



Limited liability and mechanism design in procurement[☆]

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ABSTRACT

In the presence of cost uncertainty, limited liability introduces the possibility of default in procurement. If financial soundness is not perfectly observable, then financially weaker contractors are selected with higher probability in any incentive compatible mechanism. Informational rents are associated with the probability of default. By selecting the financially weakest contractor, stronger price competition (auctions) may not only increase the probability of default but also the contractors' expected rents. Thus, weak conditions are sufficient for auctions to be suboptimal. In particular, we show that pooling at higher net worth may reduce the cost of procurement even when default is costless for the sponsor.

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1. Introduction

Contractor's default, in particular in the construction industry, is common. In the USA alone, during the period 1990–1997, more than 80,000 contractors filed for bankruptcy leaving unfinished private and public construction projects with liabilities exceeding \$21 billion (Dun & Bradstreet Business Failure Record). Default or bankruptcy can be costly for the sponsor implying delays in the completion of the project, litigation costs,¹ the cost of the new procurement process, etc. The empirical literature points to budgetary issues as one of the most important factors in explaining company failure accounting for 60.2% of company failures in the US construction industry (see *Arditi et al., 2000*). Bidders with a large budget are less likely to default, however they are also less likely to submit the winning bid in a procurement auction (see *Calveras et al., 2004*). In an environment with uncertainty over the cost of the project to be procured, limited liability cuts off the potential downside losses making bidders bid more aggressively. This effect is stronger for financially weaker bidders since they face a stochastically better distribution of gains for every fixed price.²

Public procurers have taken two different paths to deal with this problem. One approach is to attempt to screen the financial soundness of potential contractors to sort out risky contractors in competitive procurement. Requirements of surety

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¹ *White (1989)* reports direct administrative costs for liquidating a firm in the US for which bankruptcy courts keep record: these make up 7.5%–21% of the liquidation proceeds.

² Notice that this phenomenon is unrelated to the winner's curse effect, since it arises even if bidders' beliefs over the distribution of their cost of the project are independent of rivals' information.

bonds or other type of insurance to participate in the auction fall into this category (see Calveras et al., 2004). To be successful this approach requires a well functioning insurance market. Moreover, the cost of insurance falls onto the sponsor. Another (possibly complementary) approach is to alter the procurement mechanism to introduce features that aim at selecting less risky contractors. Examples are the disqualification of “abnormally low tenders (ALTs)”, i.e., bids below a certain endogenous threshold, or the non-monotone selection of bids, e.g. the selection of the bid closest to the average (see, for instance, Decarolis, 2009, and Engel and Wambach, 2006). These practices indicate that contractors have private information about their likelihood of default at the contracting stage and that this likelihood is asymmetric across contractors.

In this paper we argue that these asymmetric likelihoods of default are rooted in differences in financial conditions across contractors and study how mechanism design theory can guide a procurer who cannot or does not want to rely on indirect instruments to screen the financial soundness of firms. We model the procurement design problem of an indivisible contract assuming that potential contractors are protected by limited liability and have private information about their financial condition.³ A contractor knows the value of her assets, and defaults if completing the project would lead to losses that exceed that value. We assume that the cost of completing the project is common to all contractors, but is uncertain at the time of contracting and that ex-post costs are not contractible or too costly to verify.⁴ In other words, we assume cost uncertainty but abstract from efficiency differences in terms of production costs. This allows us to concentrate on the implications of the asymmetry of default risk across contractors, which is represented by the maximum losses that the contractor is willing/able to incur instead of defaulting.

We first study the set of feasible “winner-pays” mechanisms, that is, what the procurer may accomplish in terms of selecting financially sound firms by manipulating a contracting mechanism in which only payments from and to the selected contractor are allowed.⁵ Our first result is that feasible allocations are monotone in the “wrong” way: no matter how clever the procurer’s design is, a contractor with more assets cannot be selected with a higher probability than a contractor with fewer assets. Also, the contracted price in any feasible mechanism is weakly increasing in the value of the assets. Therefore, with ex-ante symmetric contractors in terms of efficiency, in any feasible mechanism the riskiest deal will be at least as likely to be selected as any other less risky one.

