma} A_0} - 1 \right) Y_{i,t} = 0,
\]

which depict two alternatives that will give rise to two model regimes:

1. output in the informal sector is positive and marginal cost, \(w_{l,t}/(\tilde{\gamma} A_0)\), is equal to the (unitary) price of output (or, equivalently, marginal productivity of labor in the informal sector is equal to the low-skilled wage) ;

\(^4\)Our model does not account for brain waste, which may be responsible for employment of educated workers in informality.

\(^5\)For simplicity, we omit the constraint \(Y_{f,t} \geq 0\) because it is never binding in equilibrium.
2. Firms in the informal sector produce no output and marginal cost exceeds the price of output (marginal productivity of labor in the informal sector is smaller than the low-skilled wage).

Moreover, we assume that total factor productivity (TFP) $A_t$ in the formal sector is endogenous. It is a concave function of the skill ratio in the formal sector.\(^6\) For simplicity and in reference to the AK model, the elasticity of TFP to the skill ratio equals $1 - \alpha$, i.e.,

$$A_t = A_0 \left( \frac{H_t}{L_{f,t}} \right)^{1-\alpha}.$$ (8)

### 2.2 Preferences

Each adult of type $k \in \{h, l\}$ at period $t$ chooses consumption $c_{k,t}$ and the proportion $q_{k,t} \in [0, 1]$ of children sent to college to maximize utility. The utility function is logarithmic and depends on consumption $c_{k,t}$ and the average future wage $\bar{w}_{k,t+1}$ of children,

$$U_{k,t} = \ln(c_{k,t}) + \beta \ln(\bar{w}_{k,t+1})$$ (9)

where $\beta$ is the rate of preference for the income of children, and the average future wage of children is

$$\bar{w}_{k,t+1} = (1 - q_{k,t})w_{l,t+1} + q_{k,t}w_{h,t+1} = w_{l,t+1}(1 + q_{k,t}\sigma_{t+1}),$$ (10)

which depends on the value of the skill premium $\sigma_{t+1} = (w_{h,t+1} - w_{l,t+1})/w_{l,t+1}$ in the next period.

Educating a child incurs a monetary cost $\check{e}$.\(^7\) Non-educated children can work in the informal sector as long as the informal sector exists, whereas educated children go to school and have no time left to work. In the informal sector, children receive a fraction $\eta \in [0, 1]$ of the low-skilled wage rate because they lack experience and physical strength compared to adults. The budget constraint is

$$c_{k,t} = w_{k,t} - n_k q_{k,t} \check{e} + n_k (1 - q_{k,t}) \eta w_{i,t} d_t,$$ (11)

where $n_k$ is the number of children of a $k$-type adult, and $d_t$ is a dummy variable equal to 1 if some output is produced in the informal sector, and 0 otherwise.

Plugging (10) and (11) into (9) and maximizing utility with respect to $q_{k,t}$, we obtain

$$\hat{q}_{k,t} = \frac{\beta \sigma_{t+1}(w_{k,t} + n_k \eta w_{i,t} d_t) - n_k (\check{e} + \eta w_{i,t} d_t)}{(1 + \beta)n_k (\check{e} + \eta w_{i,t} d_t)\sigma_{t+1}}.$$ (12)

\(^6\)This assumption implies that the proportion of high-skilled individuals generates a positive externality on the aggregate productivity. It is a particular case of Lucas’ model (Lucas 1988) and is also related to other AK models as the ones presented by Romer (1986) and Rebelo (1991).

\(^7\)As we will observe later, equilibrium high-skilled wages will be constant. Hence, a constant education cost is equivalent to education costs being proportional to high-skilled wages, which implies that education is more difficult to obtain for low-skilled than for high-skilled workers.
Therefore, the desired level of education is

\[
q_{k,t}^* = \begin{cases} 
0 & \text{if } \hat{q}_{k,t} < 0 \\
\hat{q}_{k,t} & \text{if } 0 \leq \hat{q}_{k,t} \leq 1 \\
1 & \text{if } \hat{q}_{k,t} > 1.
\end{cases}
\]  

\[\text{(13)}\]

2.3 Dynamics and competitive equilibrium

In the previous section we obtained adults’ optimal decision on the proportion of children to be educated. Hence, given the proportion \(h_t\) of high-skilled workers in period \(t\), fertility rates \(n_h\) and \(n_l\), and the equilibrium condition (13) we can compute the proportion \(h_{t+1}\) of high-skilled workers in the next period. For simplicity, we assume that high-skilled parents educate all their children, i.e., we assume that parameters are such that \(q_{h,t} \geq 1\), which implies that \(q_{h,t}^* = 1\).\(^8\) On the contrary, low-skilled parents only educate an endogenous fraction \(q_{l,t} \in [0, 1)\) of their children. Therefore, the dynamics of the skill ratio across generations is governed by

\[
h_{t+1} = \frac{n_h h_t + n_l q_{l,t}(1 - h_t)}{n_l(1 - q_{l,t})(1 - h_t)} = \frac{n}{1 - q_{l,t}} \frac{h_t}{1 - h_t} + \frac{q_{l,t}}{1 - q_{l,t}},
\]

\[\text{(14)}\]

where \(n \equiv n_h/n_l\) is the fertility ratio of high- to low-skilled workers.

