Dynamic political distortions under alternative constitutional settings*

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Abstract

This paper studies a dynamic model of electoral competition where two parties (or candidates) compete for power over redistribution and over public employment/public good provision. Parties only have diverging preferences over redistribution. Nevertheless, since public employment affects voters’ long run political preferences, they commit and implement socially suboptimal policies to improve their long term electoral strength. We investigate the non-institutional and institutional determinants of the resulting distortions in platforms and implemented policies. We find that more forward looking voters or more political persistence increase distortions. Consensual constitutions (as opposed to majoritarian) are associated with more platform divergence (only when the horizon is finite), less inefficient public good provision, and more redistribution. A mixed constitution can improve welfare over both. Finally, the model’s empirical implications—in particular, on the relationship between inequality and redistribution—are consistent with the available evidence.

Keywords: Dynamic Electoral Competition, Public Employment, Constitutions.

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Introduction

Government policies often affect structural aspects of a society, thereby influencing the electorate’s long term political attitudes, shaping the electoral environment in which political competition will take place. The electoral environment captures an array of factors that influence voters’ political preferences (that is, preferences over candidates or parties) but are taken as given by political actors when they make pre-electoral commitment to policies (i.e., they choose their political platforms). This paper studies how, in a democracy, parties can choose socially undesirable platforms in order to affect the electorate’s future political preferences, and improve their long term electoral strength. More specifically, we study the distortions generated by the presence of a dynamic link between current implemented policies and parties’ future electoral appeal, and how these distortions are affected by the type of constitution and by some key non–institutional variables (ex ante inequality, political actors’ patience, voters’ information).

A famous example of the mechanism described above is the Curley effect (Glaeser and Shleifer, 2005): a politician with a strong ethnic affiliation and a reputation for populist social spending has an incentive to manipulate redistributive programs and migration policy so as to generate a large inflow of poor immigrants of his own ethnic group, thereby shaping the electorate in his favor. As a result, socially undesirable policies arise. This mechanism is named after James M. Curley, four-terms mayor of Boston in the period 1914–1950. Curley, the son of an Irish immigrant, is remembered for pursuing aggressive redistributive policies that favored the inflow into Boston of poor, Catholic, Irish immigrants and drove out of the city some of the wealthier, Protestant, Anglo-Saxon elite. By discouraging business and reducing the city’s fiscal base, his policies hampered Boston’s economy. Nevertheless, by enlarging and consolidating his main constituency, Curley was able to build a long and successful political career.1

Similar explanations have been proposed, mostly informally, for a few instances of socially undesirable privatizations2 and subsidies to home ownership3. This paper shows that the

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1 Besides his four terms as a major of Boston, Curley also served as Governor and Senator of Massachusetts.
2 Glaeser and Shleifer (2005), who brought the Curley effect into the economic literature, argue that the same type of mechanism can explain other political failures, leading to underdevelopment and conflict in racially divided polities (for example, Coleman Young’s Detroit in the period 1973-1993, or Robert Mugabe’s Zimbabwe in the period 1987-present).
3 Biais and Perotti (2002) to explain the wave of privatizations that occurred in various European countries during the nineties: by increasing the median voters’ relative income, right wing governments implemented these policies in order to shift voters’ long term political attitudes in a conservative direction.
4 Several columnists (e.g., Becker, Stolberg and Labaton in The New York Times, December 20, 2008)
same logic behind the Curley effect—the desire of politicians to shape the future electorate in their favor—can play a role in a much broader sense than so far argued, and is the first attempt to systematically study the institutional and non-institutional determinants of this source of political failures.

In this paper, we study these distortions using a dynamic public finance model of electoral competition in which office is associated with policy-making power over public employment (which determines public good provision) and redistribution (which, by reducing the incentives to generate income, yields inefficiencies). Political actors have diverging preferences over the latter dimension and can *ex ante* commit to a platform on the public employment/public good dimension, over which they do not have intrinsic preferences. The key dynamic link of the model is generated by the assumption that a citizen’s employment status (public vs private) has a systematic effect on his beliefs about redistribution: due to their indirect exposure to the state of the private sector, public sector workers tend to systematically underestimate the social cost of redistribution with respect to private sector workers. Therefore, the size of the public sector systematically affects the electorate’s future political preferences.

Political actors, then, have an incentive to manipulate their platforms on public employment in order to improve their future electoral strength. The presence of Downsian voters pushes political platforms towards the socially optimal level. Nevertheless, political actors optimally trade-off current electoral strength for a better future electoral environment, resulting in socially undesirable platforms and implemented policies (dynamic political distortions).

The first contribution of the paper is to show how distortions arise because of the interplay of three key elements. First, political actors are differentiated (Krasa and Polborn, 2009): either they have different preferences over some policy dimension (in this model, over redistribution), or they have different abilities in delivering utility to voters. Second, have also suggested that the increase in the subsidy to home ownership implemented in the last decades in the United States was motivated by the goal of shifting the electorate in a conservative way. Regarding home ownership, Ortalo-Magne and Prat (2011) is a first attempt into jointly investigating the economic and political consequences of home ownership subsidies in a dynamic setting, but their policy implications are mostly normative.

5As extensively discussed in the rest of the paper, this assumption is not, by any mean, the only possible way to generate these effects.

6We model this lack of direct expertise as more noisy information.

7Voting follows one of the standard version of the probabilistic voting model, pioneered by Lindbeck and Weibull (1987) and Dixit and Londregan (1996) and extensively applied in Persson and Tabellini’s *Political Economics* (2002).
announced political platforms are related to implemented policies in a systematic way. In this paper, a commitment assumption on platforms for public employment and a *constitution* (defined as a mapping from electoral outcomes into policy-making rights) provide such a systematic relationship. Third, there is a link between implemented policies and future political preferences. In this paper, asymmetric information provides such link: since employment status (private vs public) affects the precision of one’s information about the inefficiency of redistribution, public employment influences political actors’ future *ex ante* (that is, before the announcement of political platforms) electoral strength.

This highlights two fundamental differences between this paper and Glaeser and Shleifer (2005). First, in the latter distortions arise because politicians exploit an underlying ethnic conflict. In this paper, politicians “spread” conflict from a “conflict” policy dimension (redistribution, which has, unlike a purely ethnic conflict, a clear economic rationale) to a “common value” policy dimension (public good/public employment). As a consequence, this paper shows how political representation can generate a conflict on a policy dimension absent any underlying disagreement among voters. Second, the dynamic link between policies and voters’ electoral attitudes is derived from information, rather than assumed through preferences, which makes this framework useful for studying this mechanism in other policy settings.

The second contribution of this paper is to study how the type of constitution affects these distortions. Building on Lijphart’s *Patterns of Democracy* (1999), majoritarian (M) and consensual (C) constitutions are compared. In the former, the majority winner obtains full policy-making rights; in the latter, political actors negotiate policies with bargaining power proportional to their vote share. Two types of dynamic political distortions are considered. At the platform level, we consider platform divergence on the public good/public employment dimension (interpreted as a measure of political polarization). At the implemented policy level, we focus on the expected quadratic deviation (public employment inefficiency) from the socially optimal public employment level. The paper also studies how constitutions affect implemented redistribution.

The main results are that majoritarian constitutions display higher public employment inefficiency (because of the absence of the moderating effect of bargaining), and lower redis-

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8 A systematic link between implemented policies and future political preferences can be guaranteed by several mechanisms other than informational asymmetry, namely ideology or “technological” differentiation among political actors (Krasa and Polborn, 2009). The Robustness Section of the paper describes and discusses these alternative settings and shows how most of the results of this paper are preserved.

9 This mechanism is similar to Morelli and Van Weelden (2011), but comes from a completely different source.
trubution. Nevertheless, in a finite horizon model, majoritarian constitutions display lower platform divergence (because the marginal cost of distortions is higher than under consensual constitutions). In an infinite horizon model, instead, platform divergence is independent of the constitution. This interesting long term neutrality property is due to the stationary nature of the equilibrium.

It is then natural to ask what exactly drives the constitutional comparison: is it the allocation of power over public good provision (the common value dimension, about which parties do not intrinsically care), or over redistribution (the conflict dimension, about which parties care)? The paper shows that differences are essentially driven by the allocation of power over the former dimension: a mixed constitution (with consensual allocation of power over public good provision and majoritarian over redistribution), displays the same platforms and implemented public employment as a consensual constitution. Moreover, semi-consensual constitutions welfare-dominate both consensual and majoritarian constitutions under a utilitarian criterion. Interestingly, a majoritarian constitution is dominated under any welfare criterion by either semi-consensual or consensual.

The third contribution of the paper is to study the relationship between dynamic political distortions and other important non-institutional factors, such as political persistence and far sightedness (of both voters and political actors). Several authors have identified short termism as a primary source of political failures, especially in contexts involving redistributive politics (Acemoglu and Robinson, 2001; Dixit and Londregan, 1995; Kundu 2007). Recent contributions to this literature have explored political failures in the various policy areas In all these papers, the source of the distortions lies in the inability of the policy maker (or legislative proposer) to be dynamically consistent. Since future political power is uncertain and current payoffs are fully appropriable, having more persistence in political power and/or more far sighted political actors would mitigate these distortions. This paper shows how this logic is completely reversed in a setting with dynamic political distortions: more patient political actors have a stronger incentive to manipulate public employment to improve their future electoral strength. More surprisingly, more patient voters or more

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10 Short termism is broadly defined as the inability of incorporating future consequences of current policies.
11 Battaglini and Coate, 2008, focuses on public debt; Azzimonti, 2009, on excessive investment taxation; Besley and Persson, 2010, on the development of fiscal capacity; Aht and Dutta, 2007, on long term public investment; Acemoglu et al, 2009, on labor supply distortions induced by redistribution.
12 Political persistence has slightly different meanings in each of these papers: in Battaglini and Coate’s model, it is related to the degree of autocorrelation in the identity of the proposer in their legislative bargaining game; in Aht and Dutta’s, it means ensuring that the identity of the politician in charge is the same in every period; in Azzimonti’s and Besley and Persson’s, persistence is simply the level of incumbent’s advantage.
political persistence also lead to more severe distortions.

Finally, the paper delivers various empirical implications relating inequality, redistribution, and political polarization. First, higher inequality increases polarization. Second, higher inequality decreases public good provision. Third—and most interesting—when redistribution entails significant inefficiencies and the constitution is majoritarian, inequality and implemented redistribution are inversely related. The reason is that, while higher inequality always reduces the electoral appeal of leftist parties, under a majoritarian constitution this will translate into significantly lower chances of having redistribution implemented. Each of these predictions is line with several recent empirical contributions (De Mello and Tiongson, 2003; McCarty, Poole and Rosenthal, 2008; Persson and Tabellini, 2003), thereby providing a connection between theory and data. In particular, an account for the inverse relationship between inequality and redistribution is still an open question (Campante, 2011; Kelly and Enns, 2010).

The paper is organized as follows: the first section reviews the relevant literature; the second describes the basic economic environment, voting behavior, and the political process. Section 3 analyzes the outcome of a majoritarian constitution under different assumptions on the time horizon (two-period vs infinite horizon) and derives the main comparative statics results. Section 4 describes the solution of the two-period and infinite horizon model of consensual democracy. Section 5 compares the two constitutional settings. In Section 6, the main empirical implications of the model are discussed. Section 7 examines how the model is robust to various changes to the assumptions of the baseline model. Section 8 concludes. All proofs are relegated to the Appendix.

1 Related literature

This paper is related to a large literature on the emergence and persistence of inefficient policies. Virtually all papers focus on dynamic commitment problems that political agents face, and most of them are based on the presence of some underlying conflict in the society that directly generates these inefficiencies. In Acemoglu and Robinson (2001), Kundu (2007), Glaeser and Shleifer (2005), a polity is exogenously divided into cleavages and political actors have incentive to manipulate their relative size to improve their electoral success. In the first two papers (which develop the idea originally illustrated in Dixit and Londregan (1995)), the relevant cleavages are farmers vs manufacturing workers, in Glaeser and Shleifer (2005), Irish/Catholic vs Protestant/Anglo-Saxons citizens.

In all these contributions, inefficiencies arise because political actors can successfully ex-
ploit an existing conflict of interest that feeds back into political preferences. In Glaeser and Shleifer (2005) Irish workers are more likely to vote, for exogenous reasons, for an Irish candidate. In the other two papers, inefficient redistribution to agricultural workers arise because it is politically unfeasible (due to lack of dynamic commitment to future redistribution) for parties to induce a transition to a more efficient equilibrium, which requires existing workers to suffer a short term loss. In this paper, instead, neither the rich or the poor have anything to gain from a suboptimal level of public employment: inefficiencies arise because of the effect asymmetric information between public and private sector workers on their expected political preferences. As a consequence, any institutional device that mitigates dynamic commitment problems will not reduce the inefficiencies associated with this channel of political failure.

Like Acemoglu et al. (2011), this paper features a redistributive conflict between rich and poor that feeds back into inefficiencies in some aspect of the public sector. Unlike this paper, in Acemoglu et al. (2011) inefficiencies are observed in the composition of the bureaucracy (rather than its size) and can only arise in a non-democratic context (rather than the endogenous outcome of electoral competition). Most important, these inefficiencies arise because there is an underlying interest in the society (the rich) benefitting from these policies, which is absent in our paper (where inefficiencies arise because of the Machiavellian considerations of political actors).

Krussell and Rios-Rull (1999), and Hassler et al. (2005) are also related to this paper. The first looks at a dynamic version of the Meltzer-Richard model to study the dynamics of redistribution within the context of a neoclassical model. The second studies the evolution of preferences over redistribution in an OLG economy in which young agents have to undertake an investment that improves their future expected productivity and how this interacts with redistributive politics to generate inefficiencies.

By studying the dynamics in public employment, output, and redistribution, this paper is connected to a dynamic public finance literature focusing on exogenous changes in power and inefficient volatility in output and consumption. Acemoglu et al. (2009) explore the effect of stochastic power fluctuations on the allocation of resources in a dynamic production economy where groups differ in their labor-leisure preferences. From a more general perspective, Bai and Lagunoff (2008) consider an environment in which policy-making exhibits “Faustian” dynamics: in every period the identity of the (weighted) median voter depends on the wealth distribution. As a consequence, the policy that maximizes the immediate payoff of the current median voter also shifts away political power from him. This literature shares with the present paper the idea that uncertainty over future allocation of political power constitutes
an independent channel for political failures. While in Acemoglu et al. (2009) more patient actors help mitigating these issues, in Bai and Lagunoff they make these distortions more pronounced.

The present paper is also related to a series of papers where the presence of a dynamic linkage in policies interacts with the political process to generate distortions. Battaglini and Coate (2008) look at a dynamic legislative bargain model of redistributive politics where the presence of public debt create incentives to shift costs towards future periods. Legislators bargain on a public good, whose marginal utility is stochastic, district-specific transfers, taxation, and debt. The political equilibrium features regime switches between phases in which policy-making is Pareto efficient and a “business-as-usual” phase in which inefficiently high debt and positive pork spending arise. An important result is that more persistence in power reduces pork and the inefficient accumulation of debt. This idea is completely reversed in the present paper.

Besley and Coate (1998) look at a two-period citizen-candidate model in which the political process might hamper efficient public investment. In one of the examples concluding the paper, they analyze productivity enhancing investment with a large, but non majoritarian, high productivity group and two smaller low-productivity groups. A policy that increases at no cost the productivity of one of the two low productivity groups is not implemented because of its consequences on the distribution of future political power. The group that would benefit from the investment and is in favor of redistribution will switch its political preferences against redistribution in the following period. As a consequence, it cannot find support from the other two groups to enact its preferred policy (investment and redistribution). Aghion and Bolton (1990) and Milesi-Ferretti and Spolaore (1994) look at a similar trade-off in the case, respectively, of public debt and the size of government.

The neoclassical economy in Azzimonti (2009) features two groups, each expressing a party, competing for political power over the allocation of local public goods, which is funded by an investment tax. Holding power is also associated with an exogenous incumbency advantage, the distortions associated with political competition result in inefficiently low investment rates and excessively large local public good provision. Moreover, political persistence (proportional to the size of the incumbent advantage) and low polarization (defined as a lower marginal utility for local public goods) are associated with higher level of investment and lower governments. The second result is in line with the findings on the effect of higher inequality in this paper. The first result, instead, is based on a similar logic to Battaglini and Coate (2008) and several other papers in dynamic political economy: more political persistence mitigates the dynamic commitment problem and reduces distortions. In
this paper, we show that asymmetric information, lack of commitment and differentiation in political actors can generate the opposite effect, because political actors’ long term goals might be less aligned with voters’ long term goals.

This paper is also related to a small literature on short termism as an equilibrium response to some underlying friction in the policy-making process. Garr’ı (2009) explains the policy bias toward the short term public goods using a reputational argument, leading to the conclusion that political short termism might be welfare improving because it enhances selection of congruent politicians. In this paper, short termism is beneficial absent any independent of reputation and selection motives.

The idea that political actors can manipulate policies to improve their future electoral strength is also featured in Hodler et al. (2010), where different politicians are associated with different maps between policies and outcomes. For this reason, an incumbent has incentives to provide inefficient policies in order to shift the salience towards the dimensions in which he thought to be more productive than his competitor. Policy manipulation, then, creates an endogenous incumbency advantage. The present paper shows that a similar manipulation can arise even at platform level and, more importantly, without technological differences between politicians.