The result is intuitive and robust to modeling details. A contractor in a healthy financial situation has more to lose in case of bankruptcy. Put in other words, a contractor in a weak financial situation is more protected by limited liability. Thus, she is always more willing to trade price for probability of winning than a financially sounder firm.

We show that the monotonicity of prices and win probabilities that we have just discussed implies that mechanisms that select bidders that offer lower prices with higher probability also imply higher probability of bankruptcy. Thus, the mechanism that minimizes the probability of bankruptcy, and therefore the expected costs associated to bankruptcy, is a random allocation. This can be achieved, in the cheapest way, by posting a sufficiently high price.⁶ However, we also show that no matter how large these costs associated to bankruptcy are, the expected cost for the procurer is lower when allowing for some probability of default.

In the other extreme, the mechanism that minimizes the *contracted* price is the one that selects the contractor in the weakest financial situation with probability one. We show that an auction is one such mechanism. However, minimizing the contracted price is not the same as minimizing the informational rents. In this paper, we assume that in case of default the procurer seizes the assets of the failing contractor and incurs the realized cost of the project plus any costs associated to bankruptcy. The contractor defaults only if the realized cost of the project is larger than the price plus the value of her assets. Thus, in case of bankruptcy the total cost for the procurer, even net of bankruptcy costs, is higher than the contracted price. We show that mild conditions are sufficient to guarantee that an auction does not minimize the cost for the procurer even if bankruptcy costs are zero.

The paper uses mechanism design techniques in a setup where bidders’ preferences are not separable in money to establish two further results for any log-concave distribution function of costs: (i) in the optimal mechanism the payment and the allocation functions only depend on the financial assets of the winning firm. Hence, mechanisms such as second price auctions are suboptimal in this context. (ii) A revenue equivalence theorem holds: informational rents are uniquely determined by the rents of the financially strongest type and the allocation (the win probabilities). However, this relationship is highly nonlinear, and as a consequence the optimal contract will not be a simple one even under stringent conditions on the primitives of the model. Therefore, the practical implications from solving for the optimal mechanism might be rather limited.

Perhaps a more interesting exercise, in terms of practical interest, is the analysis of how simple distortions affect the expected total cost for the procurer. In order to discuss this, we analyze a discrete type model. We show that randomizing allocation among financially stronger types may result in a lower cost for the procurer even when bankruptcy brings no additional cost. Randomizing among the financially weakest types, on the contrary, cannot reduce the expected cost for

³ While some information about the capital stock/financial condition of the contractor may be publically available or available at a small cost, there is sufficiently residual uncertainty in applications, validating the assumption of private information.

⁴ If contracting on the realized cost was feasible, the problem would be trivial: default risk could be eliminated without cost.

⁵ This restriction which excludes all pay mechanism and hence entry fees is justified at length in Section 2 when we compare our paper to Chillemi and Mezzetti (2010).

⁶ Therefore, in our model, a sufficiently high posted price is an optimal mechanism for a procurer who cares about total surplus, and not about the cost of procurement.

the procurer. This indicates that commonly used distortions, like price floors, are not the type of distortions that help in reducing informational rents.

The remainder of the paper is organized as follows. In the next section we discuss the related literature and justify our restriction to winner-pays mechanisms. In Section 3 we present the model. Section 4 contains our main results for the continuous type model. In Section 5 we discuss the optimality of auctions in the absence of default costs and posted prices in the presence of large default costs. We also study the discrete case in this section. Section 6 concludes. All proofs are relegated to an online appendix.

2. Related literature

There is a sizable literature studying the effect of limited liability on the performance of first or second price auctions (e.g. [Waehrer, 1995](#); [Parlane, 2003](#); and [Board, 2007](#)).⁷ [Parlane \(2003\)](#) and [Board \(2007\)](#) compare the cost of procurement with limited liability in first and second price auctions, and show that the second price auction induces higher prices and higher bankruptcy rates. [Engel and Wambach \(2006\)](#) discuss the poor performance of mechanisms like average pricing, and provide an illustrative comparison between a lottery, a second price auction with a multi-source and a second price auction without switching costs.