In addition, the labor-market-clearing conditions are

\[
H_t = \overline{H}_t,
\]

\[\text{(15)}\]

the supply and demand of high-skilled workers should be equal in equilibrium. In the next sections we use \(H\) to denote the equilibrium amount of high-skilled workers. And

\[
L_{f,t} + L_{i,t} = \overline{L}_t + \eta n_l(1 - q_{l,t})\overline{L}_t,
\]

\[\text{(16)}\]

low-skilled workers in formal and informal sectors should be equal to low-skilled adults and the efficiency units of children who work. Moreover, we impose the following extra condition:

\[
L_{i,t} > \eta n_l(1 - q_{l,t})\overline{L}_t \text{ whenever } L_{i,t} > 0.
\]

\[\text{(17)}\]

Some adult workers are required for the functioning of the informal sector. This is a reasonable assumption since we are imposing that children need some infrastructure provided by adults to the informal sector in order to operate. We now define an equilibrium for our economy:

\(^8\)An alternative assumption to ensure that \(\hat{q}_h \geq 1\) is to assume that \(h\) can not be higher than \(\bar{h} < \alpha\) and parameters are such that \(\left(\frac{\alpha \omega}{\varepsilon \omega} - 1\right) \beta \geq 1 + \frac{(1 - \alpha)\beta}{\alpha \bar{h}}\). de la Croix and Docquier (2012) use the same simplifying assumption.
Definition 1 Given an initial population size $N_0$ and an initial number $H_0$ of high-skilled workers, an intertemporal equilibrium consists of sequences of prices $\{w_{h,t}, w_{l,t}\}$, aggregate quantities $\{N_t, \overline{H}_t, \overline{L}_t, H_t, L_{f,t}, L_{i,t}\}$, and household’s decisions $\{c^j_t, q^j_t\}$ for $j = h, l$ and for all $t$ such that:

1. the household’s decisions $c^j_t$ and $q^j_t$ maximize utility (9) subject to the constraints (10) and (11);
2. the firms’ choices $H_t$, $L_{f,t}$, and $L_{i,t}$ maximize profits (3) and (4) subject to the constraint $Y_{i,t} \geq 0$;
3. the prices $w_{h,t}, w_{l,t}$, and aggregate quantities $\overline{H}_t, \overline{L}_t$ are such that markets clear, i.e., (15) and (16) hold;
4. aggregate variable $N_t$ evolves according to (14);
5. $\overline{L}_t$, $L_{i,t}$, and $q^l_t$ satisfy (17).

3 Implications of informality

In this section we characterize the existence of two possible transitory regimes, and then study the implications of informality for human capital accumulation and long-run growth.

3.1 The formality and informality regimes

Two regimes arise as a consequence of informality. On the one hand, the informality regime arises if the formal and informal sector co-exist. On the other hand, the formality regime arises if all low-skilled adults opt for the formal sector and the informal sector disappears.

The formality regime is characterized by the absence of an informal sector. Then, plugging (8) into (5) - (7), wages and the skill premium in the formality regime are

$$w_{h,t} = A_0 \alpha,$$
$$w_{l,t} = A_0 (1 - \alpha) \frac{h_t}{1 - h_t},$$
$$\sigma_t = \frac{\alpha (1 - h_t)}{(1 - \alpha) h_t} - 1.$$

Hence, in the formality regime, the skill premium $\sigma_t$ decreases with the proportion of high-skilled workers in the economy, and the limit of the skill premium equals infinity when $h_t$ tends to zero. A model with a single formal sector predicts huge wage disparities when human capital is low.
However, production in the informal sector becomes profitable if the marginal productivity of labor is not lower than the low-skilled wage. Combined with the assumption of perfect mobility of low-skilled workers across sectors, this implies that the number of low-skilled workers in the formal sector is proportional to the number of high-skilled workers in the economy, i.e., $L_{f,t} = \gamma H_t$ where $\gamma \equiv (1 - \alpha)/\bar{\gamma}$ is a simple combination of parameters. Again, plugging (8) into (5) - (7) and taking into account that $Y_{i,t} > 0$, wages and the skill premium in the informal regime are

$$
\begin{align*}
  w_{h,t} &= A_0 \alpha, \\
  w_{l,t} &= \frac{A_0 (1 - \alpha)}{\gamma}, \\
  \sigma_t &= \frac{\alpha \gamma}{1 - \alpha} - 1 = \bar{\sigma}.
\end{align*}
$$

Therefore, the skill premium $\sigma_t$ is constant when the informal sector is at work, so it does not depend on the proportion $h_t$ of high-skilled workers. Informality explains why skill premia are limited in developing countries where the proportion of college graduates is low, as illustrated by stylized fact SF4.

The following lemma characterizes the emergence of the informal regime in terms of the proportion of high-skilled workers in the economy:

**Lemma 1** The informal regime (resp. formality regime) arises when the proportion of high-skilled workers is not too large (resp. large enough), i.e., when $h_t < 1/(1 + \gamma)$ (resp. $h_t \geq 1/(1 + \gamma)$).

**Proof.** Low-skilled adults work in the informal sector if and only if (19)<(22). Therefore, the informal regime arises if $h_t < 1/(1 + \gamma)$. ■

Informality was modeled in the production section as an alternative for low-skilled adults to supply their working-hours. Moreover, we observe that informality arises in economies with low levels of human capital. Let us denote GDP per capita and recorded GDP per capita by $y_t = Y_{i,t}/N_t$ and $y_{f,t} = Y_{f,t}/N_t$. Consistently with stylized fact SF5, our model predicts that the elasticity of formal output to human capital is equal to unity, as stated in the following proposition:

**Proposition 1** In the formality regime, GDP per capita is proportional to the share of high-skilled workers in the labor force, i.e., $y_t = A_0 h_t$, and recorded GDP is equal to GDP per capita, i.e., $y_{f,t} = y_t$. Meanwhile, in the informality regime, GDP per capita exceeds recorded GDP per capita, $y_t > y_{f,t}$, and recorded GDP per capita is proportional to the share of high-skilled workers, $y_{f,t} = A_0 h_t$.

**Proof.** It follows from equation (8). ■

In the informality regime, wages are constant. Hence, $q_{t,1}$ is equal to:

$$
q_{t,1}^* = \frac{\beta (1 - \alpha) (1 + \eta n_t)}{(1 + \beta) [\epsilon \gamma + \eta (1 - \alpha)] n_t} - \frac{1}{(1 + \beta) \sigma_{t+1}}.
$$
Note that in case that next period proportion $h_{t+1}$ of high-skilled workers is not high enough so as to achieve the threshold proportion $1/(1 + \gamma)$ that defines informality, then $q_{t,t}$ is constant and equal to:

$$q_{t,t}^* = \frac{\beta [\alpha(1 + \gamma) - 1] (1 - \alpha) (1 + \eta n_t) - n_t (1 - \alpha) [e\gamma + \eta(1 - \alpha)]}{(1 + \beta) [e\gamma + \eta(1 - \alpha)] [\alpha(1 + \gamma) - 1] n_t} \equiv \bar{q}_t,$$

where $e = \tilde{e}/A_0$. Moreover, $q_{t,t}^* \leq \bar{q}_t$ because $\sigma_{t+1} \leq \sigma$.