This paper also contributes to a large body of literature investigating the relationship between constitutional features and public finance outcomes, such as public good provision, transfers, government size (Persson and Tabellini, 2004; Persson, Roland and Tabellini, 2005; Lizzeri and Persico, 2001; Milesi-Ferretti, Perotti and Rostagno, 2002). In a recent paper, Battaglini (2010) extends the setting of Battaglini and Coate (2008) to a a multiple-district environment with probabilistic voting. His main finding is that the basic prediction of the static literature on electoral outcomes on public finance (PR gives leads to overspending and less transfers with respect to majoritarian systems) does necessary hold in a dynamic setting with endogenous pubic debt. The reason is that, while proportional representation generates more incentives to overspending, it also results in faster accumulation of public debt, which in turns generates a tighter endogenous constraint on spending. The dynamic linkage in Battaglini’s paper is purely economic (public debt), while in our paper is essentially political (because of the effect of public employment on perceived redistribution). Moreover, our paper focuses on a broader institutional comparison, as in Ticchi and Vindigni (2010), which explicitly compares consensual and majoritarian constitutions in a redistributive setting, but focuses on the conditions promoting the *ex ante* adoption of either type, rather than the associated inefficiencies. Baron, Diermeier and Fong (2011), instead, focus on parliamentary form of government with proportional electoral rule. With respect to the current paper,
their model features a richer description of the institutional structure and a more stylized underlying economic environment. Nevertheless, the idea that parties currently endowed with proposal power have incentive to manipulate policies to improve their future bargaining position in future government coalitions is closely related to our model of consensual constitution.

Kalandrakis (2009) builds a reputational theory of two party competition in which voters are uncertain about whether a party is controlled by extremists or moderate agents. In his setting, far sightedness of partisan agent has two set of consequences. On a static level, it encourages the adoption of moderate policies for electoral purposes. On a dynamic level, it pushes towards extreme policies because of their impact on reputation: government parties pursue extreme policies to avoid losing elections almost for sure against an opponent on moderate platforms. Although the setting and the source of the intertemporal trade-off are very distant from our model, this is one of the few papers that shares with ours the idea that far-sighted politicians can be potentially detrimental for voters, due to a complementarity in current and future distortions.

The empirical implications of our paper relate the model to two important bodies of literature in economics and political science. The first investigates the negative relationship between inequality and redistribution, documented empirically, for example, in Enns and Kelly (2010), De Mello and Tiongson (2003), Perotti (1996). These results seriously challenged the conclusion that redistribution increases with inequality, which comes from a theoretical literature started by Meltzer and Rich ard (1981). Therefore, in recent years there have been several attempt to produce theories yielding the opposite prediction (Moene and Wallerstein, 2001; Bénabou, 2000; Bénabou and Ok, 2001; Iversen and Soskice, 2006). Iversen and Soskice (2006), in particular, focuses on constitutional differences and argues that countries with PR are more likely to have center left governments and, as a consequence, higher redistribution and lower inequality than countries with majoritarian electoral systems. Unlike the present paper, their focus is cross sectional, rather than dynamic. Moreover, differences in outcomes are generated by the different incentives to form coalitions, rather than different incentives to distort platforms. The second relevant body of literature investigates the relationship between inequality and political polarization. The evidence is presented in a comprehensive fashion in McCarty, Poole and Rosenthal (2008). This paper contributes to each of these literatures by showing the existence of a novel channel that can help explaining how the channel between inequality, redistribution and political polarization is affected by institutional factors.
2 Economic and electoral environment

Economic environment: A society is composed of a unit-mass continuum of citizens and lasts $T$ periods. In every period, each citizen $i$ is endowed with one unit of labor (the only input in the economy), which can be supplied in either the public sector or in the private sector. The marginal product of labor in the private sector, whose size is denoted by $1 - x_t$, is given by $A \in (0,1)$. The labor market in the private sector is competitive, and there is full mobility of labor. As a result, in every period the gross wage in both sectors equals $A$. Since the private sector labor demand for wage $A$ is undetermined, the public sector labor demand, $x_t$, pins down the labor market equilibrium.

The public sector turns each unit of labor into a unit of public good, $g_t$. The public sector is financed by a proportional labor income tax $\tau_t$ levied on every worker. Therefore, the budget constraint for the public sector is $\tau_t = x_t$. Inefficiencies associated with labor taxation are captured by a quadratic term $\frac{\tau_t^2}{2}$, which decreases citizens’ payoff. The public good enters voters’ payoff linearly. As a result, regardless of where she works, a citizen’s $x$-related payoff is given by $A(1 - x_t) + g_t - \frac{\tau_t^2}{2}$. After substituting the budget constraint and the production function for $g_t$, it becomes

$$A + (1 - A)x_t - \frac{x_t^2}{2}$$

In every period, half of the citizens are endowed with an ownership share of the private sector, which generates a per period non-labor income $\omega_t > 0$, which is assumed, without loss of generality, to be stationary. The government can redistribute non-labor income from rich to poor citizens using a proportional tax on the non-labor income $\theta_t$ to finance a per capita transfer $b_t$. Redistributive taxation also entails inefficiencies. To capture this idea in a reduced form, we assume that for every consumption unit redistributed, the government has to take $1 + q_t$ consumption units from the rich. The term $q_t$ is stochastic and incorporates a wide array of factors that influence, in a given period, the elasticity to taxation of the non-labor income. For example, $q$ can capture the relative technological advantage with respect to some foreign country or the marginal return of entrepreneurial effort. Redistribution must satisfy the following constraint

$$\omega \theta_t = (1 + q_t)b_t$$

These assumptions imply that the $x$-related payoff for each voter is quadratic and strictly concave in $x$, which makes the mathematical structure of the model conveniently simple. Numerical simulations of the model under a more general formulation of the production function of the public good (of the form $g^\alpha$) yields very similar qualitative insights.
Taking both $x$-related payoff and non-labor income into account, the per period indirect utility of a rich and a poor citizen are, respectively,

$$v^r(x_t, b_t) = A + x_t(1 - A - x_t/2) + \omega - b_t(1 + q_t)$$

$$v^p(x_t, b_t) = A + x_t(1 - A - x_t/2) + b_t$$

**First Best.** The utilitarian social welfare function is given by

$$W(x_t, b_t) = A + x_t(1 - A - x_t/2) + 1/2 \{\omega - q_t b_t\}$$

As a consequence, a utilitarian social planner would choose, in every period, $x^* = 1 - A$ and $b^* = 0$. Due to separability of $W(x_t, b_t)$, every citizen’s preferred level of public good provision is $x^*$; as a consequence, it is easy to show that any social planner attaching arbitrary weights $1 - \alpha$ and $\alpha$, respectively, to the poor and to the rich, would also choose the same level of public good provision. $x^*$ is then a natural benchmark for the outcome of the political game. The paper focuses on two measures to quantify inefficiencies in platforms (which captures political polarization\[^{14}\]) and in implemented policies: expected policy distortion ($\Sigma^{(M)}$), that is expected quadratic deviation of the implemented policy from $x^*$, and platform divergence ($\Delta$), that is the difference between $L$’s and $R$’s proposed public employment levels.

**Political process.** In each period two political actors $R$ and $L$ compete for office. Being in office is associated with policy-making power over $b$ and $x$. Political actors do not intrinsically care about the relative size of each sector and differ in their preferred level of redistribution: $R$ prefers a more efficient economy with no redistribution, while $L$ prefers a fully egalitarian society\[^{15}\] At the beginning of each period $t$, political actors commit policy platforms for $x_t$. On the other hand, no commitment is possible for $b_t$. $R$, if alone in power, would then set $b_t = 0$ and $L$, if alone in power, would implement $\bar{b}_t$, which solves $\omega - \bar{b}_t(1 + q_t) = \bar{b}_t$, that is

$$\bar{b}_t = \frac{\omega}{2 + q_t}.$$ 

Political actors derive a payoff (normalized, in every period, within the unit interval) that is linear in the distance from their preferred transfer level: $R$’s per period payoff ranges

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\[^{14}\]See, on this the recent work of McCarty and Shor on quantifying the amount political polarization using data from surveyed pre-electoral candidates’ commitments.

\[^{15}\]In this setting, egalitarian and rawlsian preferences have the same induced preferences over redistribution. Also, all the results in the paper still hold if one assumes that $L$’s target redistribution is a fraction $\alpha$ of a rich’s net capital income.
between 1 (when \( b_t = 0 \) is implemented) and 0 (when \( b_t = \bar{b}_t \) is implemented), while the opposite is true for \( L \).

The non commitment assumption’s main role is to make the exposition of the model simpler: in the Robustness Section we show that removing the non commitment assumption generates the same qualitative insights of the baseline model and that distortions are even larger than in the baseline model. The assumption is nevertheless motivated by the idea that pre-electoral commitment to a certain tax rate (or public employment level) is easier than pre-electoral commitment to transfers, since redistribution can be implemented through a large variety of means and is often delegated to lower level government officials, whose functions are not determined until after the elections.

The assumption that \( R \) and \( L \) have preferences over redistribution is, instead, a substantive one: removing it would eliminate any source of differentiation between them, thereby eliminating the possibility of current policies to affect future political preferences. This assumption seems also quite natural and captures the idea that parties and candidates typically differ in how they balance the trade-off between inequality and efficiency that virtually every polity faces. Empirically, this idea is supported by evidence that cross-national differences in redistribution can be explained by differences in partisan composition of the government (Boix, 1998; Bradley et al., 2003).

**Timing and information.** Every period begins with a given sectorial distribution \( x_{t-1} \). Workers in both sectors have a prior \( F_q \), with support \([q^l, q^h] \subset [0, \infty)\), about the value of \( q_t \), while political actors have a more precise prior \( \hat{F}_q \), which is their private information. Before choosing platforms \((x_t^R, x_t^L)\), political actors and private sector workers observe \( q_t \), while public sector workers observe a noisy signal \( s_t = q_t + \varepsilon_t \), where \( \varepsilon_t \) has zero mean and support \([-e, e] \). After observing \( s_t \), they form a posterior \( F_{q|s} \) using Bayesian updating. The assumption captures the idea that private sector workers have a more direct exposure to the array of factors (mostly related to the competitive environment of a firm) affecting \( q_t \) than public sector workers. As a consequence, the former will have a more precise conjecture about the inefficiency of redistribution than the latter. As it will become clear later, this assumption generates the dynamic linkage between current implemented policies and future electoral environment, which is the key of the paper. Moreover, we will also argue (and formally show in the Robustness section) that there are several other ways to generate such

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\(^{16}\)As a result, in every period the sum of the political actors’ payoffs equals 1.

\(^{17}\)For the equilibrium analysis, We do not need to assume that this knowledge about \( q_t \) is necessarily *ex post* accurate, but only that current private sector workers have an a priori more precise understanding of the extent of the future moral hazard problem in the private sector.
linkage in very similar frameworks, but that the one we chose has the double advantage of keeping the analysis cleaner and generating empirically plausible implications.

**Voting behavior.** Each voter $i$ computes the expected per period payoff associated with each actor’s announced $x$ and anticipated $b$, that is $v(x_i^R, \tilde{b}_t^R)$ and $v(x_i^L, \tilde{b}_t^L)$. Voters know that $b_t^R = 0$ and $b_t^L = \bar{b}_t$. The latter depends on $q_t$, which can only be conjectured by public sector workers. Voting behavior is probabilistic: $i$ votes for $R$ iff

$$v(x_i^R, \tilde{b}_t^R) > v(x_i^L, \tilde{b}_t^L) + \xi_t + \delta_t^i$$

where $\xi_t$ is the realization of a stationary zero-mean aggregate preference shock $\xi$ and $\delta_t^i$ is the realization of a stationary zero-mean, idiosyncratic preference shock $\delta$. As in one of the standard formulations of the probabilistic voting model\footnote{See Lindbeck and Weibull (1987) and Persson and Tabellini (2002).} the realizations are $iid$ over time and drawn from uniform distributions. More precisely, the aggregate shock is uniformly distributed over the interval $[-1/2\psi, 1/2\psi]$ and the idiosyncratic shock is uniformly distributed over the interval $[-1/2\varphi, 1/2\varphi]$.

Both shocks capture attributes\footnote{more precisely, the utility differential between $L$’s and $R$’s attributes.} over a vector of dimensions not explicitly modeled and assumed to be orthogonal to public good provision and redistribution. Examples of such dimensions are abortion, race issues, illegal immigration, foreign policy, personal charisma. In every period $\xi_t$ can be interpreted as a measure of how much the median voter prefers the $L$-candidate over the $R$-candidate irrespective of announced $x$ and conjectured $b$. $\delta_t^i$, instead, measures the individual-specific deviation of voter $i$ from the median bias.

Without knowing how electoral outcome maps into policies, it is not easy to evaluate the strength of the assumptions on voting behavior. As it will become fully clear in the rest of the paper, they are arguably quite natural under each constitutional setting considered: a voter will try to push the implemented policies in the direction that she expects to benefit her the most\footnote{Moreover, voting behavior in this model is compatible with the assumption, often employed in political economy, that a voter votes as if she was pivotal.}. Furthermore, since they only look at the current period’s payoff, voters are myopic. This assumption will be relaxed in the analysis, leading to the surprising result that having more forward looking voters increases distortions.

For simplicity, assume that share ownership is independent of past sectorial affiliation\footnote{Since payoffs are linear, any correlation structure would lead to the same expression for the voting shares.}.

For the $1 - x_{t-1}$ workers who start period $t$ in the private sector (and therefore observe $q_t$), the expected payoff from $R$’s platform is $v(x_t^R, 0) = A + x_t^R(1 - A - x_t^R/2) + \omega/2$, the one
from L’s is \( v(x^R_t, \bar{b}_t) = A + x^L_t(1 - A - x^L_t/2) + \bar{b}_t \). Their difference is then be given by

\[
d(x^R_t, x^L_t) + I(q_t)
\]

where \( d(x^R_t, x^L_t) = x^R_t(x^* - x^R_t/2) - x^L_t(x^* - x^L_t/2) \) is the \( x \)-related payoff differential and \( I(q_t) = \omega q_t/[2(2 + q_t)] \) is the *redistribution inefficiency* associated with L’s egalitarian redistribution. \( I(.) \) is strictly increasing and strictly concave in \( q_t \). This is not an artifact of the particular structure of this model, but captures a more general idea: when redistribution becomes more inefficient, the egalitarian policy involves a lower level of redistribution, partially compensating the direct effect of a higher \( q_t \). R’s realized vote share among the private sector workers is then

\[
\Pr[\delta < d(x^R_t, x^L_t) + I(q_t) - \xi_t] = 1/2 + \varphi[d(x^R_t, x^L_t) + I(q_t) - \xi_t]
\]

Similar computation lead to conclude that R’s realized vote share among the \( x_{t-1} \) workers who start period \( t \) in the public sector is

\[
1/2 + \varphi[d(x^R_t, x^L_t) + \bar{I}(s_t) - \xi_t],
\]

where \( \bar{I}(s_t) = E_{F_{q_t}|s_t}[I(q_t)|s_t] \) is the expectation of the redistribution inefficiency conditional on \( s_t \). R’s total realized vote share is then the sum of four distinct components

\[
\hat{\pi}_t = \frac{1}{2} + \varphi[ d(x^R_t, x^L_t) + I(q_t) - \lambda_t x_{t-1} - \xi_t ]
\]

where \( \lambda_t = I(q_t) - \bar{I}(s_t) \), which is called *informational wedge*, is the marginal effect of a change in the initial size of the public sector on R’s electoral strength. Since redistribution entails inefficiencies, R receives, in expectation, always more votes than L when their platforms deliver voters the same payoff. More important, the expected difference in votes depends on platforms and on the initial sectorial distribution. Therefore, when political actors choose

Before solving for the equilibrium, we need to verify the internal consistency of the information partition. That is, we need to make sure that public sector workers, who are imperfectly informed about \( q_t \), cannot improve their knowledge from the observed equilibrium platforms. As will become apparent in the next section, this is ensured by the fact that they do not know \( \hat{F}_q \). By using their prior \( (F_q) \), public sector workers would conjecture that \( \tilde{\lambda} = 0 \) and will not be able to extract information about \( q_t \) from equilibrium platforms. In other words, under the knowledge of equilibrium platforms and \( F_q, q_t \) is not identified.
a platform for $t$ and face at least another election in the future, they observe (and take as given) $\lambda_t$, but must conjecture the value of $\lambda_{t+1}$ in order to evaluate the effect of their platforms on the future electoral environment. The following lemma states that political actors’ conjecture of $\lambda_{t+1}$ is positive, thereby generating the dynamic link at the heart of the distortions analyzed in this paper.

**Lemma 1** Political actors’ expectation of the informational wedge, $\bar{\lambda}$, is strictly positive.

The intuition for this lemma is that, due to the concavity of the expected aggregate welfare in $q$, noisy information on the latter more likely to underestimate the inefficiency of redistribution. As a consequence, $R$ will has a stronger expected electoral advantage over $L$ when the initial size of the public sector (hence, the number of voters with noisy information over $q$) is small.