In all these papers, private information is about the expected cost, and not about the maximum losses for the contractor. Auctions select the (ex-ante) most efficient firm even under limited liability. However, little is known about the optimal auction format since informational rents are just as complex, and are subject to the same amount of endogenous non-monotonicity as in our model (see [Parlane, 2003](#)).

[Zheng \(2001\)](#) assumes, as we do, that bidders' private information refers to their "budgets", i.e., the maximum losses they are willing/able to incur. He studies first price auctions and considers the effect of costly borrowing on the performance of the auction. [Chillemi and Mezzetti \(2010\)](#) analyze practically the same model as our paper but allow the procurer to use all pay contracts, in particular they study the use of entry fees. As [Parlane \(2003\)](#) had anticipated, this certainly alleviates the bankruptcy problem. [Chillemi and Mezzetti's \(2010\)](#) model assumes implicitly that "entry fees" do not aggravate the financial situation of contractors. Under this assumption they show that "selling fair lottery tickets" where the prize is equal to the maximum deviation of the ex-post cost of the project from its expected value trivially eliminates the problem of default. Since the exposure to default is the only source of asymmetric information, rent extraction is not a problem and this is indeed an optimal mechanism. If this sort of lotteries are feasible, the procurer should certainly use them to subsidize the selected contractor and reduce the bankruptcy problem. However, there are important issues that limit the applicability of [Chillemi and Mezzetti's \(2010\)](#) result. First, there are dynamic problems that have to do with the survival of firms that do not win in a short series of "lotteries" of this sort. Second and more importantly, even in a static context, entry fees have to be paid by the contractors and should therefore affect the contractor's financial situation. If the uncertainty about the cost is large and the number of contractors is small, avoiding default would mean very large "ticket" prices, which may exceed the value of the assets of at least some firms. Hence, a rigorous analysis of the problem requires to model the relationship between up-front payments and bidder's wealth which complicates the analysis considerably since this would imply that a bidder's wealth plays two roles in the problem: it plays a role for default and acts as a budget constraint during the bidding process. For the sake of tractability we abstract from this second role by studying winner-pays mechanism only which is a useful benchmark and also of particular practical interest for many procurement problems.

Our paper is also related to [Manelli and Vincent \(1995\)](#) in a subtle but meaningful way. That paper studies the design of procurement mechanisms in an environment where the valuations of the uninformed buyer over the potential suppliers' goods (quality) are positively correlated with the suppliers' opportunity cost of supplying the good. As in the present paper, auctions or price competition lead to awarding the project to the least preferred supplier and mechanisms with some randomization in the winning probabilities may be optimal. In our setting, bidders with worse financial status are those that would result in higher default probability for the procurer and then, other things (contract price) equal, are less attractive if default is costly. Although the relationship is less direct in our case, we can think of this bankruptcy exposure as a sort of lower quality from the procurer's viewpoint. However, the parallel breaks when we consider that even with no bankruptcy costs price competition might still be undesirable for the procurer. Indeed, in our model informational rents are more subtle and are linked to the probability of default, whether default is per se costly or not. This probability is higher when price competition is more intense. Thus, even under regularity assumptions on the inverse hazard rate, it may be in the interest of the procurer to blunt price competition.

3. The model

A risk-neutral buyer (sponsor) procures an indivisible contract for which he is willing to pay V , which we assume large enough as to make the possibility of no contracting unattractive. There are N risk-neutral potential contractors all with the

⁷ [Waehrer \(1995\)](#) studies a model of "liquidated damages" which can be interpreted as a model of procurement with limited liability by reversing the roles of seller and buyers. The maximum losses for a buyer, the contractor in our case, is determined by the seller, the procurer in our case, by including liquidated damages or a deposit in the contract. The paper characterizes equilibria in first and second price auctions and studies how the level of liquidated damages affects the seller's payoff.

same cost, c , unknown at the time of signing the contract. Thus, c is a random shock which we assume to be distributed on $[0, 1]$ with density $g(c)$ and distribution function $G(c)$. We assume that G is log-concave. That is, the reverse hazard rate g/G is non-increasing.⁸

Potential contractors differ in their initial financial status. Let $A_i \geq 0$ denote the value of the assets of potential contractor $i = 1, 2, \dots, N$. A_i is contractor i 's private information. Each A_i is an independent realization of a random variable with support $[\underline{A}, \bar{A}]$, with $\underline{A} \geq 0$, density function $f(\cdot)$, and distribution $F(\cdot)$. We denote by A the vector of contractors' types, $A = (A_1, A_2, \dots, A_N)$.⁹

Contractors have limited liability, i.e. the losses of contractor i cannot be larger than A_i . Therefore, if awarded the project and after c is realized contractor i will close down if A_i plus the net profits from undertaking the project, $(P - c)$, fall below 0.