We define the following parameter condition:

**Condition 1** $1 - n \geq \bar{q}_t(1 + \gamma)$.

This condition is satisfied if the fertility ratio $n$ is low enough, and the relative productivity $\tilde{\gamma}$ of the informal sector so as the education cost $\tilde{e}$ are sufficiently high. Moreover, it ensures that $h_{t+1} < 1/(1 + \gamma)$ in the *informality* regime.

In line with some empirical papers as Buehn et al. (2010) or Schneider (2005), we also define the informality level as the proportion of value added in the informal sector with respect to the value added in the official GDP, i.e., $I_t = Y_{i,t}/Y_{f,t}$. Note that $I_t \equiv 0$ in the *formality* regime. Consistently with stylized fact SF1, we have:

**Proposition 2 (Short-run effects of informality)** The informal sector increases low-skilled workers’ wage, whereas high-skilled workers’ wage is not modified. Moreover, the informality level $I_t$ shows a decreasing and convex relationship with respect to the proportion of high-skilled workers in the labor force in the informality regime if Condition 1 is satisfied.

**Proof.** From (18) and (21) we can see that high-skilled wages are equivalent in both regimes. From (19) and (22), low-skilled wages in the *informality* regime are at least as high as in the *formality* regime if and only if $h/(1 - h) < 1/\gamma$, and, by Lemma 1, the informality regime exists if and only if $h < 1/(1 + \gamma)$, which is equivalent to $h/(1 - h) < 1/\gamma$. Moreover, if Condition 1 holds, in the *informality* regime

$$I_t = \frac{Y_{i,t}}{Y_{f,t}} = \frac{1 - \alpha (1 + \eta n_t(1 - \bar{q}_t) - h_t(1 + \gamma + \eta n_t(1 - \bar{q}_t))}{\gamma}.$$

Therefore,

$$\frac{dI_t}{dh_t} = \frac{1 - \alpha + \eta n_t(1 - \bar{q}_t)}{\gamma h_t^2} < 0$$

and

$$\frac{d^2I_t}{dh_t^2} = \frac{1 - \alpha + \eta n_t(1 - \bar{q}_t)}{\gamma h_t^3} > 0$$

This result follows from equation (14) and $q_{t,t} \leq \bar{q}_t$. 

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The existence of the informal sector reduces inequality differences and it can be good for growth for developing countries, as many studies point out. However, informality allows firms to recruit illegal labor, which includes children of poor households. The following result makes the link between child labor and informality, consistently with stylized fact SF3:

**Corollary 1 (Child labor)** The proportion of children who work decreases as the proportion of high-skilled workers in the labor force increases in the informality regime. Hence, the proportion of children who work increases as the informality level increases.

**Proof.** The proportion of children who work is

\[ CL(h) = \frac{(1 - q_l)(1 - h)n_l}{hn_l + (1 - h)n_l} = \frac{1 - h}{nh + (1 - h)}. \]

Hence, taking the derivative with respect to \( h \) we obtain

\[ CL'(h) = -(1 - q_l)n/(nh + (1 - h))^2 < 0. \]

Moreover, from Proposition 2 we know that \( I \) increases as \( h \) decreases, which implies that the proportion of children who work increases with informality. □

### 3.2 Effect on long-run growth

Now we study the long-run effects of informality, in particular, we study its effects on human capital accumulation. We distinguish two important channels. First, as informality limits the returns to schooling, it is likely to reduce the incentive to acquire human capital. Second, informality allows firms to recruit illegal labor, which includes children of poor households.

In the formality regime, i.e., \( h_t \geq 1/(1 + \gamma) \), substituting wage rates (18)-(20) into (13) yields:

\[ q_{t,t}^* = \frac{\beta(1 - \alpha)h_t}{(1 + \beta)en_t(1 - h_t)} - \frac{\alpha - h_{t+1}}{(1 + \beta)(1 - \alpha)h_{t+1}} \equiv q_t(h_t, h_{t+1}). \]  

(24)

Moreover, human capital dynamics for an economy without informality is governed by

\[ h_{t+1} = \frac{n}{1 - q_t(h_t, h_{t+1})} \frac{h_t}{1 - h_t} + \frac{q_t(h_t, h_{t+1})}{1 - q_t(h_t, h_{t+1})} \equiv \varphi(h_t, h_{t+1}). \]  

(25)

Therefore, plugging (24) into (25) characterizes human capital dynamics. To simplify these two expressions let \( z_t \) be \( h_t/(1 - h_t) \). This variable transformation allows us to write equations (24) and (25) as follows:

\[ q_{t,t}^* = \frac{\beta(1 - \alpha)}{(1 + \beta)en_t} z_t - \frac{(1 - \alpha)z_{t+1}}{(1 + \beta)\alpha(1 + z_{t+1}) - z_{t+1}} \equiv q_t(z_t, z_{t+1}) \]  

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and
\[ z_{t+1} = \frac{n}{1 - q_l(z_t, z_{t+1})} z_t + \frac{q_l(z_t, z_{t+1})}{1 - q_l(z_t, z_{t+1})} \equiv \varphi(z_t, z_{t+1}). \]

Moreover, the properties of the dynamical system are not modified by this transformation. The following proposition describes the long-run convergence of human capital in the formality regime:

**Proposition 3 (Long-run convergence in the formality regime)** The dynamical system characterized by (24) and (25) displays a stable steady state \( h^{s\text{stat}} > 0 \) and an unstable one \( h^{u\text{stat}} = 0 \) in \( h \in [0, 1] \) if and only if parameters satisfy that \((1 + \alpha \beta)en_t < \alpha((1 - \alpha)\beta + (1 + \beta)en_t)\).