Three important observations are in order. First, there are several other types of informational asymmetries that can generate a dynamic linkage between public employment and preferences over redistribution. As long as the posterior distributions of beliefs over $I(q_t)$ depends on the employment status, the linkage will generally exist. Second, we could employ several other, non-informational channels to generate the same dynamic linkage. The Robustness Section discusses two of these alternative channels, and shows how the model can accommodate them. Third, we have chosen to present the model in its current form because it has the advantages of (i) keeping the analysis clean (that is, showing how politicians spread conflict from one policy dimension to another that is, from the point of view of voters, fully orthogonal), and (ii) generating empirically plausible voting patterns. (The model predicts that public sector employees are more favorable to redistribution, as shown, for example, in Guillaud, 2011, and a much weaker correlation between income and voting for rich voters, as, for example, in Gelman et al., 2007.)

In order to keep the political actors’ problem well behaved, we need to assume that $\varphi$ and $\psi$ are related in such a way that ensures that, for every initial value of $x$ and $q$, both actors have a positive probability of obtaining a majority of the votes. A detailed description of this assumption, which is almost equivalent to the one in Persson and Tabellini (2002) is contained in the Appendix.

---

23 As argued earlier, the concavity is not specific to the particular setting of this model.

24 For example, the one of public sector workers first order, or second order, stochastically dominates the one for private sector workers.

25 That is, in order to ensure continuity and differentiability in the objective functions.
3 Policy making in a majoritarian democracy

The only component of the model that needs to be described is a rule mapping electoral result into an allocation of policy-making power over \( x \) and \( b \), that is a constitution. In this paper we focus, following Arend Lijphart’s approach, on the distinction between consensual constitution, from now onward \((C)\), and majoritarian constitution, from now onward \((M)\). Although this dichotomy has already been employed in economics (Ticchi and Vindigni, 2010; Herrera and Morelli, 2010), the majoritarian is simpler and more natural in terms of modeling choices. For this reason, this section describes the \((M)\)-electoral game and develops comparative statics whose intuition, for the most part, also applies to the consensual constitution case. The subsequent section describes the equilibrium of the \((C)\)-game, and the one that follows compares the two constitutions.

**Majoritarian Constitution.** In a majoritarian democracy the majority winner, denoted by \( W_t \), gets full policy-making rights over \( b \) and \( x \), that is implements his announced platform in the latter dimension and his preferred policy on the former. Under \((M)\), therefore, the payoffs in every period are \( 1_{\{R=W\}} \) and \( 1_{\{R=W\}} \). Political actors’ per period expected payoff is then the probability of winning a majority of the votes. For \( R \), this is given by

\[
p_t = \Pr \left[ \hat{\pi}_t > \frac{1}{2} \right] = \frac{1}{2} + \psi [d(x_t^R, x_t^L) + I(q_t) - \lambda_t x_{t-1}] \tag{1}
\]

and for \( L \) by \((1-p_t)\). Throughout the paper the equilibrium concept employed for two-period models is pure strategy subgame perfect Nash, and for infinite horizon models we restrict to equilibrium strategies that are differentiable, stationary Markov Perfect (Maskin and Tirole, 2001). As a result, players’s strategies will be a pair of platform functions of the form

\[
X_t^j : [\mu^l, \mu^h] \rightarrow [0, 1] \quad j \in \{R, L\}, \quad t \in \{1, 2\}
\]

which depend on the payoff-relevant state \( \mu_t = I(q_t) - \lambda_t x_{t-1} \). \( \mu_t \), naturally interpreted as the average voters’ perceived redistribution inefficiency. The range of feasible states is bounded by \( \mu^l = I(q^l), \mu^h = I(q^h) \).

3.1 Two-period model

In the two-periods version of the \((M)\)-game, the economy starts with an initial sectorial distribution, \( x_0 \), and the first elections take place at the end of \( t = 0 \). The following

\[
W_t = \begin{cases} 
R & \text{if } \hat{\pi}_t \geq 1/2 \\
L & \text{if } \hat{\pi}_t < 1/2 
\end{cases}
\]
proposition describes the unique equilibrium of the game.

**Proposition 1** In the unique equilibrium of the two-period \((M)\)-game

i) In \(t = 2\) both platforms converge to the efficient level: \(X^R_T = X^L_T = x^*\).

ii) In \(t = 1\) platforms are given by

\[
X^R_1 = x^* - \frac{\Delta(M)}{2} - \frac{\psi \Delta(M)}{1 + \psi \Delta^2(M)} \mu_1 \quad ; \quad X^L_1 = x^* + \frac{\Delta(M)}{2} - \frac{\psi \Delta(M)}{1 + \psi \Delta^2(M)} \mu_1
\]

and platform divergence, that is the difference between \(L\)'s and \(R\)'s platform (denoted by \(\Delta(M)\)), solves

\[
\Delta[1 + \psi \beta \lambda \Delta] - \beta \lambda = 0 \quad \text{(2)}
\]

iii) Implemented public good provision at \(t = 1\), is, in expectation, lower than the efficient level \((x^*)\).

In the last period, as in a standard static Downsian model, political competition drives both platforms to the efficient level. This is not true in the first period, when \(R\) announces an inefficiently low public good provision and \(L\) does the opposite. They both do that in order to secure themselves a better future electoral environment. As a result, at \(t = 1\) there is either underprovision of public good with no redistribution or overprovision of public good and fully egalitarian redistribution. Moreover, since \(R\) is more likely to win the elections, in expectation there will be underprovision of public good. Regardless of the specific welfare criterion adopted, political competition then delivers a second best outcome. The reason is that at \(t = 1\), political actors face a trade-off, which can analyzed by decomposing the FONC for the optimal choice of \(X^R_1\):

\[
\frac{dp_1}{dX^R_1} + \beta \frac{dp_1}{dX^R_1} E[p_2(\mu_2|X^R_1) - p_2(\mu_2|X^L_1)] + \beta p_1 \frac{dE[p_2(\mu_2|X^R_1)]}{dX^R_1} = 0
\]

- **Downsian** (centripetal)
- **Legacy** (centripetal)
- **Curleyan** (centrifugal)

On the one hand, setting \(X^R_1 = x^*\) maximizes the chances of winning the upcoming elections, which is valuable for two reasons. First, it allows \(R\) to implement the favorite redistribution level today (“Downsian” component). Second, it maximizes the impact of \(R\)'s platform on tomorrow’s electoral environment (“Legacy” component). On the other hand, marginally distorting \(X^R_1\) downward increases \(R\)'s electoral chances in period 2 (“Curleyan” component). \(L\) faces an analogous trade-off, with the only difference that he gains from distorting
his platform upward. The point at which this trade-off is balanced generates distortions at both platform and implemented policy level. Finally, the volatility of the aggregate shock ($\psi^{-1}$) only affects platform distortion through the Legacy component, reflected in the middle term of (2): larger volatility means weaker Legacy component. One might then conjecture that aggregate volatility increases platform divergence. As the next section shows, this is indeed the case.

**Comparative statics.** The following proposition summarizes how the size of the distortions changes with the main parameters of the model.

**Proposition 2**  

In the unique equilibrium of the two-period ($M$)-game  

i) Platform divergence is increasing in political actors’ discount factor ($\beta$), in the expected informational wedge ($\bar{\lambda}$), in inequality ($\omega$), and in the volatility of the aggregate shock ($\psi^{-1}$), and is independent of the initial state ($\mu_1$).

ii) $\Sigma_1^{(M)}$ is increasing in policy divergence, initial state, discount factor, and inequality; it is ambiguous in aggregate volatility and on the initial size of the public sector ($x_0$).

Political actors’ patience, then, increases dynamic distortions: as they care relatively more about the future, their incentive to manipulate $x$ to improve their future electoral appeal increases. Glaeser and Shleifer (2005) obtain a result that, although in a different environment, has a similar logic. A larger $\bar{\lambda}$ has the same effect as a larger $\beta$: increasing the expected informational wedge directly increases the marginal benefit from platform distortion.

The effect of a larger initial state (i.e., larger average perceived inefficiency) is to increase expected policy distortion: as $\mu_1$ gets larger $R$ becomes, ceteris paribus, more likely to win the elections. Therefore, the centrifugal component in his intertemporal trade-off gets larger, and pushes his platform towards a more severe underprovision of public good. $\mu_1$ is a linear combination (weighted by $x_0$) between two a priori unordered terms, $I(q_1)$ and $\tilde{I}(s_1)$. The effect of $x_0$, the initial size of the public sector, is then ambiguous: when the realization of the signal $s_1$ is large, public workers are more hostile to redistribution than private sector workers and, as a consequence, more likely to vote for $R$. For low and intermediate values of $s_t$, the opposite is true.

Part ii) of the Proposition also implies that, as $q_1$ increases, $R$’s platform becomes more extreme. This means that the electoral process has a “spiraling effect” on dynamic distortions: as the inefficiency associated with $L$’s redistribution scheme increases (that is, as $R$’s structural advantage increases), $R$’s public good underprovision becomes more severe and
his equilibrium probability of winning increases. Voters then become less likely to have an inefficient redistribution scheme implemented, but also more likely to suffer from a larger underprovision of public good.

This result distinguishes this paper from most of the recent literature on dynamic political economy: $q_1$ increases, ceteris paribus, $R$’s probability of winning. It can, therefore, be related to a measure of political persistence. This paper shows that, unlike in the previous dynamic political economy literature, more political persistence can increase, rather than decrease, the severity of political failures. This result is, to the best of our knowledge, novel. The Robustness Section shows how the same result can be derived using alternative concepts of political persistence.

The effect of inequality goes as expected: on the one hand, it increases $\lambda$, thereby increasing the expected informational wedge and the incentive to distort platforms. On the other hand, it also increases $\mu_1$, thereby increasing $R$’s ex ante strength and the severity of the underprovision of public good.

Finally, the effect of $\psi^{-1}$ on policy distortions is, in principle, ambiguous: as aggregate volatility increases, the connection between platforms and electoral result weakens, but so does the connection between current platforms and future electoral environment. Which effect dominates crucially depends on $\mu_1$, the perceived inefficiency of the average voter. When $\mu_1$ is large, the a priori initial advantage of $R$ is high enough, and he will be able to afford large platform inefficiencies and, yet, win with a substantial probability.

**Forward looking voters.** As already mentioned, an important assumption in the baseline model is that voters are myopic. One might then suspect that the interaction between far sighted politicians and fully myopic voters might play a key role in generating the distortions analyzed in this paper. Looking at a two period model with forward looking voters, it is possible to show that distortions not only persist, but are stronger than in the baseline specification. Voters at $t = 1$ now take into account the effect of implemented policies on their expected payoff at $t = 2$. Since in the second period platforms converge to $x^*$, the only relevant effect is on implemented redistribution, that is $b_2$. Since implemented redistribution depends on $p_2$, which, in turn, depends on $x_1$, forward looking voters have an additional term in their payoff, reflecting the effect of current policies on future redistribution. $R$’s realized vote share among private sector workers is then

$$1/2 + \varphi[d(x_1^R, x_1^L) + I(q_1) + \psi(x_1^L - x_1^R)\beta^\gamma\gamma_1 - \xi_1],$$
where $\beta''$ denotes voter’s discount factor and $\gamma_1 = E[I(q_2)\lambda_2|q_1] = E[p_2(x^R_1) - p_2(x^L_1)|q_1]$. R’s realized vote share among public sector workers is

$$1/2 + \varphi[d(x^R_1, x^L_1) + \tilde{I}(s_t) + \psi(x^L_1 - x^R_1)\beta''\tilde{\gamma}_1 - \xi_1],$$

where $\tilde{\gamma}_1 = E[I(q_2)\lambda_2|s_1]$. Although the specific values of $\gamma_1$ and $\tilde{\gamma}_1$ depend on the time correlation structure of $q_t$ (which in the model is left unspecified), it possible to show that they are both (weakly) positive.\(^{27}\) Solving the model yields the following equilibrium policy functions:

$$
\begin{align*}
X^R_1 &= x^* - \frac{\Delta(M)}{2} - \frac{\psi\Delta(M)}{1 + \psi\Delta^2(M)}\mu - \beta''[\gamma_1(1 - x_0) + \tilde{\gamma}_1x_0] \\
X^L_1 &= x^* + \frac{\Delta(M)}{2} - \frac{\psi\Delta(M)}{1 + \psi\Delta^2(M)}\mu - \beta''[\gamma_1(1 - x_0) + \tilde{\gamma}_1x_0]
\end{align*}
$$

Having more forward looking voters creates an additional channel through which initial sectorial allocation affects policies: since voters care also about the second period’s inefficiency, the effect of $q_1$ becomes more relevant. This increases the *ex ante* advantage of $R$ and, as a consequence, has the same effect of an increase in political persistence: by pushing both platforms downwards, having more forward looking voters increases the inefficiencies at the implemented policy level.\(^{28}\)

### 3.2 Infinite horizon model

The analysis of infinite horizon version of the (M) game confirms some of the insights of the two-period model, but also uncovers other important aspects. It also highlights an important virtue of the model: the ability to explicitly solve for the stationary Markov perfect equilibrium, which allows us to obtain a straightforward comparison between two-period and infinite horizon models.

Given its recursive structure, we denote by $X^R(\mu)$, $X^L(\mu)$ the equilibrium platform given an initial state $\mu = I(q) - \lambda x$

**Proposition 3** *In the unique stationary differentiable MPE of the infinite horizon (M)-game*

i) **Platform divergence**, $\Delta(M)_{-\infty}$, equals $\beta\bar{\lambda}$, and platforms are given by

$$
\begin{align*}
X^R(\mu) &= x^* - \frac{\Delta(M)_{-\infty}}{2} - \frac{\psi\Delta(M)_{-\infty}}{1 + \psi\Delta^2(M)_{-\infty}}\mu \\
X^L(\mu) &= x^* + \frac{\Delta(M)_{-\infty}}{2} - \frac{\psi\Delta(M)_{-\infty}}{1 + \psi\Delta^2(M)_{-\infty}}\mu
\end{align*}
$$

\(^{27}\)More specifically, in the simple time iid case, $\gamma_1 = \tilde{\gamma}_1$.

\(^{28}\)Simple inspection of equilibrium platforms also shows that having more far sighted voters does not affect political polarization.
ii) Platform divergence is increasing in $\mu$, $\beta$, $\bar{\lambda}$, $\omega$, and independent of $\psi^{-1}$.

iii) Platform divergence and policy distortion are larger than in the two-period model.

Most of ii) and iii) are quite intuitive: as the time horizon increases to infinity, the marginal value of platform distortion increases. As a consequence, the size of the distortions on both platforms and implemented policies increase. A less obvious (which will turn out to be very important) fact is that policy divergence is no longer dependent on the variance of the aggregate shock. To develop an intuition for why that is the case, the reader must notice that in the infinite horizon equilibrium platform divergence is constant over time. As argued before, aggregate volatility only affects platform divergence through the Legacy component, which represents the incentive of each actor to maximize the influence of his own platform on the second period electoral environment. Since in the second and last period there is no incentive to maximize the influence of his own platform on future electoral environment, the Legacy component enters only one side of the political actors’ dynamic trade-off. More generally, in every finite horizon model, the Legacy component will change over time, generating a change in the incentives for political divergence across periods. In a stationary equilibrium, instead, the Legacy component must be constant over time. In the baseline setting of model, where payoffs are quadratic, the Legacy component must then disappear from the dynamic trade-off. This component is captured by the term $\psi \beta \bar{\lambda} \Delta^2$ in (2); removing it yields precisely $\Delta = \beta \bar{\lambda}$.

**Keeping the intertemporal trade-off constant.** To properly compare the distortions in the two-period model and in the infinite horizon model, one should control for the strength of the agents’ intertemporal trade-off, rather than keeping constant the discount factor. To achieve that goal, we consider a pair $(\beta, \bar{\beta})$ such that $\bar{\beta} = \sum_{t=1}^{\infty} \beta^t$ and compare a two-period model where actors have discount factor $\bar{\beta}$ to an infinite horizon model where actors have discount factor $\beta$. The following proposition describes the comparison:

**Proposition 4** If one keeps the intertemporal trade-off constant, platform divergence and expected policy distortion are lower in the infinite horizon model.

When the intertemporal trade-off is the same across the two models, the only difference between two-period and infinite horizon is that, in the latter, $R$’s and $L$’s future platforms will diverge. The location of these platforms with respect to $x^*$ depends on the future electoral environment: the more the latter is favorable to $R$, the more inefficient his platform, and the closer $L$’s one is to $x^*$. As a result, there is a compensating effect of future diverging
platforms that mitigates the impact of future electoral environment on electoral outcome, thereby decreasing the marginal gain from platform distortion in the infinite horizon model.

4 Consensual constitution

The idea of consensual (also known as consociational) democracy, introduced by Lijphart, is based on the observation that in several countries (especially in northern and central Europe) constitutional rules, rather than assigning policy making power to a majority winner like in Anglo-Saxon countries, prescribe a division of political power between different groups within a society, with a weight that depends on their electoral strength. In his 1977 book, *Democracy in plural societies* Lijphart identifies the main features of this type of democracy:

Consociational Democracy can be defined in terms of four characteristics. The first and most important element is government by a grand coalition of the political leaders of all significant segments of the plural society. (...) The other three basic elements are (1) the mutual veto (...) (2) proportionality (...), and (3) a high degree of autonomy for each segment.

In an effort to adhere as much as possible to this definition (and, at the same time, maintaining comparability with the majoritarian setting), we model consensual democracy as a stylized post-electoral bargaining game between $R$ and $L$. More specifically, the two actors negotiate over the implemented $x$ and $b$ (where $b$ can be anything and $x$ should be between the two announced platforms) with bargaining power proportional to their vote share. The default option is to bargain separately over each dimension, which implies that the following policies would arise

$$X_t^{(C)} = \tilde{\pi}_t x_t^R + (1 - \tilde{\pi}_t) x_t^L; \quad b_t^{(C)} = (1 - \tilde{\pi}_t) \bar{b}_t.$$

If, on the other hand, there exists a set of Pareto improving pairs $(x_{pr}^R, b_{pr})$ that would allow a randomly determined proposer to strictly increase his expected payoff with respect to the default option, he will choose his preferred pair within that set and the other will accept it.