The sponsor chooses a ("winner-pays") procurement mechanism to award the contract.¹⁰ After the contract is awarded at a price P , the cost c is realized and publicly observed and the selected contractor either finishes the project or declares bankruptcy. Bankruptcy may imply extra costs for the sponsor. We summarize these costs by a constant C_B , with $C_B \geq 0$. Thus, when the selected contractor declares bankruptcy the sponsor has to bear the realized cost c plus this bankruptcy cost, but can liquidate and seize the assets of the firm at that moment, $A_i + P$. Thus, if the sponsor signs a contract with contractor i (with assets A_i) at a price P and the realized cost is c , then the utility of the sponsor is

$$U^S = \begin{cases} V - P & \text{if } P - c + A_i \geq 0, \\ V - c + A_i - C_B & \text{otherwise.} \end{cases} \quad (1)$$

These payoffs can be considered as the reduced form payoffs of a continuation game where the sponsor asks for bids from the rest of contractors once the realized cost c has been revealed. We now summarize the timing of the model:

1. Nature chooses the net worth A_i of each firm. Each firm privately learns its net worth.
2. The sponsor announces the procurement process.
3. Contractors submit their "bids" or messages, and the project is awarded according to the rules announced by the sponsor. The price P is set according to these rules.
4. The cost parameter c is realized. If the net worth of the selected firm i is such that $A_i + P - c \geq 0$, then the firm finishes the project. Otherwise it declares bankruptcy.
5. The sponsor and the contractors realize their payoffs. The selected contractor retains a financial value of $\max\{0, A_i + P - c\}$, all other firms retain their assets, and the sponsor obtains a payoff defined in (1).

In the next section we determine the choice set for stage 2, that is, the set of incentive compatible mechanisms. We also discuss the constraints that incentive compatibility imposes on the *shape* of these mechanisms.

4. Implementable mechanisms

A mechanism in this model is a pair (σ, P) , where $P : [\underline{A}, \bar{A}]^N \rightarrow R^N$, and $\sigma : [\underline{A}, \bar{A}]^N \rightarrow [0, 1]^N$ and $\sum_{i \in N} \sigma_i = 1$. We interpret $\sigma_i(A)$ as the probability that contractor i is assigned the project when A is this vector. $P_i(A)$ represents the price of the contract if the vector of net worths (types) is A and contractor i is assigned the project. Thus, $P_i(A)$ is not the unconditional expected payment to contractor i . In an auction design problem without limited liability and default risk, that would be all that would matter both to the contractor and to the sponsor. On the contrary, in our problem it is the joint distribution of contract allocation and payments that matters. Note that we are implicitly assuming that the price is not random, once we condition on the types of all contractors and the identity of the winner. That is, we consider mechanisms as the second price auction, but not mechanisms where the price depends on random devices that are unrelated to the primitives of the problem. This is only for simplicity. In fact, in the proof of Proposition 1 we extend the definition to include this latter class of mechanisms and then show that the restriction to these mechanisms is without loss of generality.

The sponsor faces participation or individual rationality constraints, IR: for all i and for all A_i ,

$$U_i(A_i; \sigma, P) \equiv E_{A_{-i}} \sigma_i(A) \left[\int_0^{\min\{1, P_i(A) + A_i\}} (P_i(A) + A_i - c) g(c) dc - A_i \right] \geq 0. \quad (2)$$

Also, the sponsor faces incentive compatibility constraints, IC, which in this setting means that for all A_i and \hat{A}_i , and all i ,¹¹

⁸ This assumption is needed for Proposition 1. We will discuss the role of this assumption after establishing the proposition.