**Proof.** The proof is divided into three steps.

**Step 1:** there exists a function \( \psi \) that determines \( z_{t+1} \) given \( z_t \) and its slope is positive for all \( z_t \geq 0 \), i.e., \( z_{t+1} = \psi(z_t) \) and \( \psi'(z_t) > 0 \).

Let \( F \) be a function \( F: \mathbb{R}^2 \to \mathbb{R} \) such that \( F(z_t, z_{t+1}) = \varphi(z_t, z_{t+1}) - z_{t+1} \). The vectors \((z_t, z_{t+1})\) such that \( F(z_t, z_{t+1}) = 0 \) characterize human capital dynamics. Taking partial derivatives we obtain the Jacobian

\[ DF(z_t, z_{t+1}) = \begin{bmatrix} \frac{\partial \varphi(z_t, z_{t+1})}{\partial z_t} & \frac{\partial \varphi(z_t, z_{t+1})}{\partial z_{t+1}} - 1 \end{bmatrix} = \frac{1}{(1 - q_l)^2} \left[ n(1 - q_l) + q_1(1 + nz_t), q_2(1 + nz_t) - (1 - q_l)^2 \right] = [DF_1, DF_2], \]

where \( q_l = q_l(z_t, z_{t+1}), q_1 = \partial q_l(z_t, z_{t+1})/\partial z_t = \beta(1 - \alpha)/(en_t(1 + \beta)) > 0 \), and \( q_2 = \partial q_l(z_t, z_{t+1})/\partial z_{t+1} = -\alpha(1 - \alpha)/((1 + \beta)(1 + z_{t+1}) - z_{t+1})^2 < 0 \) for all \( z_t \). Since \( DF_2 < 0 \), by the Implicit Function Theorem there exists a function \( z_{t+1}(z_t) = \psi(z_t) \) in a neighborhood of \( z_t \) (for all \( z_t \)) and

\[ z'_{t+1}(z_t) = \psi'(z_t) = -\frac{n(1 - q_l) + q_1(1 + nz_t)}{q_2(1 + nz_t) - (1 - q_l)^2}. \]

Moreover, \( \psi \) is increasing for all \( z_t \geq 0 \), i.e., \( \psi'(z_t) > 0 \), because the numerator is strictly positive if \( z_t \geq 0 \), while the denominator is negative.

**Step 2:** the dynamical system displays two steady state values in \( z \geq 0 \): 0 and \( z_+ > 0 \), and they are the only ones.

The steady state values are the vectors \((z_t, z_{t+1})\) such that \( z_t = z_{t+1} \), or the values of \( z \) such that \( F(z, z) = 0 \). Note that (24) and (25) become

\[ q_l(z, z) = z \frac{1 - \alpha}{1 + \beta} \left( \frac{\beta}{en_t} - \frac{1}{\alpha(1 + z) - z} \right) \quad (26) \]
and
\[ z = \frac{n z + q_1(z, z)}{1 - q_l(z, z)} \]  \hspace{1cm} (27)
respectively. Plugging (26) into (27) and rearranging terms we obtain
\[ F(z, z) = q_l(z, z)z + q_1(z, z) - (1 - n)z. \]

Clearly, \( z = 0 \) satisfies \( F(0, 0) = 0 \) because \( q_l(0, 0) = 0 \). Since we are interested in the remaining solutions to the problem \( F(z, z) = 0 \), we substitute \( q_l \), divide by \( z \), and equalize to 0. The solutions to the resulting equation can be rewritten as the roots of the following grade 2 polynomial of \( z \):
\[ a_2 z^2 + a_1 z + a_0 = 0, \]
where
\[ a_0 = -\left(1 + \alpha \beta + \frac{(1 + \beta)(1 - n)en_l \alpha}{1 - \alpha}\right), \]
\[ a_1 = (1 + \beta)(1 - n) - (1 - \beta + 2\alpha \beta), \]
\[ a_2 = (1 - \alpha)\beta. \]

Since \( a_0 < 0 \) and \( a_2 > 0 \), the roots of the polynomial are \( z_- < 0 \) and \( z_+ > 0 \). Hence, the steady state values of the dynamical system are \( z_- \), 0, and \( z_+ \).

**Step 3:** \( \lim_{z \to +\infty} \psi'(z) = 0 \).

Rewrite \( \psi'(z_t) \) as
\[ \psi'(z_t) = \frac{n(1 - q_1)}{1 + n z_t} + \frac{q_1}{-q_2 + \frac{(1 - q_1)^2}{1 + n z_t}} \]
and note that the denominator goes to infinity when \( z_t \) goes to infinity whereas the numerator goes to 0 or to a constant because \( q_1 \) is a constant,
\[ -\infty < \lim_{z_t \to +\infty} \frac{n(1 - q_1(z_t, \psi(z_t)))}{1 + n z_t} = -n \frac{\beta}{1 + \beta} \frac{1 - \alpha}{en_l} < +\infty, \]
\[ 0 \leq \lim_{z_t \to +\infty} -q_2 = < +\infty, \]
and
\[ \lim_{z_t \to +\infty} \frac{(1 - q_1)^2}{1 + n z_t} = +\infty. \]

From **Steps 1** and **2** we know that the system is well defined and displays two different steady state values in \( z \geq 0 \): 0 and \( z_+ > 0 \). A necessary and sufficient condition for the instability of the 0 steady state is \( \psi'(0) > 1 \), which is equivalent to \( (1 + \alpha \beta)en_l < \alpha((1 - \alpha)\beta + (1 + \beta)nen_l) \). Moreover, **Step 3** ensures that
\[ z_{t+1} = \psi(z_t) < z_t \] for all \( z_t > z^*_t \), and we can conclude that \( z^*_t \) is stable because necessarily \( 0 < \psi'(z^*_t) < 1 \). ■

In the informal regime, i.e., \( h_t < 1/(1 + \gamma) \) or \( z_t < 1/\gamma \), we have \( q_{t,t}^* = \bar{q}_t \) if Condition 1 is satisfied, and the dynamics is governed by

\[ z_{t+1} = \frac{n}{1 - \bar{q}_t} z_t + \frac{\bar{q}_t}{1 - \bar{q}_t} \equiv \phi(z_t), \tag{28} \]

where \( \phi(z_t) \) is a linear function of \( z_t \) with \( \phi(0) > 0 \) and a slope smaller than one if \( n < 1 - \bar{q}_t \).