This assumption captures the idea that, lacking a different agreement between the two political actors, the constitution prescribes that each political actor will have an influence

---

29 The comparison between this type of “Westminster democracy” and consensual democracies is the main theme of Lijphart’s classic book, *Patterns of Democracy.*
on each policy dimension that is proportional to his electoral strength.\footnote{In Western democracies, it is possible to find several formal and informal mechanisms explicitly tying the number and type of cabinet positions to a party’s vote share. For example, the so called Cencelli manual, used to distribute cabinet positions in pre-1994 Italy.}

**Lemma 2** Under the assumptions, in a consensual constitution bargaining separately over each dimension has no Pareto improvement.

As a consequence, under (C), R’s per-period realized payoff is \( \hat{\pi}_t \) and L’s payoff is \( (1-\hat{\pi}_t) \). The expected payoffs are then, respectively, \( \pi_t \) and \( (1-\pi_t) \), where

\[
\pi_t = E_t[\hat{\pi}_t] = 1/2 + \varphi [d(x^R_t, x^L_t) + I(q_t) - \lambda_t x_{t-1}]
\]

is the expected vote share.

### 4.1 Two-period model

This section describes the unique equilibrium of the two-period version of the (C)-game and compares it with the results from the two-period version of the (M)-game.

**Proposition 5** In the unique equilibrium of the two-period (C)-game

i) In \( t=2 \) both platforms converge to the efficient level: \( X^R_2 = X^L_2 = x^* \).

ii) In \( t=1 \) platform divergence, \( \Delta(\text{C}) \), solves

\[
\Delta[1 + \varphi \beta \lambda \Delta] - \beta \lambda = 0 \tag{3}
\]

and platforms are given by

\[
X^R_1 = x^* - \frac{\Delta(\text{C})}{2} - \frac{\varphi \Delta^2(\text{C})}{1 + \varphi \Delta^2(\text{C})} \mu_1; \quad X^L_1 = x^* + \frac{\Delta(\text{C})}{2} - \frac{\varphi \Delta^2(\text{C})}{1 + \varphi \Delta^2(\text{C})} \mu_1
\]

iii) The implemented policy at \( t=1 \), \( X^{(\text{C})}_1 \), is a uniform centered in \( x^{(\text{C})}_1 \) = \( x^* - \frac{2\varphi \Delta(\text{C})}{1+\varphi \Delta^2(\text{C})} \mu_1 \), whose variance increases in \( \psi^{-1} \).

iv) \( \Delta(\text{C}) \) is increasing in discount factor, expected informational wedge, wealth inequality, and in the variance of the idiosyncratic shock \( (\varphi^{-1}) \).

v) \( \Sigma^{(\text{C})}_1 \) is increasing in policy divergence, initial state, discount factor, aggregate volatility, and wealth inequality; it is ambiguous in \( (\varphi^{-1}) \) and on the initial size of the public sector \( (x_0) \).
The key difference between consensual and majoritarian case is that, rather than only on aggregate volatility ($\psi^{-1}$), platforms depend on idiosyncratic volatility ($\varphi^{-1}$); while political actors in a majoritarian constitution only care about going above 50% of the votes, in a consensual democracy every vote has the same marginal effect on the future implemented policy, since the actors’ objective function depends on their expected vote share. The second key difference is that aggregate volatility affects the distribution of implemented policies (more specifically, its variance) under (C). The rest of the comparative statics has similar intuition to the (M) case.

4.2 Infinite horizon model

In this subsection we consider the infinite horizon version of the (C)-game and compare it to its two-period version. As for (M), let $X^R(\mu), X^L(\mu)$ denote the equilibrium platform for a given state $\mu$.

**Proposition 6** In the unique stationary differentiable MPE of the infinite horizon (C)-game

i) Platform divergence, $\Delta(C)_{-\infty}$, equals $\Delta(M)_{-\infty} = \beta \lambda$ and platforms are given by

$$X^R(\mu) = x^* - \frac{\Delta(C)_{-\infty}}{2} - \frac{\varphi \Delta(C)_{-\infty}}{1 + \varphi \Delta^2(C)_{-\infty}} \mu$$

$$X^L(\mu) = x^* + \frac{\Delta(C)_{-\infty}}{2} - \frac{\varphi \Delta(C)_{-\infty}}{1 + \varphi \Delta^2(C)_{-\infty}} \mu$$

ii) Platform divergence is increasing in $\beta, \lambda, \mu$, independent of $\varphi^{-1}$ and larger than in the two-period (C) game; policy distortion is also larger than in the two-period (C) game.

iii) If one keeps the intertemporal trade-off constant, platform divergence and expected policy distortion are larger in the two-period model.

The key difference between two-period and infinite horizon is then that platform divergence no longer depends on the idiosyncratic volatility. The intuition is similar to the majoritarian case: in a stationary equilibrium, platform divergence is constant over time. As a consequence, the current and future Legacy components in the actors’ dynamic trade-off offset each other.

5 Constitutional comparison

The following proposition compares the two-period equilibria of the two constitutions.
Proposition 7 In the two-period model, a consensual constitution is associated with
i) Larger platform divergence
ii) Smaller expected policy distortion
iii) Larger expected redistribution
Moreover, larger $\beta$ or $\bar{\lambda}$ increase constitutional differences at both the platform and the implemented policy level.

A consensual constitution is then associated with more political polarization at the platform level and, in expectation, with more redistribution and less underprovision of public goods. These results echo several theoretical and empirical findings on parliamentary form of government (Gerber and Ortúñor Ortín, 1998; Persson, Roland and Tabellini, 2000; Persson and Tabellini, 2003 and 2004) and proportional electoral rules (Austen-Smith and Banks, 1988; Milesi-Ferretti, Perotti and Rostagno, 2002; Persson and Tabellini, 2003 and 2004), two constitutional features that Lijphart explicitly associates with consensual democracy. The key difference with the previous literature is that, in this paper, the predictions are derived in a dynamic general equilibrium environment where voters are a priori identical. Moreover, the modeling of institutional details is as simple as possible and, more important, the source of political failure is the dynamic informational consequence of public employment.

The proposition also shows how that, as the expected reward from distorting the platform increases, constitutional differences become more pronounced: when political actors become more patient, or the expected informational wedge increases, platform divergence in $(M)$, which is smaller than in $(C)$, increases more slowly, and the expected inefficiency of implemented public good provision in $(M)$, which is larger than in $(C)$, increases faster.

The following corollary fully illustrates the comparison of the infinite horizon equilibria.

Corollary 1 In the infinite horizon model
i) Platform divergence is the same across constitutions.
ii) Expected policy distortion is larger under $(M)$.
iii) Expected redistribution is larger under $(C)$.

This corollary leads to the surprising conclusion that political polarization is, in the long run, independent on the constitution. Mathematically, the reason is that, when the horizon is infinite, the volatility of the relevant shocks (aggregate shock for $(M)$ and idiosyncratic shock for $(C)$) no longer affects the trade-off between current electoral strength and future electoral strength. On a more substantive level, the fact that there exists a substitution effect between initial electoral environment and future platform divergence plays an important role.
Consider a two period model: as one increases the time horizon, the compensating effect must be stronger under a consensual constitution, which exhibits a higher platform divergence in period 1. As a consequence, polarization in a majoritarian democracy must be more reactive to increases in the time horizon. In the baseline model this higher responsiveness exactly compensates, as the horizon goes to infinity, the initial larger polarization of consensual democracy. This is due to the simple structure of the baseline model. But it should be clear that the general intuition behind this long term neutrality result is robust to more general payoff structures.\footnote{A more thorough discussion on this point can be found in the Robustness Section.}

The second part of the corollary follows from combining its first part with the previous proposition: policy distortions in the two-period model are lower under (C), despite a larger platform divergence. As a consequence, this must also hold in the infinite horizon model, where platform divergence is the same. The third part echoes the findings in Ticchi and Vindigni (2010) and Iversen and Soskice (2006), who find that consensual constitutions and proportional systems tend to give left-wing parties more power.

5.1 Semi-consensual constitution

In order better to understand the observed differences between the two constitutions, in this section we consider a hybrid type, called semi-consensual (denoted by (S)), in which the allocation of policy-making power over redistribution is majoritarian\footnote{Formally, \( b_{t}^{(S)} = \tilde{b}_t I_{\{L=W\}} \).} and the one over public good provision is consensual.\footnote{Formally, \( X_{t}^{(S)} = \hat{\sigma}_t x_t^R + (1 - \hat{\sigma}_t) x_t^L \).} This exercise is valuable because (C) and (M) differ in the allocation of power over two separated dimensions: public good provision and redistribution. One’s first conjecture might be that, since actors only care about redistribution, it is the difference in the allocation of power over this dimension that drives the difference in outcomes. Therefore, the two-period equilibrium of the (S) game should look quite similar to the one of the two period (M)-game. The following proposition shows that the opposite is true.

**Proposition 8** In \( t = 1 \) the equilibrium platforms in (S) are the same as in the consensual constitution.

This proposition suggests that the equilibrium is mostly driven by the way the constitution allocates policy-making power over public good provision, over which political actors have no preferences but can credibly commit. As a consequence, a constitution determines how distortions are transmitted across policy dimensions. It must be stressed that is not...
a consequence of the lack of commitment assumption on redistribution. As the Robustness Section suggests, removing it yields a very similar outcome.

The proposition also implies that policy distortions in (S) are as large as in (C) and smaller than in (M). Furthermore, implemented redistribution under (S) is majoritarian, which suggests that semi-consensual has lower average redistribution than (C). As a consequence, one would conjecture that, from a utilitarian welfare perspective, (S) dominate both (C) and (M). The following proposition shows that this is true, and that something can be said even when one adopts a non-utilitarian welfare criterion, in which the relative weight of the rich and poor are, respectively, $\alpha$ and $1 - \alpha$, with $\alpha \in (0, 1)$.

**Proposition 9**

i) Under a utilitarian welfare criterion, a semi-consensual constitution dominates both consensual and majoritarian.

ii) Under any generic welfare criterion, a majoritarian constitution is always dominated.

The normative implication is that a majoritarian allocation of power on policy dimensions over which political actors have only induced preferences, coupled with a consensual allocation of power on policy dimensions over which political actors intrinsically care about minimizes the overall distortions associated with policy-making. The second part of the proposition essentially follows from the fact that, under a more generic welfare criterion, redistribution might be socially desirable, provided that poor citizens have enough weight. Under that circumstance, (C) is unambiguously better than (M). Finally, these results do not depend on the time horizon: both propositions can also be obtained for the infinite horizon case.

### 6 Empirical implications

This section discussed three empirical predictions that the model delivers about the relationship between *ex ante* inequality, political polarization, and redistribution. These topics have received a lot of attention from researchers across various disciplines in the social sciences. More important, they are also at the center of a debate that goes well beyond academia. It is then important to illustrate how this work, which is theoretical in nature, can also contribute to that debate.

The first two results come from the comparative statics of the model. The first is that, by increasing the inefficiency associated with $L$’s proposed redistribution, inequality increases

34It is also possible to show that the opposite configuration (majoritarianism on public good provision and consociativism over redistribution) would be, under a utilitarian criterion, dominated by both (M) and (C)
polarization under both constitutions. As a consequence, the model suggests a positive relationship between inequality and political polarization, as widely documented by the work of McCarty, Poole, and Rosenthal (McCarty et al., 2006), among others. The suggested channel for that relationship is that, as inequality rises, so does the perceived inefficiency of redistribution, which, in turn, magnifies the effect of asymmetric information. Political actors have, then, a stronger incentive to distort their platforms. The second result, which follows from the first, is that higher inequality is associated with lower public good provision (and higher inefficiency public good provision). More specifically, more inequality is associated with more severe underprovision of public goods and more volatility with the electoral cycle.

To develop the third implication, we make use of the following lemma.

**Lemma 3** R’s equilibrium probability of winning and expected vote share are increasing in $\omega$ in both the two-period model and the infinite horizon model. Moreover, under a majoritarian constitution, when the cost of redistribution is high enough, higher inequality decreases expected redistribution; under a consensual constitution, redistribution and inequality are directly related.

The intuition for the first part of the lemma is the following: as inequality increases, so does the inefficiency of L’s preferred redistribution (the reason is that payoffs are linear). As a consequence, conditional on the left winning, redistribution increases. At the same time, the Left, despite choosing a less distorted platform on public employment, becomes electorally weaker, which decreases expected redistribution. Depending on the constitution and on the value of $q$, the effect of higher inequality on (period 1) implemented redistribution is ambiguous: when the effect of more efficient platform does not prevent Left’s winning probability to fall too much, expected redistribution decreases. In the (M) case, when the cost of redistribution is high and initial size of the private sector is large enough, the effect of reduced electoral strength more than offsets the effect of L’s increase in his preferred redistribution.

The existence of an inverse relationship between inequality and redistribution has been empirically documented, but it is still a fairly open question theoretically. (See, for example, De Mello and Tiongson, 2003, for cross country evidence, Campante, 2011, Moene and Wallerstein, 2001, or Bénabou, 2000, for some proposed theoretical explanations.) Furthermore, the work by Enns and Kelly (2010) has recently documented an inverse relationship between inequality and political support to redistribution among all income levels, which sharply contrasts with the prediction from any standard Meltzer-Richard models of public finance, as well as most of the more recent proposed theoretical explanations. This pa-
per suggests a potential channel through which interpret this recent empirical evidence: as inequality increases, the inefficiencies associated with redistribution also increase, thereby leading to a lower demand for redistribution across income groups.

In this paper, the above described effect can only happen in a majoritarian democracy, thereby yielding an interesting prediction which, to our knowledge, has not been tested yet. It must be also stressed that this result does not directly depend on the averaging effect of policy-making under (C), but rather on how political competition and the constitutional type influence the fundamental trade-off of the model (current vs future electoral strength).

7 Robustness

This section illustrates how the results of this paper are robust to alternative specifications of various components of the model. Since the setting is meant to capture, albeit in a stylized way, several aspects of an economic and a political system, it is quite natural to wonder how many of the modeling choices play a crucial role in the derivation of the results. This section addresses some of the main potential sources of concern.

Term limits, general heterogeneity in the actors’ intertemporal trade-off. The outcome under (C) critically depends on the fact that the agents cannot find any Pareto improvement from bargaining jointly on both dimensions. The reason is that, in the baseline setting, the agents have the same intertemporal trade-off. Without this symmetry, the comparison between (M) and (C) can change quite dramatically. There are at least three different perturbations of the model that generate a failure in such symmetry. First, the presence of term limits might create asymmetries in political actors’ time horizon. Second, political actors might have different discount factors: candidates typically differ in age and, more broadly, in the expected length of their political career. Analogously, parties are also typically ruled by different waves of top executives, who often belong to different generations. Third, and more disturbing, if one removes the normalization in the actors’ payoffs within the unit interval, the two actors’ intertemporal trade-offs are, in general, asymmetric. Assume that $L$’s payoff is simply linear in the value of the implemented redistribution, and ranges between 0 and $\bar{b}_t$, while $R$’s payoff ranges between $\omega/2$ and $[\omega - q_t\bar{b}_t]$. It is easy to see that the ratios between current and future payoffs are, respectively,

$$\frac{2 + q_t}{2 + E[q_{t+1}]} : \frac{E[q_{t+1}](2 + q_t)}{(2 + E[q_{t+1}])q_t}$$

Depending on whether $q_t$ is above or below its expected value, $L$ cares relatively more or
relatively less than $R$ about the future. In this setting, under a consensual constitution the two actors can improve their expected utility by bargaining on both dimensions jointly, and splitting power across them asymmetrically. For example, when future payoffs are relatively larger for $L$, his induced preferences are such that he cares relatively more about public good provision. He finds then profitable to trade policy-making power over redistribution for policy-making power over public good provision, and $R$ finds it profitable to do the opposite. As a consequence, the allocation of policy-making power differs from the one that arises in the baseline case.

How does this asymmetry affect the incentives to distort platforms? It is possible to show that the *ex ante* incentive for policy distortions increase. The reason is that the connection between the vote shares and each actor’s payoff is weaker than in the symmetric case: in the latter, vote share is essentially the actors’ payoff, in the former it only affects their reversion points. As a consequence, consensual democracy not only allocates power over redistribution overwhelmingly towards the “short sighted” actor and over public good provision towards the “far sighted” actor, but displays an even larger platform divergence than in the symmetric case.

The effect of such asymmetry is very different under a majoritarian constitution, because actors cannot trade power through bargaining. The only effect of the asymmetry is given by a differential incentive to distort policies (larger for the far sighted side, lower for the short sighted side). Given the strict concavity of the actor’s payoffs over $x$, the asymmetry actually reduces equilibrium platform divergence, thereby improving the outcome under (M). A qualitatively similar effect is observed in the (S) constitution, where bargaining over both dimensions is not possible. As a consequence, when the agents’ intertemporal trade-off is not symmetric the (M) constitution can dominate the (C) constitution, but not the (S) constitution.

In conclusion, consensual constitutions are fragile to heterogeneity in the intertemporal trade-off among actors, while majoritarian and semi-consensual are not only robust to it, but do benefit from such asymmetries. This fact also helps explaining why term limits tend to be more often observed and less controversial in majoritarian systems: they not only reduce the incentive for a successful incumbent to distort policies, but also reduce platform divergence.

**Alternative channels linking current policies to future political preferences.** The mechanism of the model is crucially based on two features: the presence of an informational link between the two policy dimensions and the fact that political actors are associated with different levels of redistribution. This is not the only possible source of dynamic political
distortions. For example, working in the private sector might directly affect the distribution of workers’ skills in the society, increasing inequality and, therefore, the social cost of redistribution. Below, we briefly discuss two other potential channels that would generate an endogenous link between implemented policies and future electoral environment.

**Ideology (rather than asymmetric information).** Suppose that there are two preference types: most voters are like in the baseline model (rational type, $r$), but a measure $\varepsilon_t$ of them derives a disutility term from inequality (egalitarians, type $e$). Further assume that the public sector has a larger share of egalitarian types (that is $\varepsilon^G_t > \varepsilon^P_t$), and one’s type can switch after interacting with a new working environment. More specifically, assume that a worker who moves from one sector to the other takes the type of the first co-worker she interacts with, which is randomly drawn. The law of motion of the share of types $e$ in this simple environment is then given by

$$\varepsilon_{t+1} = (x_{t+1} - x_t)(\varepsilon^G_t - \varepsilon^P_t) + \varepsilon_t$$

The effect of expending the public sector is then to increase the chance of ideological switches from $r$ to $e$ and vice versa. As a consequence, expanding the public sector today produces an electorate that is more favorable (or less hostile) to redistribution, thereby increasing $L$’s appeal.

**Specialized candidates (rather than policy motivated).** Recent theoretical work in political economy has analyzed electoral competition model in which candidates (Krasa and Polborn, 2009, 2010) differ in their ability to provide policies across different dimensions, or in some fixed characteristic that affect the payoff voters obtain from policies. Suppose that political actors have full commitment and are purely office motivated, and that the marginal social value of redistribution is positive, provided $b$ is close enough to zero. Further assume that $R$ and $L$ have different abilities in implementing redistribution, namely that $L$ has, for various reasons, more expertise in implementing redistributive programs. As a consequence, $L$’s electoral strength increases when the demand for redistribution increases. The latter is related to the perceived inefficiency of redistribution, which depends on public employment. As a consequence, $L$ and $R$ will have the same ex ante incentive to manipulate public good provision as in the baseline model.

**Alternative specification of Consensual Democracy.** Some authors (for example, Battaglini, 2010, or Herrera and Morelli, 2010) have modeled proportional electoral systems (which are a prominent feature of consensual constitutions) as games in which a party gets to implement his platforms with a probability proportional to their realized vote share.
Although the setting presented in the baseline better captures Lijphart’s concept, it is quite natural to ask to what extent the results derived in this paper are sensitive to the modeling assumptions of (C).

To partially address this question, let’s consider an alternative version of consensual democracy, denoted by (C2) where, as in Battaglini (2010), a political actor gets full policy-making rights with probability \( \hat{\pi}_t \). It is easy to show that equilibrium strategies are exactly as in the standard model, and the implemented policy is a lottery with probabilities \( \hat{\pi}_t \) and \( 1 - \hat{\pi}_t \).

Since in a two-period model platform divergence is larger under (C2) than under (M), one would expect that policy distortion should not necessarily be larger under the latter, as in the baseline model. It is possible to show that, when the initial state \( \mu_1 \) is large enough, platform distortion is larger under (M), while the opposite it true for low values of \( \mu_1 \). The intuition is that under (C2) platform divergence is larger, but the probability of having the most extreme platform (that is, \( R \)’s) implemented is lower. On the other hand, in the infinite horizon policy distortion is larger under a majoritarian constitution, since platform divergence is the same under each constitution but the average implemented policy is closer to \( x^* \) under (C2).

In conclusion, although the constitutional comparison in the two-period model is affected by changes to the modeling assumptions of a consensual constitution, the basic message of the paper remains.

**Full commitment on redistribution.** Assuming that political actors can fully commit to a certain redistribution level does not eliminate dynamic distortions. To see that, consider a two-period majoritarian democracy. Denote by \( b^R_t \) and \( b^L_t \) the redistribution levels chosen by the two actors. In the second period, \( R \)’s and \( L \)’s platforms solve, respectively

\[
\max_{x^R,b^R} \left\{ p_2 \frac{\bar{b}_2 - b^R}{b_2} + (1 - p_2) \frac{\bar{b}_2 - b^L}{b_2} \right\}
\]

\[
\max_{x^L,b^L} \left\{ p_2 \left[ I_{\{b^R \leq b_2\}} \frac{b^R}{b_2} + I_{\{b^R > b_2\}} \frac{\bar{b}_2 - b^R}{b_2} \right] + (1 - p_2) \left[ I_{\{b^L \leq b_2\}} \frac{b^L}{b_2} + I_{\{b^L > b_2\}} \frac{\bar{b}_2 - b^L}{b_2} \right] \right\}
\]

where \( p_2 = 1/2 + \psi d(x^R, x^L) + \psi \bar{q}_2 (b^L - b^R) \) and \( \bar{q}_2 = q_2 (1 - x_1) + E(q_2|s_2)x_1 \). The dependence on \( \bar{b}_2 \) comes from the normalization of the payoffs within the unit interval. \( b^R_2 \), \( x^R_2 \) and \( x^L_2 \) are, like in the baseline model.\(^{35}\) The optimal choice of redistribution for \( L \) is

\(^{35}\)That is, respectively 0, \( x^* \), and \( x^* \).
given by

\[
b^L_2 = \begin{cases} 
\bar{b}_2 & \text{if } \frac{1}{2} - \psi \bar{b}_2 \bar{q}_2 > 0 \\
\frac{1}{2\psi \bar{q}_2} & \text{otherwise}
\end{cases}
\]

Under the assumptions, the corner solution is the only feasible one: (4) implies \( \psi^{-1} > 2[\mu^h + d^h] \), which is sufficient to guarantee the choice of \( \bar{b}_2 \). As a consequence, the informational wedge enters the actors’ problem in period one exactly as in the baseline mode, thereby yielding the same qualitative insights. In period 1, the choice of \( b^L \) can be interior or, as in the baseline model, equal to \( \bar{b}_1 \).[36] It is important to stress that, in the former case, the incentives to manipulate public employment are even larger. To see the intuition for this result, consider the extreme case in which, \( b^L_1 = 0 \). In this case, regardless of \( x^R_1 \) and \( x^L_1 \), in period 1 \( R \) will capture the whole surplus, and platforms for \( x \) will only be relevant for their effect on period 2’s electoral environment. This will result in a larger incentive to distort public employment with respect to the baseline case. To summarize, assuming full commitment on redistribution does not alter the basic tradeoff of the model and, under certain conditions, yields exactly the same solution as the two-period model with no commitment. When the two solutions are different, polarization and distortions are larger than in the baseline model.

**Alternative measures of political persistence.** One of the most important results that distinguish this paper from the existing literature in dynamic political economy is the fact that an increase in political persistence worsens, rather than improving, the extent of political distortions. In the paper, an increase in the inefficiency of redistribution is interpreted as an increase in political persistence. A more standard measure of political persistence, such as a super majority requirement for \( L \) to win the elections, would yield the same result. More formally, if one increases the winning threshold of \( L \) from 1/2 to \( q > 1/2 \), it is possible to show that \( R \)’s equilibrium platform becomes more extreme and his chances to win the elections increase. As a consequence, a more severe underprovision of the public good is observed in equilibrium.

Properly assessing the welfare consequences can only be done numerically.

**More general production technology.** In public finance models, a more general

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[36] The choice will be interior when the following condition holds

\[
\frac{1}{4\psi} > \mu_1 \frac{1 + \beta \Delta \psi}{1 - \beta \lambda \Delta \psi}
\]

that is, when the initial environment is favorable enough to the left.
and standard\textsuperscript{37} formulation of voters’ payoff involves a strictly concavity in the public good, which can also capture in a more flexible way the inefficiencies associated with redistribution. If one specifies the $x$-related component of voters’ payoff as $A(1 - x_t) + H(x_t)$ (with $H(.)$ satisfying the Inada conditions) rather than $A + (1 - A)x_t - \frac{x_t^2}{2}$, it is possible to show that most of qualitative results of the paper for two period models still hold, although an explicit solution is no longer available and the existence of a stationary Markov perfect equilibrium is harder to prove (although uniqueness can still be guaranteed). Numerical simulations, using a power function of the form $H(x) = x^\alpha$, with $0 < \alpha < A$, suggest that the equilibrium still exists, and that most of the results derived in the paper hold. The long run neutrality of the constitution on platform divergence holds in a weaker sense: rather than eliminated, the effect of the constitution becomes less pronounced. In other words, as one moves from two-period to infinite horizon, platform divergence becomes more similar across constitutions, but (C) remains associated with higher divergence. The reason is that the quadratic structure of the baseline model eliminates second order effects that are present in a more general setting.

**Microfounding the inefficiency of labor tax and redistribution.** The baseline model contains two important reduced form assumptions about the inefficiency of redistribution and labor tax: the fact that the former is concave in $q_t$ and the latter is strictly concave and quadratic in $x$ play a crucial role, respectively, in generating the dynamic incentive to distort platforms and ensuring the analytical tractability of the model. One might then wonder whether the main structure of the model can be obtained by making explicit assumptions about the source of these two inefficiencies. To obtain a strictly concave and quadratic $x$-related payoff, one could explicitly include an elastic labor supply, using preferences of the form

$$c_t + q_t - \frac{l^2}{2}$$

where $l$ is the number of hours worked by each worker in both sectors, (recall that equilibrium workers get the same wage in each sector). Each worker’ consumption equals the net wage, that is $Al(1 - \tau)$. The workers’ optimal labor supply is given by $l^\ast(\tau) = A(1 - \tau)$ and the budget constraint for the public sector is as in the baseline model. Re-expressing everything in terms of $x$ yields the following indirect utility

$$A^2(1 - x)^2 + xA(1 - x) - \frac{A^2(1 - x)^2}{2}$$

which is strictly concave and quadratic in $x$. The only difference with respect to the baseline

\textsuperscript{37}For example, in Persson and Tabellini, (2002).
model is that efficient level \( x^* \) is given by \((1 - A)/(2 - A)\).

To generate a microfounded \( I(q_t) \), consider a setting in which only a share \( \omega \) of citizens own the private sector, and that the additional income generated depends linearly on entrepreneurial effort \( e \), which costs \((1 + q)^{-1}e^{q+1}\) and yields \( e \) units of additional income. The optimal effort is given by \( e^* = (1 - \theta)^{1/\tau} \), and the rich’s payoff is \( \omega e^* \theta = (1 - \omega)b \). R’s implemented redistribution yields an expected payoff of \( \omega \frac{q}{1+q} \). After a few steps of algebra, it is also possible to verify that L’s implemented redistribution \( \bar{b} \), yields an expected payoff of \( \frac{q}{1+q} \left( \frac{\gamma}{1+\gamma} \right) \), where \( \gamma = \frac{1+q}{q} \frac{\omega}{1-\omega} \). As a consequence, one obtains

\[
I(q) = \frac{q}{1+q} \left[ \omega - \left( \frac{1+q}{q} \frac{\omega}{1-\omega} \right) \left( \frac{1+q}{q} \right) \right]
\]

It is possible to show that this function is strictly concave in \( q \).

**Unified government budget.** In the baseline model, redistribution and public good provision have two separate budgets. This assumption allows us to keep the two policy dimensions (\( x \) and \( b \)) separated in a very strong sense, but it is not crucial for the results derived in the paper. To see this, consider a model with a unique tax rate \( \tau \) for both labor and non labor income. The government budget constraint is then \( \tau[A(1 - x) + \omega/2] = A(1 - \tau)x + b(1 + q)/2 \). The uniqueness of an optimal level of \( x \) is ensured by a strictly convex technology \( g_t = H(x_t) \) for the public good. After substituting the tax rate as a function of \( b \) and \( x \), the inefficiency of L’s redistribution becomes

\[
I(q_t, X_t^L) = q_t \frac{\omega A(1 - X_t^L) + \omega^2/2}{2A + \omega(1 + q_t)}
\]

the fact that the inefficiency also depends on L’s platform makes the analysis more complicated, but it does not eliminate the presence of an incentive to manipulate public employment: being \( I \) strictly concave in \( q_t \), a public employee is \textit{ex ante} less hostile to redistribution, like in the baseline model. The difference is that now, since taxation is also redistributive, there is a substitution effect between \( b \) and \( x \), which adds a static trade-off to L’s platform choice, but does not affect the presence of the dynamic trade-off.

**Quantifying the impact of asymmetric information on distortions.** The model analyzed in this paper is not designed to quantitatively match empirical phenomena. Nevertheless, the reader might wonder how large are the implied inefficiencies. To answer this question, one needs to quantify the informational wedge \( \overline{\lambda} \) (which constitutes an upper bound
for political polarization). Since the latter does not have an analytical expression (not even for a simple uniform case), we rely of numerical simulations to provide a rough idea that they are sizeable. The table below lists the values of the ratio $\bar{\lambda}/\omega$ as a function of its key determinants and assuming that the relevant distributions, $F_q$, $\hat{F}_q$ and $F_\varepsilon$ are truncated normals.

<table>
<thead>
<tr>
<th>$[q^l, q^h]$</th>
<th>$\hat{\sigma}_q = 0.01$</th>
<th>$\hat{\sigma}_q = 0.1$</th>
<th>$\hat{\sigma}_q = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0, 1]</td>
<td>0.0089</td>
<td>0.0028</td>
<td>0.0003</td>
</tr>
<tr>
<td>[0, 2]</td>
<td>0.0205</td>
<td>0.0147</td>
<td>0.0029</td>
</tr>
<tr>
<td>[0, 3]</td>
<td>0.0209</td>
<td>0.0211</td>
<td>0.0070</td>
</tr>
</tbody>
</table>

Table 1: Simulated\textsuperscript{39} values of $\bar{\lambda}/\omega$

To give a rough idea of what these numbers imply in terms of public employment levels, one needs to construct a reasonable empirical analog of $\omega$. According to the US Census\textsuperscript{40}, the ratio between the average income of the top half and of the income distribution in 2009 is roughly 4 times the corresponding value for the bottom half. As a consequence, the simulation suggests that the scope of the disagreement between $R$ and $L$ is between .844 and .012 percentage points of total employment. Given that over the period 1970 - 2010 the share of public employees over the employed population in the United States has been between 15% and 19%, the inefficiencies generated by the model are sizeable.

8 Conclusion

This paper studies dynamic electoral competition in a setting where current policies affect political actors’ future electoral environment and studies how the associated distortions depend on various institutional (constitutions) and non-institutional factors. Unlike most of the recent literature on dynamic political failures, inefficiencies decrease with political actors’ and voters’ patience. Moreover, as political persistence increases, distortions are magnified.

The paper shows how that these type of political failures are not unique to a few specific policy contexts so far analyzed in economics (that is, migration policy and the Curley effect), nor do they depend on ideology associated with the presence of ethnic cleavages. Rather,

\textsuperscript{38}Assuming uniform distributions yields values in the same order of magnitude.

\textsuperscript{39}The parameterized distributions are $\varepsilon \sim N(0, 1)$ truncated at $-1$ and 1, $q \sim N(q^h/2, 1)$ truncated at 0 and $q^h$, and $\hat{q} \sim N(q^h/2, \hat{\sigma}_q)$ truncated at 0 and $q^h$. Each number is the average over 100 samples of 500000 observations drawn from the multivariate distributions of $(q, q + \varepsilon)$ and $(\hat{q}, \hat{q} + \varepsilon)$.

\textsuperscript{40}Data available at www.census.gov/hhes/www/cpstable032010/lhiinc/new06_000.htm
this type of political failures can arise in a relatively standard dynamic public finance setting with rational agents and asymmetric information.

The paper considers two types of dynamic distortions: at the platform level, capturing political polarization, and at the implemented policy level, quantifying inefficiencies in public good provision. The second contribution of the paper is to show how these distortions depend on the constitutional setting. In a finite horizon model, majoritarian constitutions display lower polarization but more inefficient public good provision. Moreover, the expected implemented redistribution is lower than in consensual constitutions. In an infinite horizon model, public good provision is even more inefficient relative to consensual constitutions. On the other hand, platform divergence no longer depends on the type of constitution. This suggests an interesting result: while long term inefficiencies are very sensitive to the constitutional setting, long term polarization does not depend on institutional factors.

The differences in outcomes between constitutions critically depends on the allocation of policy-making rights over the public good/public employment dimension, and is less sensitive to the allocation of power over redistribution (which is, quite paradoxically, the only dimension over which political actors have preferences). From a utilitarian perspective, a consensual allocation of power over public good provision combined with a majoritarian allocation of power over redistribution welfare-dominates both consensual and majoritarian democracy. This result implies that, when drafting a constitution, consensualism is more beneficial on dimensions over which parties are not intrinsically in conflict, as opposed to dimensions over which parties have induced preferences.

The third contribution of the paper is to study how non institutional variables (political actors’ and voters’ discount factor, political persistence, time horizon, strength of informational asymmetry, voters’ ideological volatility) affect dynamic political distortions. The main result, which goes against the intuition suggested by the previous literature on dynamic electoral competition, is that more political persistence and more far sighted voters can increase, rather than decrease, the extent of these distortions.