**Proposition 4 (Long-run effects of informality)** There are poverty traps in the informal regime if and only if Condition 1 holds. Hence, if the initial proportion of high-skilled workers is not high enough, \( h_0 < 1/(1 + \gamma) \), the long-run proportion \( h^{stst} \) of high-skilled workers remains low.

**Proof.** Human capital dynamics is determined by (28). Thus, a stable poverty trap with informality emerges if and only if \( \phi(1/\gamma) \leq 1/\gamma \), and \( n/(1 - \bar{q}_t) < 1 \). The former condition is equivalent to Condition 1. In addition, this condition ensures that \( 1 > \bar{q}_t + n \). Hence, the former condition is sufficient for the latter condition to be satisfied. Therefore, there exists a steady state level of human capital such that \( h^{ss} < 1/(1 + \gamma) \) if and only if \( 1 - n \geq \bar{q}_t(1 + \gamma) \). ■

The previous two proposition characterize the equilibrium path of the skill ratio. Figure 2(a) shows the dynamics with and without informality. The solid line corresponds to an economy with informal sector if the skill ratio is lower than \( z_0 = 1/\gamma \), while the dashed line corresponds to one without informality. For high enough levels of human capital there is not an informal sector and both lines coincide. As predicted by Proposition 3, without informal sector the skill ratio converges to the point \( A_1 \) as long as the initial skill ratio is larger than 0. However, if the informal sector is at work, Proposition 4 states that there can be poverty traps as the one presented in Figure 2(a). The linear part of the solid line crosses the 45° line and the skill ratio converges to the point \( A_2 \) if the initial skill ratio is lower than \( z_0 \).

Figure 2(b) presents three different possibilities of skill ratio dynamics with informality. In all cases there is a jump from the formal to the informal regime due to child labor in the informal sector. Dynamic \( B \) is a possible situation without poverty trap. It might happen if, for example, the education cost \( \tilde{e} \) is low enough. Dynamic \( A \) is a case with a poverty trap in the informality regime, and convergence to a high proportion of high-skilled workers in the formality regime. Whereas Dynamic \( C \) corresponds to a case where parameters satisfy that the stable steady state is 0 in the formality regime, or a case where there is a stable steady state greater than 0 but lower than \( 1/\gamma \). Because of the existence of the informal sector the poverty trap makes the economy to converge to the point \( C \), which is characterized by a low proportion of high-skilled workers in the economy.
(a) Dynamics with and without informality

(b) Different configurations with informality

Figure 2: Dynamics of human capital accumulation with informality
4 Quantitative assessment

We have shown that informality may slow down income convergence across countries or be the source of a poverty trap depending on the fact that the model exhibit multiple equilibria or uniqueness. In this section, we confront data to theory, calibrate the model, and discriminate between these two hypotheses. We use the stylized facts presented in the introduction and other consensual parameters found in the empirical literature.

4.1 Parametrization

Our parametrization strategy is based on the following principles:

- Parameters are calibrated so as to be compatible with developed and developing economies’ observations. In particular, we require our calibrated model to be compatible with the stylized facts described in the introduction.

- The United States situation is considered as a possible steady state without poverty-induced informality.

- Lest developed countries might be out of steady state and are characterized by the informality regime.

The model is calibrated under the assumption that one period (or generation) represents 30 years, and that individuals become high-skilled after 10 years of education. As far as production is concerned, we use recent data on skill premium from Hendricks (2004). The average return to one year of college for France, Germany, Japan, the United Kingdom and the United States males was around 6 percent in the late nineties. Assuming that high-skilled workers have 10 years of education, we have $\sigma_{t}^{US} = 0.79$ for the United States economy. According to Barro and Lee data, the United States proportion of workers with at least one year of college was around 50 percent in 2000, so $k^{US} = 0.5$. Barro and Lee report that between 1950 and 2000 the percentage of population with some college studies increased from 13 percent to 48.5 percent, whereas from 2000 to 2010 it just increased to 51.8 percent. It seems reasonable to assume a steady state value of 50 percent of high-skilled workers in developed countries. Using (20), we obtain the parameter $\alpha$ to be 0.64. In addition, Hendricks (2004) reports a return to schooling of 15 percent in the least developed countries, or equivalently $\sigma_{i}^{Poor} = 3.04$. From (23), this requires $\gamma$ to be equal to 2.27, which implies that the relative total factor productivity in the informal sector $\gamma$ is 0.16, and that the threshold proportion of college graduates below which the informality regime is observed is 30.6 percent.

So far we have obtained the main parameters from the production side, now we turn to the parameters that affect household’s decisions. The fertility ratio $n$ of high-to low-skilled workers is set to 0.57 from Kremer and Chen (1995). They show that
$n$ does not vary that much with the level of development, it is stable across countries and over time. Moreover, as we observe in the United States and other developed economies, we assume no population growth, which implies $n_h = 0.73$ and $n_l = 1.27$.\footnote{We assume these parameters to be constant because we depart from fertility decisions although poor countries have higher fertility rates than developed ones in the data so as in the model.} Haveman and Wolfe (1995) and Knowles (1999) suggest the education cost is around 15 percent of time endowment of parents while children live with parents. This implies that if children live 15 years with parents, then $e = 0.048$.\footnote{For example, de la Croix and Doepke (2003) assume that children live 15 out of 30 years with parents.} The remaining parameters are the weight of children’s income on utility and child labor productivity. Assuming that United States economy is in the steady state, from (13) and (14) we obtain $\beta = 0.26$. And the relative productivity $\eta$ of children compared to parents is 0.37 to match the empirical evidence presented by Horrell and Humphries (1995) who claim that 25 percent of family income comes from child labor.\footnote{We obtain a relative productivity of children compared to parents higher than Doepke and Zilibotti (2005) who obtain 0.1 to match the same empirical fact. However, Goldin and Sokoloff (1984) claim that that the relative productivity of children and females compared to males rose from around 0.3 in the North (.58 in the South) to .5 from 1820 to 1850, which is in line with our value.}

### 4.2 The case for multiplicity

Figure 3(a) shows human capital dynamics with parameter values obtained in the parametrization subsection. As predicted by Proposition 4, a poverty trap emerges in the presence of informality because the informal sector does not allow high-skilled wages increase enough so as to encourage education. Moreover, the existence of informality opens the door to child labor.