Finally, the paper delivers implications that show how three important empirical regularities, which have received a lot of attention in the literature, can be interpreted in light of the presence of dynamic political distortions. First, higher inequality is associated with higher political polarization. Second, higher inequality lower public good provision. Third, an inverse relationship between inequality and redistribution can arise under a majoritarian constitution, thereby suggesting a new explanation for an existing theoretical puzzle. Although these interpretations require serious empirical validation, we are not aware of similar arguments in previous formal literature. The results discussed in this paper are analytically
derived from the model, which is simple enough not to require numerical simulations even for the infinite horizon version.

This paper shows that more theoretical and empirical work is needed to cast light on the presence of dynamic political distortions in other important policy domains. This approach has been almost completely neglected so far, but it can help our understanding of the dynamics of policy-making on other important areas, such as subsidies to home ownership and agriculture, or the public funding of religious education.

Appendix

Proof of Lemma 1

\( \bar{\lambda} \) can be rewritten as \( \int_{q}^{q_h} \left\{ I(\gamma) - \int_{-\epsilon}^{\epsilon} I(\gamma + u) dF(\epsilon) \right\} dF_{q}(\gamma), \) with \( I(s) = \int_{q}^{q_h} I(z) dF_{q|s}(z). \)

The proof has two steps. First, I establish that \( \tilde{I}(s) < I(s) \) \( \forall s, \) where \( \tilde{I}(s) = \int_{q}^{q_h} I(z) dF_{q|s}(z). \)

To see why the latter must hold, notice that, for every realization of \( s, F_{q|s} | s \) is second order stochastically dominated by \( \hat{F}_{q|s}: \) the two distributions have the same mean but the first has higher variance. Since \( I(.) \) is strictly concave, it follows that \( \tilde{I}(s) < I(s). \) The second step is to show that \( I(\gamma) - \int_{-\epsilon}^{\epsilon} I(\gamma + u) dF(\epsilon) = 0. \) This follows from the fact that the difference can be rewritten as \( E_{\hat{F}_{q}}[q] - E_{\hat{F}_{q}}[E_{\hat{F}_{q|s}}[I(q)]] \). Using the law of iterated expectation, \( E_{\hat{F}_{q}}[E_{\hat{F}_{q|s}}[I(q)]] = E_{\hat{F}_{q}}[q], \) which completes the proof.

Bounds on \( \varphi \) and \( \psi \)

In order to make players’ objective functions continuous and differentiable, we need to make assumptions on the relative size of the state space, \( \varphi \) and \( \psi. \) This is a standard in this type of models.\(^{41}\) Given the structure of the model (and, in particular, the different types of constitution considered), we need both political actors to be competitive in every election. That implies that the range of the realized vote share must include, for every realization of the state \( \mu_t \) and any platform profile \( (x^R, x^L), \) the value 1/2.

More formally, given

\[ \hat{\pi}(\xi, d, \mu) = 1/2 + \varphi[d + \mu + \xi] \]

we must have \( \max_{\xi} \hat{\pi}(\xi, d, \mu) \in (1/2, 1), \) \( \min_{\xi} \hat{\pi}(\xi, d, \mu) \in (0, 1/2) \) \( \forall \mu, d, \) where \( \mu \in [\mu^l, \mu^h], \)

\( d \in [d^l, d^h], \) \( d^l = \min\{0, 1 - A - 1/2\} - (1 - A)^2/2, \) and \( d^h = (1 - A)^2/2 - \min\{0, 1 - A - 1/2\}. \) The two conditions yield 4 equations that are equivalent to \( \min\{1/\varphi - 1/\psi, 1/\psi\} >

\(^{41}\)See Persson and Tabellini (2002), Chapter 3.
2 \max\{d^l + \mu^l, d^h + \mu^h\}$, which, given that $|d^l + \mu^l| < d^h + \mu^h$, is equivalent to

$$\min\{1/\varphi - 1/\psi, 1/\psi\} > 2(d^h + \mu^h) \quad (4)$$

**Proof of Proposition 1**

i) and ii). In $t = 2$ equilibrium policies solve $X_2^R \in \arg \max p_2(x_1), X_2^L \in \arg \max \{1-p_2(x_1)\}$, where

$$p_2(x_1) = 1/2 + \psi[d(x_2^R, x_2^L) + I(q_2) - \lambda x_1].$$

The FONC of the problem are also sufficient and fully define the solution. $X_1^R$ and $X_1^L$, instead, solve

\[
\begin{align*}
X_1^R & \in \arg \max_{x \in [0,1]} \left\{ p_1(x_0) + \beta p_1(x_0)p_2^*(x) + \beta(1 - p_1(x_0))p_2^*(X_1^L) \right\} \\
X_1^L & \in \arg \max_{x \in [0,1]} 1 + \beta - \left\{ p_1(x_0) + \beta p_1(x_0)p_2^*(x) + \beta(1 - p_1(x_0))p_2^*(X_1^L) \right\}
\end{align*}
\]

where, denoting $E[I(q)]$ by $\bar{I}$, $p_2^*(x) = E[p_2(x)] = 1/2 + \psi \bar{I} - \psi \bar{\lambda} x$ follows from the observation that $d(X_2^R, X_2^L) = 0$. The FONC are of the problem (which are also sufficient under the assumptions) define the following system

\[
\begin{align*}
\frac{d}{\delta X_1^R} p_1(x_0)[1 + \beta(p_2^*(X_1^R) - p_2^*(X_1^L))] + \beta p_1(x_0) \frac{d}{\delta X_1^R} p_2^*(X_1^R) & = 0 \\
\frac{d}{\delta X_1^L} p_1(x_0)[1 + \beta(p_2^*(X_1^R) - p_2^*(X_1^L))] + \beta(1 - p_1(x_0)) \frac{d}{\delta X_1^L} p_2^*(X_1^L) & = 0
\end{align*}
\]

subtracting the first from the second gives \[2\], while summing them gives

$$\psi(2x^* - (X^R + X^L))(1 + \beta \psi \bar{\lambda} \Delta(M)) = \psi \bar{\lambda} \beta(2p_1(x_0) - 1) = \psi^2 \bar{\lambda} \beta[\Delta(M)((X^R + X^L)) - 2x^* + 2\mu] \forall \mu$$

which implies that the unique pair of equilibrium strategies must be linear in the state and takes the form displays in the statement of the proposition. Part iii) follows from the binary outcome of the $(M)$-game and the fact that \[3\] can be re-written as

$$X_1^R = x^* - \Delta(M)p_1(X_1^R, X_1^L, \mu) ; X_1^L = x^* + \Delta(M)(1 - p_1(X_1^R, X_1^L, \mu)) \quad (7)$$

and that, in equilibrium

$$p_1 = 1/2 + \mu_1 \psi(1 + \psi \Delta^2(M))^{-1} > 1/2 \quad (8)$$
Proof of Proposition 2

For i), start observing that, since the RHS of (2) is supermodular in $\beta \lambda$, the positive solution of that equation shifts to the right as $\beta \lambda$ increases. Then notice that $\lambda$ can be re-written as $\omega \int_0^\omega \{I_\omega(\gamma) - \int \omega \tilde{I}_\omega(\gamma + u) dF_\omega(u)\} d\tilde{F}_\omega(\gamma)$ where $I_\omega(q) = q/[2(2 + q)]$ and $\tilde{I}_\omega(s) = E_F[I_\omega(q)|s]$ are independent of $\omega$. For ii), notice that $\Sigma_1(M) = E[X_1(M) - x^*]^2$, using (7) and (8), simplifies to

$$\Sigma_1(M) = \Delta^2_{(M)} \left[ 1/4 + 3(\psi \mu_1)^2(1 + \psi \Delta^2_{(M)})^{-2} \right]$$

which is increasing in $\Delta^2_{(M)}$ and $\mu_1$. Notice that $\mu_1 = I(q_1) - \lambda_1 x_0$ and that $\lambda_1$ can be positive or negative. To see that $\frac{d}{d\Delta_{(M)}} \Sigma_1(M) > 0$, notice that its derivative is certainly positive if $1 > (\psi \Delta^2_{(M)})^2$, which is guaranteed by rewriting (2) as $\frac{\Delta_{(M)}}{\beta_1} + \psi \Delta^2_{(M)} = 1$ To see that it ambiguous in $\psi$, notice that $\frac{d}{d\psi} \Sigma_1(M) = \frac{\partial}{\partial \psi} \Sigma_1(M) + \frac{\partial}{\partial \Delta} \Sigma_1(M) \frac{d}{d\psi} \Delta_{(M)}$ becomes

$$\frac{2\Delta^2_{(M)}}{(1 + \psi \Delta^2_{(M)})^3} \left[ 3\mu_1^2 \psi - (\Delta_{(M)}(1 + \psi \Delta^2_{(M)})/2)^2 - 3(\mu_1 \psi \Delta_{(M)})^2 \left( \frac{1 - \psi \Delta^2_{(M)}}{1 + \psi \Delta^2_{(M)}} \right) \right]$$

which, depending on the value of $\mu_1$, can be positive or negative.

Proof of Proposition 3

i) Since the game has a fixed total value $\tilde{V} = (1 - \beta)^{-1}$ for each player, the recursive formulation of the problem solved by $R$ and $L$ under (M) is given by:

$$V^R(\mu) = \max_{x^R \in [0,1]} \{ p(x^R, x^L, \mu)[1 + \beta(E[V^R(\mu^R)] - E[V^R(\mu^L)])] + E[V^R(\mu^L)] \} \quad (10)$$

$$V^L(\mu) = \max_{x^L \in [0,1]} \{ \tilde{V} - p(x^R, x^L, \mu)[1 + \beta(E[V^R(\mu^R)] - E[V^R(\mu^L)])] - E[V^R(\mu^L)] \} \quad (11)$$

where $\mu^R = I(q) - \lambda x^R$ and $\mu^L = I(q) - \lambda x^L$.

A differentiable stationary Markov perfect equilibrium (DSMPE) is a pair of differentiable value functions $V^R(\mu)$, $V^L(\mu)$ and differentiable policy functions $X^R(\mu)$, $X^L(\mu)$ such that

1. given $x^L = X^L(\mu)$, $V^R(\mu)$ solves (10) and, given $x^R = X^R(\mu)$, $V^L(\mu)$ solves (11)
2. $X^R(\mu)$ attains the RHS of (10) and $X^L(\mu)$ attains the RHS of (11)

To see that the two platforms constitute a DSMPE, start with two affine guesses of the form $h^R(\mu) = h^R_0 + h^R_1 \mu$, $h^L(\mu) = h^L_0 + h^L_1 \mu$ and plug them into the problem. In Step 1 we
verify that the value functions are affine in $\mu$, in Step 2 we solve for the coefficients, in Step 3 we show it is the unique DSMPE.

**Step 1.** A few lines of algebra allow to verify that $p(h_R(\mu), h_L(\mu), \mu)$ is an affine function of $\mu$

$$p(h_R(\mu), h_L(\mu), \mu) = \bar{p}(\mu) = h_p + h_p\mu$$

Then the value functions can be re-expressed in the following way:

$$V^R(\mu_0) = \bar{p}(\mu_0)(1 + \beta(\bar{p}(\mu_1|h_R(\mu_0)) - \bar{p}(\mu_1|h_R(\mu_0)))) + \beta\bar{p}(\mu_1|h_R(\mu_0)) + ...$$

$$V^L(\mu) = (1 - \beta)^{-1} - V^R(\mu)$$

where

$$\bar{p}(\mu_1|h_R(\mu_{t-1})) = E[\bar{p}(I(q) - \lambda h_L(\mu_{t-1}))] = \bar{p}(E[I(q)] - \lambda h_L(\mu_{t-1}))$$

Moreover, $\bar{p}(\mu|h_R(\mu)) - \bar{p}(\mu|h^R(\mu))$ does not depend on $\mu$. Therefore $\bar{p}(\mu)(1 + \beta(\bar{p}(\mu|h_R(\mu)) - \bar{p}(\mu|h^R(\mu)))$ is affine in $\mu$ and, for the same reason, all subsequent terms of the summation are also affine in $\mu$. Denote by $V_1$ the slope coefficient of $V^R$.

**Step 2.** The FONCs are (the equilibrium must be interior)

$$\begin{cases} 
\quad \frac{dp^R}{dx}[1 + \beta\bar{V}_1(x_R - x^L)] = \beta p^R V_1 \bar{\lambda} \\
\quad \frac{dp^L}{dx}[1 + \beta\bar{V}_1(x_L - x^R)] = \beta(1 - p^L) V_1 \bar{\lambda}
\end{cases}$$

where $p^R = p(x_R, h_L(\mu), \mu)$ and $p^L = p(h_R(\mu), x_L, \mu)$; the envelope conditions yield

$$V_1 = \frac{dp^R}{dx}[1 + \beta\bar{V}_1(x_R - x^R)] - \beta(1 - p^R) \bar{V}_1 h_1$$

$$V_1 = \frac{dp^L}{dx}[1 + \beta\bar{V}_1(x_L - x^R)] - \beta p^L \bar{V}_1 h_1$$

re-expressing these 4 equations as functions of $h_0^L - h_0^R = \Delta_M - \infty$, $h_0^R + h_0^L$, and $h_1$ yields a unique solution, given in the proposition. To obtain the solution, impose equilibrium, which gives $p^R = p^L$, $h^R = x^R$, $h^L = x^L$, then solve for $\Delta_M - \infty$ summing the two FONCs to get $V_1^{-1}$ and equating the resulting expression to the $V_1^{-1}$ obtained from each envelope condition. After that, sum the two first order conditions and obtain an equation in $h_0^R + h_0^L$, and $h_1$, which is affine in $\mu$. Setting $\mu = 0$ gives $h_0^R + h_0^L = 2x^*$, which then allows us to solve for $h_1$. The slope of the value function is then

$$V_1 = \frac{\psi}{1 - \psi(\beta\bar{\lambda})^2}$$

**Step 3a.** To show that this is the unique differentiable MPE, we first show that the value function must be unique by showing that the operator defining it is a contraction.
Subsequently, we show that no pair of policy functions other than the one previously derived can generate the same value function.

First, for every bounded, continuous, and increasing value function of the game $v(\mu)$ define

$$P[v](x^R, x^L, \mu) = p(x^R, x^L, \mu) + \beta \int_{\mu^l}^{\mu^h} v(\mu') f(\mu'|x^R, x^L, \mu) d\mu'$$

where, denoting by $q'$ and $s'$ the future values of $q$ and $s$, $f(\mu'|x^R, x^L, \mu) = p(x^R, x^L, \mu)f(q', s')$ if $\mu' = I(q') - \lambda x^R$, $f(\mu'|x^R, x^L, \mu) = f(q', s')(1 - p(x^R, x^L, \mu))$ if $\mu' = I(q') - \lambda x^L$, and zero otherwise.

Using the results from Step 1, we show that the (M)-game has the minimax property by showing that Theorem 4.2 in Jaskiewicz and Nowak (2006) holds: for their theorem to apply, we need that:

1. $p(x^L, x^R, \mu)$ and $f(\mu'|x^R, x^L, \mu)$ are continuous
2. There exists $U(\mu) : |p(x^L, x^R, \mu)| < U(\mu) \forall (x^R, x^L, \mu)$
3. The mapping $(x^L, x^R, \mu) \mapsto \int_{\mu^l}^{\mu^h} U(\mu') f(\mu'|x^R, x^L, \mu) d\mu'$ is continuous
4. There exists a Borel function $\delta : [0, 1]^2 \times [\mu^l, \mu^h] \mapsto [0, 1]$ and a probability measure $\varphi(\mu)$ such that
   i. $f(M|x^R, x^L, \mu) \geq \delta(x^R, x^L, \mu) \varphi(M) \forall (x^R, x^L, \mu)$ and every Borel set $M \subset [\mu^l, \mu^h]$
   ii. $\int_{\mu^l}^{\mu^h} \inf_{x^R \in [0,1]} \inf_{x^L \in [0,1]} \delta(x^R, x^L, \mu) \varphi(d\mu) > 0$
   iii. $\varphi(U) = \int_{\mu^l}^{\mu^h} U(\mu) \varphi(d\mu) < \infty$
   iv. For some $\rho \in (0, 1)$ and for every $(x^R, x^L, \mu)$

$$\int_{\mu^l}^{\mu^h} U(\mu') f(\mu'|x^R, x^L, \mu) d\mu' \leq \rho U(\mu) + \delta(x^R, x^L, \mu) \varphi(U)$$

(12)

Define $f = \inf_{(\mu', x^R, x^L, \mu)} f(\mu'|x^R, x^L, \mu)$ and choose $U(\mu) = 1$, $\delta(x^R, x^L, \mu) = f p(x^R, x^L, \mu)$ and $\varphi(\mu)$ uniform (that is $\varphi(\mu) = 1$).

Then conditions 1-4.iii are trivially satisfied. To see why 4.iv must also hold, notice that (12) becomes $1 \leq \rho + f p(x^R, x^L, \mu)$. Since, under the assumptions, $p(x^R, x^L, \mu) > 0$, there exists $\tilde{\rho} < 1$ that satisfies (12).

**Step 3b.** Therefore, the following operator can be defined

$$Val(P[v]) = \max_{x^R \in [0,1]} \min_{x^L \in [0,1]} \{P[v](x^R, x^L, \mu)\} = \min_{x^L \in [0,1]} \max_{x^R \in [0,1]} \{P[v](x^R, x^L, \mu)\}$$

Note that, although the distribution of implemented policies is essentially a Bernoulli, the distribution of realized states has full support, due to the assumptions on $f(s', q')$.

Under the assumptions, we know that, $\forall K = (x^R, x^L, \mu), f(\mu'|K)$ has full support. As a consequence $f > 0$. 

43
To show that the value function is unique, we make use of the following Lemma.

**Lemma UB**: Fix $\mu$, then for every pair of bounded, continuous, and differentiable $P[v^1](x^R, x^L; \mu)$ and $P[v^2](x^R, x^L; \mu)$ with support in $[0, 1]^2$, we have:

$$|Val(P[v^1]) - Val(P[v^2])| \leq \max_{(x,y)\in[0,1]^2} |P[v^1](x,y) - P[v^2](x,y)|$$

**Proof.** Let $(h^R, h^L)$ be a pair of policy functions generating an MPE of $P[v^1]$ and let $(\tilde{h}^R, \tilde{h}^L)$ be its analog for $P[v^2]$. Then it must be that

$$P[v^1](h^R, h^L) \leq P[v^1](h^R, h^L) \leq P[v^1](\tilde{h}^R, \tilde{h}^L) \leq P[v^2](\tilde{h}^R, \tilde{h}^L) \leq P[v^2](h^R, h^L)$$

which implies

$$P[v^1](h^R, h^L) - P[v^2](\tilde{h}^R, \tilde{h}^L) \leq P[v^1](h^R, h^L) - P[v^2](\tilde{h}^R, \tilde{h}^L) \leq \max_{(x,y)\in[0,1]^2} |P[v^1](x,y) - P[v^2](x,y)|$$

Now, let’s define the operator $T$, mapping the space of bounded, continuous, and differentiable functions (with domain in $[\mu^l, \mu^h]$) into itself:

$$T[v](\mu) = Val \left( p(x^R, x^L, \mu) + \beta \int_{\mu^l}^{\mu^h} v(\mu') f(\mu' | x^R, x^L, \mu) d\mu' \right) = Val(P[v]).$$

For every bounded, continuous $v, v' : [0, 1] \to \mathbb{R}$, we have

$$||T[v] - T[v']||_{\infty} = \max_{\mu \in [\mu^l, \mu^h]} |Val(P[v]) - Val(P[v'])|$$

by Lemma UB, we must have

$$||T[v] - T[v']||_{\infty} \leq \max_{\mu \in [\mu^l, \mu^h]} \left\{ \max_{(x,y)\in[0,1]^2} |P[v^1](x,y) - P[v^2](x,y)| \right\}$$

which simplifies to

$$||T[v] - T[v']||_{\infty} \leq \max_{\mu \in [\mu^l, \mu^h]} \left\{ \max_{(x,y)\in[0,1]^2} \beta \int_{\mu^l}^{\mu^h} (v(\mu') - v'(\mu')) f(\mu' | x^R, x^L, \mu) d\mu' \right\}.$$
Now define \( \bar{D} = \max_{\mu' \in [\mu^t, \mu^h]} |v(\mu') - v'(\mu')| \). It must then be that
\[
||T[v] - T[v']||_{\infty} \leq \max_{\mu \in [\mu^t, \mu^h]} \left\{ \max_{(x,y) \in [0,1]^2} \beta \bar{D} \left| \int_{\mu^t}^{\mu^h} f(\mu'|x^R, x^L, \mu) d\mu' \right| \right\}
\]
since \( \beta \bar{D} \) does not depend on \( x^R, x^L \), and \( \mu \), we can move them to the left of the maximum operators. Since \( \int_{\mu^t}^{\mu^h} f(\mu'|x^R, x^L, \mu) d\mu' = 1 \forall (x^R, x^L, \mu) \), one obtains
\[
||T[v] - T[v']||_{\infty} \leq \beta \bar{D} = \beta \sup_{\mu' \in [\mu^t, \mu^h]} |v(\mu') - v'(\mu')|
\]
This implies that \( T \) is a contraction. As a consequence, the value function associated with the infinite horizon (M)-game is unique.

**Step 3c.** To complete the proof, we need to show that no other pair of policy functions \((x^R(\mu), x^L(\mu))\) can generate the value function obtained in Step 1. To see that, combining the FONCs and the Envelope Conditions of the problem with the requirement that the value function is linear yields
\[
\begin{align*}
\psi(x^* - x^R)[1 + \beta \lambda V_1(x^L - x^R)] &= \beta p V_1 \lambda \\
\psi(x^L - x^*)[1 + \beta \lambda V_1(x^L - x^R)] &= \beta (1-p) V_1 \lambda \\
V_1 &= \psi(1 + \frac{dx^L}{d\mu} (x^L - x^*))[1 + \beta \lambda V_1(x^L - x^R)] - \beta \lambda V_1 (1-p) \frac{dx^L}{d\mu} \\
V_1 &= \psi(1 + \frac{dx^R}{d\mu} (x^* - x^R))[1 + \beta \lambda V_1(x^L - x^R)] - \beta \lambda V_1 p \frac{dx^R}{d\mu}
\end{align*}
\]
substituting the first equation into the fourth and the second into the third gives, in both cases,
\[
V_1 = \psi[1 + \beta \lambda V_1(x^L - x^R)]. \tag{13}
\]
The equation, in turn, implies that the difference \( \Delta = x^L - x^R \) is independent of \( \mu \). We can then set \( x^L = x^R + \Delta \). The difference between the FONCs then becomes
\[
\psi(2x^* - \Delta - 2x^R(\mu))[1 + \beta \lambda V_1 \Delta] = \beta V_1 \lambda (2p - 1);
\]
substituting for the expression of \( p \) yields
\[
\psi(2x^* - \Delta - 2x^L(\mu))[1 + \beta \lambda V_1 \Delta] = \beta V_1 \lambda \psi(\Delta(2x^R(\mu) + \Delta) - 2x^* \Delta + \mu).
\]
Assuming that \( x^R(\mu) \) it is linear leads to the equilibrium already derived in Step 1. As a consequence, one must rule out the existence of a positive non linear component in \( x^R(\mu) \). Suppose, wlog; that \( x^R(\mu) = x_0 + x_1 \mu + x(\mu) \), where \( x(\mu) \) is a continuous, differentiable and bounded non-linear function. For that equation to hold, it must be that the non-linear coefficients on each side must be equal. That means that the following equation must hold:
\[
-2\psi[1 + \beta \lambda V_1 \Delta] = \beta V_1 \lambda \psi 2.
\]
This equation implies that \( V_1 = -(2\beta\bar{\lambda} \Delta)^{-1} \). Combining this with (13) yields \( V_1 = -\psi/2 \). The value function associated with the equilibrium obtained in Step 1, instead, is \( V_1 = \psi(1 - \psi\beta\bar{\lambda}\Delta_{\infty}) \); that is a contradiction. As a consequence, the infinite horizon of the \((M)\)-game must have only one DSMPE.

**Proof of Proposition 4**

Since \( \bar{\beta} = (1 - \beta)^{-1} \), (2) can be rewritten as

\[\Delta_{(M)}[1 - \beta + \psi\beta\bar{\lambda}\Delta_{(M)}] - \beta\bar{\lambda} = 0\]

since \( \Delta_{(M)} - \infty = \beta\bar{\lambda} \), if one proves that \( \psi\bar{\lambda}\Delta_{(M)} < 1 \), then \( \Delta_{(M)} > \Delta_{(M)} - \infty \) and, by proposition 2, \( \Sigma^{(M)} \) is larger in the two period model. To see that \( \psi\bar{\lambda}\Delta_{(M)} < 1 \), notice that, using the fact that equilibrium \( p < 1 \) and the fact that \( \psi\Delta_{(M)}^2 < 1 \)

\[2\psi\mu < 1 + \psi\Delta_{(M)}^2 < 2 \forall \mu\]

which implies that \( \psi\mu^h < 1 \). Since \( \bar{\lambda} < I(q^h) \leq \mu^h \) and \( \Delta_{(M)} < 1 \), it follows that \( 1 > \psi\mu^h > \psi\bar{\lambda}\Delta_{(M)} \). Since \( \Sigma_{1}^{(M)} \) only depends on the discount factor through platform divergence, the rest of the proposition directly follows.

**Proof of Lemma 2**

Follows from the observation that, being it a zero sum game and having actors the same discount factor and the same information, every proposed deviation from the status quo \((\alpha_x, \alpha_b)\), to which are associated implemented policies\(^{44}\)

\[X^{(C)} = \alpha_x x^R_1 + (1 - \alpha_x)x^L_1; \quad b^{(C)} = (1 - \alpha_b)b_t.\]

would weakly benefit at most one player, and would thereby not proposed in equilibrium.

**Proof of Proposition 5**

i) and ii) In \( t = 2 \) equilibrium policies solve \( X^R_2 \in \arg \max \pi_2(x_1), \quad X^L_2 \in \arg \max \{1 - \pi_2(x_1)\} \), where

\[\pi_2(x_1) = 1/2 + \varphi[d(x^R_2, x^L_2) + I(q_2) - \lambda_2 x_1].\]

The FONC of the problem define the solution. \( X^R_1 \) and \( X^L_1 \), instead, solve

\(^{44}\)It's easy to see that a pair \((\alpha_x, \alpha_b)\) would completely characterize any possible allocation of policymaking rights.
\[
\begin{align*}
X^R_t & \in \arg\max_{x \in [0,1]} \left\{ \pi_1(x_0) + \beta E[\hat{\pi}_2(X^{(C)})] \right\} \\
X^L_t & \in \arg\max_{x \in [0,1]} 1 + \beta - \left\{ \pi_1(x_0) + \beta E[\hat{\pi}_2(X^{(C)})] \right\}
\end{align*}
\]

where \( E[\hat{\pi}_2(X^{(C)})] = 1/2 + \varphi \hat{I} - \varphi \hat{\lambda} E[X^{(C)}] \) follows from \( d(X^R_2, X^L_2) = 0 \), the former is then \( 1/2 + \varphi \hat{I} + \varphi \hat{\lambda} \{ \pi_1(x_0)x^R_1 + (1 - \pi_1(x_0))x^L_1 \} \) The FONC are of the problem (which are also sufficient under the assumptions) define the following system

\[
\left\{ \begin{array}{l}
\frac{d}{dX^R_t} \pi_1(x_0)[1 + \beta \varphi \hat{\lambda}(X^R_t - X^L_t)] + \beta \varphi \hat{\lambda} \pi_1(x_0) = 0 \\
\frac{d}{dX^L_t} \pi_1(x_0)[1 + \beta \varphi \hat{\lambda}(X^R_t - X^L_t)] + \beta \varphi \hat{\lambda}(1 - \pi_1(x_0)) = 0
\end{array} \right. \tag{14}
\]

whose unique solution gives the equilibrium at \( t = 1 \), using the same steps as in (M). Part iii) follows from the observation that, once platforms are fixed, the only randomness in the implemented policy is given by the realization of the aggregate shock, \( \hat{\xi}_1 \), and the observation that \( X^{(C)} = X^L_t - \hat{\pi}_1 \Delta^{(C)} \), which yields \( [\bar{e}^{(C)}, \bar{\pi}^{(C)}] = \left[ x^{E}_{(C)} - \frac{\varphi \Delta^{(C)}}{\psi}, x^{E}_{(C)} + \frac{\varphi \Delta^{(C)}}{\psi}, \bar{\lambda}_{(C)} \right] \).

iv) The first part follows from is supermodularity of the RHS of (3) in \( \beta \hat{\lambda} \) and the same decomposition of \( \hat{\lambda} \) performed above. For v), notice that \( X^{(C)} - x^* \) is uniform in \( \left[ -\frac{\varphi \Delta^{(C)}}{\psi}, \frac{\varphi \Delta^{(C)}}{\psi} \right] \). \( \Sigma^{(C)} = E[X^{(C)} - x^*]^2 \) then becomes

\[
\Sigma^{(C)} = \Delta^{(C)} \left[ \frac{\varphi^2}{(12\psi^2)} + 4(\varphi \mu_1)^2(1 + \varphi \Delta^{(C)})^{-2} \right]
\]

which is increasing in \( \Delta^{(C)} \) (use (3) and \( \psi^{-1} \), and ambiguous in \( \varphi \) due to the increasing direct effect and the decreasing indirect effect through \( \Delta^{(C)} \).

**Proof of Proposition 6**

i). The recursive formulation of the problem solved by \( R \) and \( L \) under (M) is given by:

\[
V^R(\mu) = \max_{x^R \in [0,1]} \pi(x^R, x^L, \mu) + \beta E[V^R(\mu)|X^{(C)}] \tag{15}
\]

\[
V^L(\mu) = \max_{x^L \in [0,1]} \bar{V} - \left\{ \pi(x^R, x^L, \mu) + \beta E[V^R(\mu)|X^{(C)}] \right\} \tag{16}
\]

where

\[
\beta E[V^R(\mu)|X^{(C)}] = \beta E[V^R(I(q) - \lambda(\hat{\pi} x^R + (1 - \hat{\pi}) x^L))]}
\]

To see that the two platforms are a DSMPE, start with two affine guesses of the form
\( h^R(\mu) = h_0^R + h_1\mu \), \( h^L(\mu) = h_0^L + h_1\mu \) and plug them into the problem. In Step 1 we verify that the value functions are affine in \( \mu \), and in Step 2 we solve for the coefficients, and in Step 3 we show that this is the unique DSMPE.

**Step 1.** A few lines of algebra allow to verify that \( \pi(h^R(\mu), h^L(\mu), \mu) \) is an affine function of \( \mu \)

\[
\pi(h^R(\mu), h^L(\mu), \mu) = h_p + h_p\mu
\]

where the realized value of \( \bar{\pi}(\mu) \) is \( \pi(\mu) + \varphi \xi \). Then the value functions can be re-expressed in the following way:

\[
V^R(\mu_0) = \bar{\pi}(\mu_0) + \beta E[\bar{\pi}(\mu_1)] + \beta^2 E[\bar{\pi}(\mu_2)] + ... \\
V^L(\mu_0) = (1 - \beta)^{-1} - V^R(\mu_0)
\]

where

\[
E[\bar{\pi}(\mu_t)] = E[\bar{\pi}\{I(q) - \lambda[(\bar{\pi}(\mu_{t-1}) + \varphi \xi)h^R(\mu_{t-1}) + (1 - \bar{\pi}(\mu_{t-1})) - \varphi \xi)h^L(\mu_{t-1})]\}]
\]

simplifies to \( E[I(q)] - E[\lambda][(\bar{\pi}(h^R_0 - h^L_0) + h^L_0 + h_1\mu_{t-1})] \), which is affine in \( \mu_{t-1} \). Therefore, all the terms in the summation are compositions of affine functions, therefore affine. Denote by \( V_1 \) the slope coefficient of \( V \).

**Step 2.** The FONCs are (the equilibrium must be interior)

\[
\begin{cases}
\frac{d\pi^R}{dx^R}[1 + \beta \bar{\lambda}V_1(h^L - x^R)] = \beta \pi^R V_1 \bar{\lambda} \\
\frac{d\pi^L}{dx^L}[1 + \beta \bar{\lambda}V_1(x^L - h^R)] = \beta (1 - \pi^L)V_1 \bar{\lambda}
\end{cases}
\]

where \( \pi^R = \pi(x^R, h^L(\mu), \mu) \) and \( \pi^L = \pi(h^R(\mu), x^L, \mu) \); the envelope conditions yield

\[
V_1 = \frac{d\pi^R}{d\mu}[1 + \beta \bar{\lambda}V_1(h^L - x^R)] - \beta (1 - \pi^R)\bar{\lambda}V_1 h_1 \\
V_1 = \frac{d\pi^L}{d\mu}[1 + \beta \bar{\lambda}V_1(x^L - h^R)] - \beta \pi^L \bar{\lambda}V_1 h_1
\]

re-expressing these 4 equations as functions of \( h^R_0 - h^L_0 = \Delta_{(M)-\infty}, h^R_0 + h^L_0 \), and \( h_1 \) yields a unique solution, given in the proposition. To obtain the solution, impose equilibrium, which gives \( \pi^R = \pi^L, h^R = x^R, h^L = x^L \), then solve for \( \Delta_{(C)-\infty} \) summing the two FONCs to get \( V_1^{-1} \) and equating the resulting expression to the \( V_1^{-1} \) obtained from each envelope condition. After that, sum the two first order conditions and obtain an equation in \( h^R_0 + h^L_0 \), and \( h_1 \), which is affine in \( \mu \). Setting \( \mu = 0 \) gives \( h^R_0 + h^L_0 \), which then allows us to solve for \( h_1 \). The slope of the value function is then

\[
V_1 = \frac{\varphi}{1 - \varphi (\beta \bar{\lambda})^2}
\]
Step 3a. The proof for the uniqueness has the same structure as the one for the (M) game: for every bounded, continuous, and differentiable function \( v(\mu) \) define

\[
\Pi[v](x^R, x^L, \mu) = \pi(x^R, x^L, \mu) + \beta \int_{\mu'}^{\mu} v(\mu') g(\mu' | x^R, x^L, \mu) d\mu'
\]

where \( g(\mu' | x^R, x^L, \mu) = f(q', s') \frac{\psi}{\phi(q' - s')} \). The assumptions in Jąskiewicz and Nowak (2006) are still satisfied (following same steps as in the proof in Proposition 3, Step 3a, choose \( \delta(x^R, x^L, \mu) = \int \pi(x^R, x^L, \mu) \), and Lemma UB holds unchanged. As a consequence, following the same steps as in Proposition 3, Step 3b, the operator

\[
T[v](\mu) = \text{Val} \left( \pi(x^R, x^L, \mu) + \beta \int_{\mu'}^{\mu} v(\mu') g(\mu' | x^R, x^L, \mu) d\mu' \right) = \text{Val}(\Pi[v])
\]

is a contraction. As a consequence, the value function in the infinite horizon (C)-game is also unique.