As can be seen in Figure 3(b), human capital dynamics is driven by the proportion $q_l$ of children of low-skilled parents. In the informality regime a constant share of children is educated. While in the formality regime this share increases up to a point where parents do not find it profitable to educate so many children, and the proportion of children who provide education by low-skilled adults decreases.

These two Figures explain why the poverty trap emerges. The not high enough returns to education and the opportunity cost of sending children to school make the proportion $q_l$ to be lower in the informality regime than in the formality regime for proportions $h_t$ of high-skilled workers between 15% and 30%. Because agents do not internalize the externality of education on TFP and the low number of highly educated children, the proportion of high-skilled workers remains low and stable over time.
Figure 3: Human capital dynamics $h_{t+1}(h_t)$ and proportion $ql(h_t)$ of educated children with and without informal sector.
4.3 Removing informality

In this section we look at the transition from the low steady state to the high steady state if informality vanishes. Figure 4(a) shows the transition from the low steady state to the high steady state if we keep the parameter values but do not allow for the existence of informality. We can observe that the transition would last around 300 years (or 10 periods) to achieve the new steady state. At the same time, we can also observe that after 3 periods the proportion of high-skilled individuals is higher than the threshold value that defines the informality regime.

The question that follows is how removing informality would affect individuals in this economy. In Figure 4(b) we compute the welfare loss as the percent consumption deviation across time with respect to the consumption level observed in the steady state with informality. To be more precise, in every period $t$, we compute the percentage deviation as $100 \times (c_{k,t} - c_{k}^{\text{low}})/c_{k}^{\text{low}}$, where $c_{k}^{\text{low}}$ is the consumption level of a type-$k$ worker in the steady state with informality. As expected, the consumption level of high-skilled workers is not modified because their wage is constant along the evolution path, and they find it optimal to educate all their offspring. For the low-skilled workers, however, consumption falls by more than 50% in the first period that informality is removed and it increases as time passes to reach the consumption level of the high steady state. The first period welfare loss is related to the flow of workers to the formal sector that lowers wages of the less educated adults. Then, as the economy evolves, the higher proportion of high-skilled adults increases wages of low-skilled workers up to the new steady state because of the complementarity between high- and low-skilled workers, and the increase in TFP (equation 8). Finally, note that the average loss is driven by the low-skilled loss. As the economy evolves, the proportion of low-skilled workers is lower and their weight on the average loss is lower as well.

4.4 Implications for development policy

In the previous sections we established the result that the existence of an informal sector combined with human capital externalities can generate a poverty trap. We also showed that if informal activities were rendered illegal, low-skilled workers would suffer initially a quite dramatic drop in wages. In this section, we analyze policies that could help the economy to escape the poverty trap and converge towards the high-income steady state. We analyze the cost-efficiency of such policies under the constraint that wage losses during the transition should be avoided.

We consider the situation of a developing country trapped in the low-income steady state and assume that it will obtain a windfall gain (which might come from different sources, e.g. foreign aid or the discovery of natural resources).\textsuperscript{13} How can

\textsuperscript{13}In the case of a resource-rich country, it would have to be assumed that the natural resource sector operates independently from the rest of the economy, excluding thereby Dutch disease effects.
(a) Transition from low to high steady states

(b) Welfare loss due to transition

Figure 4: Transition from informality to formality.
such a windfall gain be used in the most efficient way in order to escape the poverty trap? We analyze different policies to answer this question; our findings can be summarized as follows. First, among four possible education and wage subsidy schemes, two policies dominate the others in terms of cost efficiency: education subsidies to low-income families and wage subsidies for low-skill jobs in the formal sector. Second, these two policies play distinct and possibly complementary roles in the transition to the high-income equilibrium. Whereas the education subsidy speeds up the accumulation of human capital, the low-skill wage subsidy reduces the threshold at which the informal sector disappears. Third, targeted education subsidies are the cheapest single policy but for medium time horizons a combination of the two policies turns out to be the most cost-efficient choice.

**Alternative policies.** On the one hand, we consider the introduction of education subsidies that are either paid unconditionally to all families or targeted to low-skill parents. The latter policy can be interpreted as the education component of existing conditional cash transfers.\textsuperscript{14} Second, we analyze wage subsidies for jobs in the formal sector, allowing for different subsidy rates for low-skill and high-skill jobs.\textsuperscript{15} To sum up, we introduce the following policy variables in the model:

- an education subsidy at rate $s_{e}^t$ (paid to all families or targeted to low-skilled parents);
- a wage subsidy for low-skilled workers in the formal sector at rate $s_{l}^t$;
- a wage subsidy for high-skilled workers in the formal sector at rate $s_{h}^t$.

From the assumptions of the model it is immediately clear that it would be inefficient to pay education subsidies to high-skilled parents since they educate all their children even without receiving any subsidies. Hence the general education subsidy is less cost-efficient than the targeted education subsidy. As we will show below, the wage subsidy for high-skilled workers has similar effects as an education subsidy to all parents. This type of wage subsidy is therefore also dominated by the targeted education subsidy.

**Policy effects in the informality regime.** In the informality regime, the introduction of subsidies does not change the income of low-skilled workers. Subsidizing low-skilled workers draws them into the formal sector but as long as the informal

\textsuperscript{14}E.g., the Oportunidades/Progresa program in Mexico or the Bolsa Familia scheme in Brazil. These programs are targeted towards low-income families and provide grants for children conditional on school attendance.