Step 3b. It remains to show that no other pair of policy functions \((x^R(\mu), x^L(\mu))\) can generate the value function obtained in Step 1. To see that, combining the FONCs and the Envelope Conditions of the problem with the requirement that the value function is linear yields

\[
\begin{align*}
\varphi(x^* - x^R)[1 + \beta \bar{\lambda} V_1(x^L - x^R)] &= \beta \pi V_1 \bar{\lambda} \\
\varphi(x^L - x^*)[1 + \beta \bar{\lambda} V_1(x^L - x^R)] &= \beta (1 - \pi) V_1 \bar{\lambda} \\
V_1 &= \varphi(1 + \frac{dx^L}{d\mu}(x^L - x^*))[1 + \beta \bar{\lambda} V_1(x^L - x^R)] - \beta \bar{\lambda} V_1 [1 + \pi] \frac{dx^L}{d\mu} \\
V_1 &= \varphi(1 + \frac{dx^R}{d\mu}(x^* - x^R))[1 + \beta \bar{\lambda} V_1(x^L - x^R)] - \beta \bar{\lambda} V_1 [1 + \pi] \frac{dx^R}{d\mu}
\end{align*}
\]

following the same method as in Proposition 3 allows us to establish that the difference \( \Delta = x^L - x^R \) is independent of \( \mu \), and subsequently \( V_1 = -\varphi/2 \). Since the value function associated with the equilibrium obtained in Step 1 is \( V_1 = \varphi(1 - \varphi \bar{\lambda} \Delta \infty) \), a contradiction is obtained. As a consequence, the infinite horizon of the (C)-game must have only one DSMPE.

Proof of Proposition 7

i) Notice that \( (4) \) implies that \( \varphi^{-1} > \psi^{-1} \); these two parameters are the only difference between \( (2) \) and \( (3) \); therefore, the result follows.

ii) To see that \( \Sigma_1^{(M)} > \Sigma_1^{(C)} \), notice that the difference can be rewritten as

\[
\frac{\Delta_{(M)}^2}{4} - \frac{1}{12} \left( \frac{\Delta(\varphi)}{\psi} \right)^2 - 4\mu_1^2 \left( \frac{\Delta(\varphi)}{1 + \varphi \Delta_{(C)}^2} \right)^2 + 3\mu_1^2 \left( \frac{\Delta(\varphi)}{1 + \psi \Delta_{(M)}^2} \right)^2
\]
Since $\Delta(C)\varphi/(1+\varphi^2(C))$ and $\Delta(M)\psi/(1+\psi^2(M))$ can be rewritten as $((\Delta(C)\varphi)^{-1}+\Delta(C))^{-1}$ and $((\Delta(M)\psi)^{-1}+\Delta(M))^{-1}$, it is possible to conclude, using the fact that $\psi\Delta(M) = 1 - \Delta(M)/\beta<0 > \varphi\Delta(C) = 1 - \Delta(C)/\beta$, that

$$\Delta(C)\varphi/(1+\varphi^2(C)) < \Delta(M)\psi/(1+\psi^2(M))$$

As a consequence,

$$\Delta^2(M) - \frac{1}{3}\left(\frac{\Delta(C)\varphi}{\psi}\right)^2 - 4\mu^2\left(\frac{\Delta(C)\varphi}{1+\varphi^2(C)}\right)^2 > 0$$

implies $\Sigma_1(M) - \Sigma_1(C) > 0$. Multiplying each side by $\Delta^{-2}(M)$ and using (4), if one proves that

$$1 - \left(\frac{\Delta(C)\varphi}{\Delta(M)}\right)^2 \left(\frac{1}{3\psi^2} + \left[\min\left\{\frac{1}{\psi}, \frac{1}{\varphi} - \frac{1}{\psi}\right\}\right]^2\right) > 0 \quad (18)$$

then it must be that $\Sigma_1(M) - \Sigma_1(C) > 0$.

**Case 1:** $\frac{1}{\psi} < \frac{1}{\varphi} - \frac{1}{\psi}$. (18) simplifies to $1 - \left(\frac{\Delta(C)\varphi}{\Delta(M)\psi}\right)^2 \frac{4}{3} > 0$, which is strictly increasing in $\psi$ (using the implicit function theorem it’s easy to verify that $d(\Delta(M)\psi)/d\psi > 0$). Using (4), the largest value that $\psi$ can take is $\psi = \varphi(1 - 2\varphi\mu h)^{-1}$ Combining this with the fact that $\frac{1}{\psi} < \frac{1}{\varphi} - \frac{1}{\psi}$, one obtains $1 - 2\varphi\mu h < 1/2$. Moreover, $\frac{\Delta(M)}{\Delta(C)}$ can be re-expressed, using (2) and (3) and $\psi = \varphi(1 - 2\varphi\mu h)^{-1}$, as

$$\frac{1}{1 - \varphi^2(C)} \frac{\varphi^2}{\varphi} \frac{\Delta^2(M)}{(1 - 2\varphi\mu h)(1 - \varphi^2(C))}$$

(19)

since $\Delta(M) < \Delta(C)$, the ratio must be below one, which implies $\Delta^2(C) < \Delta^2(M)/(1 - 2\varphi\mu h)$, that is. As a consequence, it must be that

$$\left(\frac{\Delta(C)\varphi}{\Delta(M)\psi}\right)^2 < (1 - 2\varphi\mu h) < \frac{1}{2}$$

Therefore, (18) is implied by $1 - \frac{4}{3} > 0$, which trivially holds.

**Case 2:** $\frac{1}{\psi} > \frac{1}{\varphi} - \frac{1}{\psi}$. (18) simplifies to

$$1 - \left(\frac{\Delta(C)\varphi}{\Delta(M)\psi}\right)^2 \left(\frac{1}{3} + \left(\frac{\psi}{\varphi} - 1\right)^2\right) > 0 \quad (20)$$

and can be re-expressed as $1 - 2r^2/3 - R^2 + 2rR$, where $R = \frac{\Delta(C)\varphi}{\Delta(M)\psi} > 1 > r = \frac{\Delta(C)\varphi}{\Delta(M)\psi}$. Direct inspection allows us to conclude that the expression is minimized when $R$ is maximum and $r$ is minimum, which happens when $\varphi$ is the smallest with respect to $\psi$. Combining this with the restriction $\frac{1}{\psi} > \frac{1}{\varphi} - \frac{1}{\psi}$, one obtains $2\varphi = \psi$. As a consequence $1 - \left(\frac{\Delta(C)\varphi}{\Delta(M)\psi}\right)^2 \frac{4}{3} > 0$ is a
lower bound for the LHS of (20). Using the analog of (1), one obtains that \( \frac{\Delta(M)}{\Delta(C)} \) can be re-expressed as \( \frac{1-2\varphi \Delta^2(M)}{1-\varphi \Delta^2(C)} \). Combining that and \( \frac{\Delta(M)}{\Delta(C)} < 1 \) yields \( \left( \frac{\Delta(C)}{\Delta(M)} \right)^2 < 2 \). As a consequence \( \left( \frac{\Delta(C) \varphi}{\Delta(M) \varphi} \right)^2 < \frac{1}{2} \) and (1) must hold, since
\[
1 - \left( \frac{\Delta(C) \varphi}{\Delta(M) \varphi} \right)^2 \left( \frac{1}{3} + \left( \frac{\varphi}{\varphi} - 1 \right)^2 \right) > 1 - \frac{4}{23} > 0
\]

iii) Notice that, expected redistribution under (C) and (M) are given, respectively, by \( \bar{b}(1-p_1^*) \) and \( \bar{b}(1-\pi_1^*) \). Since
\[
p_1^* > 1/2 + \mu_1 \psi(1 + \psi \Delta^2(M))^{-1} > \pi_1^* = 1/2 + \mu_1 \varphi(1 + \varphi \Delta^2(C))^{-1}
\]
\( \bar{b}(1-p_1^*) < \bar{b}(1-\pi_1^*) \) must hold.

For the last part, notice that, by applying the Implicit function theorem to (2) and (3), re-arranging, one obtains
\[
\frac{d}{d\beta \lambda} \Delta(C) = \frac{\Delta(C) 1 - \varphi \Delta^2(C)}{\beta \lambda 1 + \varphi \Delta^2(C)} \quad \frac{d}{d\beta \lambda} \Delta(M) = \frac{\Delta(M) 1 - \psi \Delta^2(M)}{\beta \lambda 1 + \psi \Delta^2(M)}
\]
Changes in \( \beta \lambda \) affect \( \Sigma_1(M) - \Sigma_1(C) \) only through \( \Delta(C) \) and \( \Delta(M) \). Since we have just proved that the former is more responsive than the latter to \( \beta \lambda \), if there exists an interval \([a, b]\) in which \( \Sigma_1(M) - \Sigma_1(C) \) is decreasing, then that difference must be decreasing also in \((b, 1)\), where 1 is the highest possible value that \( \beta \lambda \) can take. As a consequence, if one proves that
\[
\frac{d}{d\beta \lambda} (\Sigma_1(M) - \Sigma_1(C)) \bigg|_{\beta \lambda = 1} > 0 \tag{21}
\]
then the proof is complete. After computing the derivative and rearranging terms, one obtains that (21) is implied by
\[
\left( \frac{1}{2} + \frac{3}{4} \frac{k(M)}{1 + \psi \Delta^2(M)} \right) - \left( \frac{1}{12} + \frac{1}{21 + \varphi \Delta^2(C)} \frac{k(C)}{k(M)} \right) > 0
\]
where \( k(M) = \Delta(M)/(2 - \Delta(M))^{-1} \), \( k(C) = \Delta(C)/(2 - \Delta(C))^{-1} \). The expression is increasing in the ratio \( \Delta(M)/\Delta(C) \). This ratio is minimized by fixing \( \psi = \varphi(1 - 2\varphi \mu^h)^{-1} \) at this point, \( \Delta(M) \) and \( \Delta(C) \) become, respectively \( \Delta[1 - 2\varphi \mu^h + \varphi \Delta] - 1 + 2\varphi \mu^h = 0 \) and \( \Delta[1 + \varphi \Delta] - 1 = 0 \). At this point, \( \Delta(M) \) is more responsive to \( \varphi \) than \( \Delta(C) \). As a consequence, the LHS of the inequality will be larger than
\[
\lim_{\varphi \to \infty} \left\{ \left( \frac{1}{2} + \frac{3}{4} \frac{k(M)}{1 + \psi \Delta^2(M)} \right) - \left( \frac{1}{12} + \frac{1}{21 + \varphi \Delta^2(C)} \frac{k(C)}{k(M)} \right) \right\} = \frac{1}{2} - \frac{1}{12} \lim_{\varphi \to \infty} \left\{ \frac{k(C)}{k(M)} \right\}
\]
using Leibnitz rule, one can verify that
\[
\lim_{\varphi \to \infty} \left\{ \frac{k(C)}{k(M)} \right\} = \lim_{\varphi \to \infty} \frac{d\Delta(C)/d\varphi}{d\Delta(M)/d\varphi} < 1
\]
which completes the proof.
Proof of Proposition 8

The FONC of the problem define the solution. For \( t = 1 \) \( X^R_t \) and \( X^L_t \) solve

\[
\begin{align*}
X^R_t & \in \arg \max_{x \in [0,1]} \{ p_t(x_0) + \beta E[p(X^S_1)] \} \\
X^L_t & \in \arg \max_{x \in [0,1]} \{ p_t(x_0) + \beta E[p(X^S_1)] \}
\end{align*}
\]

where \( E[p(X^S_1)] = 1/2 + \psi E[I(q)] + \psi \bar{\lambda} E[X^C_1] = 1/2 + \psi E[I(q)] + \psi \bar{\lambda} \{ \pi_1 X^R_t + (1 - \pi_1)X^L_t \} \) follows from the observation that \( d(X^R_2, X^L_2) = 0 \). The FONC are of the problem (which are also sufficient under the assumptions) define the following system

\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{d}{dX^R_t} p_t(x_0) + \beta \psi \bar{\lambda} \left\{ \pi_1 + (X^R_t - X^L_t) \frac{d\pi_t}{dX^R_t} \right\} = 0 \\
\frac{d}{dX^L_t} p_t(x_0) + \beta \psi \bar{\lambda} \left\{ (1 - \pi_1) + (X^R_t - X^L_t) \frac{d\pi_t}{dX^L_t} \right\} = 0
\end{array} \right.
\]

which simplifies to

\[
\begin{align*}
x^* - X^R_t + \beta \bar{\lambda} \left\{ \pi_1 + \Delta(S) \varphi (x^* - X^R_t) \right\} &= 0 \\
x^*_t - x^* + \beta \bar{\lambda} \left\{ (1 - \pi_1) + (X^R_t - X^L_t) \varphi (X^L_t - x^*) \right\} &= 0
\end{align*}
\]

which is the same system as in (14).

Proof of Proposition 9

i) First, observe that, from the previous proposition, \( \Sigma_1^{(S)} = \Sigma_1^{(C)} < \Sigma_1^{(M)} \). Next, I show that \( b_1^{(M)} > b_1^{(S)} \), which implies \( b_1^{(C)} > b_1^{(M)} > b_1^{(S)} \). To see that the inequality must hold, notice that it is equivalent to

\[
b_1(1 - p(X^R_{(M)}, X^L_{(M)}, \mu_1)) > b_1(1 - p(X^R_{(S)}, X^L_{(S)}, \mu_1))
\]

and follows from

\[
p(X^R_{(M)}, X^L_{(M)}, \mu_1) = \frac{1}{2} + \frac{\mu_1 \psi}{1 + \psi \Delta^2_{(M)}} < \frac{1}{2} + \frac{\mu_1 \psi}{1 + \psi \Delta^2_{(C)}} = p(X^R_{(S)}, X^L_{(S)}, \mu_1)
\]

to complete the proof, notice that \( \Sigma_2^{(S)} = \Sigma_2^{(C)} = \Sigma_2^{(M)} = 0 \) and, since \( E(x_{1}^{(M)}) < E(x_{1}^{(C)}) \)

\[
E(p_2) = 1/2 + \psi[I(q_2) + \lambda_2 E(x_{1}^{(M))}] > E(p_2) < 1/2 + \varphi[I(q_2) - \lambda_2 E(x_{1}^{(C))}]
\]

\[
E(b_2^{(S)}) < E(b_2^{(M)}) < E(b_2^{(C)}).
\]

ii) The generic welfare function is \( W = A + x_t(1 - A - x_t/2) + \alpha \omega + (1 - 2\alpha - \alpha q_t)b_t \). For \( \alpha > 1/(2 + q_t) \), the welfare ranking among constitutions is the same as with the utilitarian criterion. For \( \alpha < 1/(2 + q_t) \), larger redistribution is welfare improving. As a consequence (C) becomes the best constitution.
Proof of Lemma 3

We prove the dependence of equilibrium $p$ on $\omega$. First, notice that, in equilibrium $p = 1/2 + \psi \mu (1 + \psi \Delta^2_{(M)})^{-1}$. Denote by $\mu_\omega$, $\lambda_\omega$ the ratios $\mu/\omega$ and $\lambda/\omega$, which are independent of $\omega$. The total derivative of $p$ wrt $\omega$ is then $\frac{\partial}{\partial \omega} p + \frac{\partial}{\partial \Delta} p \frac{d}{d \omega} \Delta_{(M)}$. Using the implicit function theorem, $\frac{d}{d \omega} \Delta_{(M)} = \beta \lambda [1 + \psi \Delta^2_{(M)}][1 + 2\psi \Delta_{(M)} \beta \lambda \omega]^{-1}$. After a few steps of algebra, the total derivative simplifies to

$$\psi \frac{I_\omega + \bar{\lambda}_\omega x}{1 + \psi \Delta^2_{(M)}} \left[ 1 - \frac{2\psi \Delta_{(M)} \beta \lambda \omega}{1 + 2\psi \Delta_{(M)} \beta \lambda \omega} \right] > 0.$$  

The expected redistribution is $b_1(1 - p)$. Denote by $b_\omega$ the ratio $\bar{b}/\omega$ which is independent of $\omega$. Computing the total derivative and rearranging yields

$$b_\omega \left\{ \frac{1}{2} - \psi \frac{\mu_\omega \omega}{1 + \psi \Delta^2_{(M)}} \right\} - b_\omega \psi \frac{\mu_\omega}{1 + \psi \Delta^2_{(M)}} \frac{1 - \psi \Delta^2_{(M)}}{1 + \psi \Delta^2_{(M)}}$$

which is negative iff

$$\psi^{-1} < \frac{4}{1 + \psi \Delta^2_{(M)}} \mu$$

which, for $\mu$ high enough, holds without violating (4). The analog of this condition for (C), $\varphi^{-1} < 4\mu (1 + \psi \Delta^2_{(C)})^{-1}$, does violate (4).
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