\textsuperscript{15}Equivalently the government could implement a combination of an output subsidy in the formal sector and a (progressive) tax on income from the formal sector. An output subsidy has the same effect as subsidizing high-skilled and low-skilled workers in the formal sector at the same rate. Adding a progressive income tax would be equivalent to differentiating the effective subsidy rates received by high and low-skilled workers.
sector exists, the low-skill wage is determined by the (exogenous) productivity in the informal sector. By contrast, the income of high-skilled workers is increased one-by-one by the subsidy. Hence, wages (including subsidies) and the skill premium in the informality regime are

\[
\begin{align*}
\bar{w}_{h,t} &= A_0 \alpha (1 + s^h_t) \\
\bar{w}_{f,t} &= \bar{w}_{i,t} = \frac{A_0 (1 - \alpha)}{\gamma} \\
\bar{\sigma}_t &= \frac{\alpha \gamma}{1 - \alpha} (1 + s^h_t) - 1
\end{align*}
\]

The number of low-skilled workers in the formal sector is given by \(L_{f,t} = (1 + s^l_t) H_t\).

The informal sector disappears if marginal cost exceeds the price of its output, i.e., if \(L_{f,t}/H_t \leq (1 + s^l_t)\gamma\). This condition is equivalent to

\[h_t \geq \frac{1}{1 + \gamma(1 + s^l_t)}.
\]

The role of the two types of wage subsidies in the formal sector can now be made clear. Subsidizing high-wage jobs increases the skill premium but has no effect on the allocation of workers between sectors. By contrast, a subsidy for low-wage jobs in the formal sector does not affect the skill premium but lowers the critical human capital level at which the economy leaves the informality regime.

In turn, the budget constraint of adults is modified by the introduction of an education subsidy as follows:

\[c_{k,t} = \bar{w}_{k,t} - n_k q_{k,t} \bar{e} (1 - s^l_t) + \eta n_k (1 - q_{k,t}) d_t \bar{w}_{i,t}.
\]

The proportion of children who go to school is therefore equal to

\[q^*_{k,t} = \frac{\beta (1 - \alpha) (1 + n_t \eta)}{(1 + \beta) [\epsilon \gamma (1 - s^l_t) + \eta (1 - \alpha)] n_t} - \frac{1 - \alpha}{(1 + \beta) \left[ \alpha (1 + \gamma (1 + s^h_{t+1})) - 1 \right]}.
\]

Subsidizing high-skilled workers in the next generation \((t + 1)\) has similar qualitative effects as subsidizing education for the current generation \(t\). Obviously, an expected rise in the future skill premium increases the incentive to send children to school. There is, however, a decisive difference between the two types of subsidies: an education subsidy can be targeted towards low-skilled parents and is therefore more cost-effective (since high-skilled parents educate all their children even without subsidies). Moreover, subsidizing the wages of relatively rich workers rather than the education of poor children seems politically less feasible.

The preceding results enable us to highlight the different (and possibly complementary) roles of the two most promising policies: targeted education subsidies and wage subsidies for low-skilled workers in the formal sector (see Figure 5). If the economy is initially stuck in the inferior steady-state \((B_2)\), the introduction of targeted
education subsidies increases the incentive of low-skilled parents to invest in their children’s education and the informal sector schedule shifts upwards in Figure 5. If the subsidy rate is sufficiently high, the country can escape the poverty trap with the help of this single policy instrument; the new situation of the economy could then be described by Dynamic A in Figure 2(b).

By contrast, the subsidy for low-skilled workers in the formal sector pulls workers out of the informal sector and decreases the critical skill ratio from $z_0$ to $z_1$ in Figure 5 without changing the informality schedule. It is clear that such a low-wage subsidy has no effect on human capital accumulation if it is too small or if the economy is too far below the critical skill ratio; the subsidy rate must be sufficiently high to eliminate informal sector employment entirely. Wage subsidies should therefore only be used as a temporary policy allowing to accelerate the transition to the formality regime.

As the two types of subsidies address different aspects of the transition to the high income equilibrium, they can be implemented jointly and their combined use might possibly reduce the overall cost of escaping the poverty trap. This issue will be taken up below in the simulations. In any case, we assume that subsidies are abolished as soon as the economy reaches the formality regime.\(^{16}\)

\(^{16}\)To avoid clutter, Figure 5 does not depict the policy-induced change in the dynamics of the form-
Cost-efficient policies. The calibrated model can now be used to calculate, for each policy, the minimum windfall gain necessary to enable the country to escape the poverty trap. This windfall gain (or discounted cost of policy) depends on the time horizon within which the economy leaves the informality regime. Consider a constant subsidy of each type, $s^k_t = s^k$ for $k \in \{e, l, h\}$. The horizontal axis of Figure 6 indicates the time needed to achieve a level of human capital that ensures convergence to the high steady state, or equivalently, the time $T$ needed to achieve a proportion of high-skilled workers higher than the threshold value $h_T > 1/(1 + \gamma)$ delimiting the two regimes. The vertical axis of Figure 6 shows the total discounted cost of the policy for a country with an initial population of 20 million inhabitants, an initial TFP $A_0$ of 7000, and a discount factor equal to $0.99^{120} \approx 0.2994$.

The parameter $A_0$ is set to 7000 to obtain that GDP per capita in the United States is 3500 in 2005 US$ PPP adjusted, which is close to the value in PWT 6.3. The discount factor is obtained from the literature taking into account that a period lasts 30 years and the discount factor of a quarter of year is $0.99^{120}$. The two policies have different effects on the formality schedule. Whereas education subsidies shift the formality schedule unambiguously upwards, the introduction of low-skill wage subsidies has ambiguous effects: a positive income effect (low-skill parents receive a higher income which is partly spent on education of their children) and a negative substitution effect (low-wage subsidies decrease the future wage differential, diminishing the incentive for education). These changes do not seem to have a decisive influence on the transition from the informality to the formality regime.

\[ h_T > 1/(1 + \gamma) \]

Figure 6: Total discounted cost of policies and time necessary to achieve $h_T > 1/(1 + \gamma)$.
As expected, targeted education subsidies are more cost-efficient than unconditional education subsidies or high-skill wage subsidies at any time horizon. A windfall of 100 to 150 million 2005 US$ (PPP adjusted) is needed to help a country of around 20 million inhabitants escape from the poverty trap within one or two generations (30 or 60 years). As the initial skill ratio of this economy is far below the critical level, low-skill wage subsidies are very inefficient if they are used as a single policy instrument.

Moreover, as Figure 6 makes clear, policies that take more time to leave the informality regime have lower discounted costs. Consider for example education subsidies targeted to low-skilled parents. The total discounted cost of attaining the critical human capital ratio is lower if the policy is implemented over several generations using a low subsidy rate (by opposition to a high-subsidy policy which operates within one generation). The reason for this result is twofold. First, within a generation the marginal cost of subsidizing education increases with the proportion of children that are educated. Second, targeted education subsidies have a cumulative impact over time: in each generation, they provide an incentive to low-skilled adults to educate a larger proportion of their children. In the following generation, these high-skilled children will provide education to all their offspring although they do not receive the (targeted) education subsidy.

A similar result holds for low-skill wage subsidies in the formal sector: a marginal increase in formal employment of (low-skilled) workers is obtained at the cost of paying higher subsidies to all (low-skilled) formal sector workers, including the infra-marginal workers. Therefore the cost of eliminating the informal sector is a quadratic function of low-skilled employment. Indeed, the informal sector disappears if the low-skill wage subsidy is fixed at the rate \( s_l = (1/\gamma)(L_t/H_t) \). Therefore the cost of eliminating the informal sector within the current generation is given by \( s_l L_t = (1/\gamma)(L_t^2/H_t) - L_t \).

Combination of policies. The preceding results leave scope for a cost-reducing combination of policies. As the marginal cost of a single policy increases with its rate, it might be more cost-efficient to combine two instruments using lower rates. We explore this possibility by combining targeted education subsidies and low-skill wage subsidies. As we have argued above, the latter should only be used as a transitory measure. In the simulations reported in Figure 7, low-skill wage subsidies are only used if they enable the economy to reach the formality regime within the next generation.\(^{18}\) Figure 7 shows the minimum cost of reaching the higher income equilibrium either by using only targeted education subsidies or by combining the two policy instruments. The combination of the two policies is cheaper than the single instrument for time horizons that exceed three generations (90 years). Note that for

\(^{18}\) Alternatively, one could assume that the subsidy is phased out gradually if it takes several generations to attain the critical human capital level. This possibility is disregarded in our search of the cheapest policy combination.
slightly richer countries (that are closer to the critical skill ratio), a combination of the two policy instruments is likely to be more cost-efficient even for shorter time horizons.

Figure 7: Cheapest combination of policies and time necessary to achieve $h_t > 1/(1 + \gamma)$.

5 Conclusion

This paper establishes a theoretical relation between education, child labor and the informal sector. In the data we observe a direct relation between informality and education, countries with high proportions of tertiary educated workers tend to show lower levels of informality than countries with low proportions. Moreover, child labor is related as well with the informal sector since it is one of the possible forms of it, and the data confirms that countries with more informality have more children involved in production activities. With these facts in mind we construct an overlapping generations model that is able to reproduce these relations in line with previous findings of other authors.

The model is able to explain, or to give a complementary view of, the documented fact that low-developed countries present higher levels of inequality than developed ones but much lower than standard models predict. The introduction of the informal sector in a model with complementarity between high- and low-skilled workers makes the skill premium lower and constant than without informality. In other words, we
view informality as a possible channel to reduce the skill premium in developing countries.

The reduction in inequality due to informality generates several effects in the short and the long run. On the one hand, low-skilled workers may obtain a higher salary with the existence of an informal sector than in its absence, because there is an alternative sector where they can supply their working hours. However, this sector is not controlled by state agencies and enables children to use their time to work and generate an extra source of income for the household. Hence, the model is able to replicate the relations between informality and education, and between child labor and education in line with the data, high-skilled workers are negatively correlated with informality, and informality is positively correlated with child labor. On the other hand, the model has several predictions on the long run. The trade-off between child income and future education of children is taken into account and is key to generate poverty traps due to informality and child labor. The “low” inequality observed in developing countries and the opportunity cost of sending children to work instead of going to school can make a pernicious effect on parents. They may not provide enough education to their children so as to increase the aggregate proportion of educated workers in the labor force. Parents do not internalize the positive externality of aggregate education on firms productivity. Therefore, the informal sector can make the economy not to develop as it could in the absence of informality.

The model is calibrated to reproduce several facts in the data. The model is also calibrated to evaluate different policies considered to reduce the size and effects of informality. The calibration exercise reveals that the case for the poverty-trap hypothesis is strong: although informality serves to protect low-skilled workers from extreme poverty in the short-run, it prevents income convergence between developed and developing nations. Sudden elimination of informality would induce severe welfare losses for poor people on the transition path.

Hence, we analyze policies that could help the economy to escape the poverty trap and converge towards the high-income steady state. We analyze the cost-efficiency of such policies under the constraint that wage losses during the transition should be avoided. Assuming that an inflow of resources arrives to a developing country, for example in the form of foreign aid, we analyze the effects of different subsidies. One possible way to reduce informality may come from reducing education costs or making the formal sector more attractive, as for example, increasing formal firms wages. Then, we consider four possible subsidies on education and formal firms wages. Subsidizing education is the most cost-effective policy, and it can be targeted towards low-skilled parents to reduce costs. Subsidizing high-wage jobs increases the skill premium but has no effect on the allocation of workers between sectors. Moreover, the increase in the skill premium gives similar incentives to parents on children’s education than reducing education costs. By contrast, a subsidy for low-wage jobs in the formal sector does not affect the skill premium but lowers the critical human capital level necessary to skip the poverty trap. Because of the possible
complementary effect of different subsidies, we turn to analyze the cost-efficiency of a combination of subsidies on education to low-income parents and low-skilled formal firms wages. Although targeted education subsidies are the cheapest single policy, for medium time horizons a combination of the two policies turns out to be the most cost-efficient choice.

References