⁹ This way of modeling bidder-heterogeneity is due to Che and Gale (1996) who model bidder-heterogeneity in terms of wealth instead of value in a forward auction. It is also used in Zheng (2001).

¹⁰ The restriction to these type of procurement contracts was discussed at length in Section 2.

¹¹ Note that we are implicitly assuming that bankruptcy cannot be claimed when funds are sufficient to cover the cost. That is, that assets cannot be hidden once the supplier has applied for bankruptcy.

$$U_i(A_i; \sigma, P) \geq E_{A_{-i}} \sigma_i(A_{-i}, \widehat{A}_i) \left[\int_0^{\min\{1, P_i(A_{-i}, \widehat{A}_i) + A_i\}} (P_i(A_{-i}, \widehat{A}_i) + A_i - c)g(c) dc - A_i \right].$$

The sponsor's goal is to minimize the cost of the project. That is, to minimize

$$E_A \sum_i \sigma_i(A) \left[P_i(A) + \int_{\min\{1, P_i(A) + A_i\}}^1 (c + C_B - P_i(A) - A_i)g(c) dc \right].$$

Parlane (2003) has argued that randomness does not benefit the sponsor. In our model, that is guaranteed by log-concavity of G .

Proposition 1. Assume log-concavity of $G(c)$. Then for any IC, IR mechanism (σ, P) , there exists a mechanism (σ, \bar{P}) , where $\bar{P}_i(A)$ is constant on A_{-i} , (σ, \bar{P}) is also IC and IR, results in the same expected rents for each contractor i and each value of A_i , and results in expected (weakly) lower cost for the sponsor.

Proof. See online appendix. □

Remark 1. While Proposition 1 is stated as a weak inequality, its proof is constructive and provides conditions when the inequality is strict. Indeed, the constructed mechanism (σ, \bar{P}) results in strictly lower costs for the sponsor quite generally. In particular, a second price auction results in strictly higher cost than a first price auction as long as the former has a monotone equilibrium.¹²

Proposition 1 allows us to pay attention to each contractor's expected probability of winning the contract, conditioning only on her type, what we denote by $\psi_i(A_i) = E_{A_{-i}} \sigma_i(A)$.

The intuition for the proof of Proposition 1, and so the need for our assumption of log-concavity of G , can be illustrated in a simple way.¹³ Consider a contractor in isolation with given net worth that we can simply assume equal to 0. Suppose that we start with a fixed price P and consider switching some probability (or all) to a lottery of two prices $P_1 > P > P_2$, each with equal probability, so that the contractor is indifferent between the lottery and the price P . That is, so that

$$(P_1 - P)G(P) + \int_P^{P_1} (P_1 - c)g(c) dc = (P - P_2)G(P_2) + \int_{P_2}^P (P - c)g(c) dc.$$

The right hand side is the loss for the contractor when the realization of the lottery is P_2 and the left hand side is her gain when the realization of the lottery is P_1 , both with respect to the fixed price P . Integrating by parts, this means that

$$\int_P^{P_1} G(c) dc - \int_{P_2}^P G(c) dc = 0.$$

Now, if the two mechanisms result in the same rents for the contractor, then from the sponsor's viewpoint the only difference between the two is the probability of bankruptcy. The difference between the probability of bankruptcy with the fixed price and the probability of bankruptcy with the lottery is simply

$$\int_P^{P_1} g(c) dc - \int_{P_2}^P g(c) dc.$$

If G is log-concave, that is, if G/g is nondecreasing, this difference is negative since

$$\begin{aligned} 0 &= \int_P^{P_1} G(c) dc - \int_{P_2}^P G(c) dc \\ &= \int_P^{P_1} \frac{G(c)}{g(c)} g(c) dc - \int_{P_2}^P \frac{G(c)}{g(c)} g(c) dc > \frac{G(P)}{g(P)} \left(\int_P^{P_1} g(c) dc - \int_{P_2}^P g(c) dc \right). \end{aligned}$$

¹² This can be easily shown as a corollary of our Propositions 4 and 5 below.

¹³ We thank the Advisory Editor for pointing out a mistake in a previous version and inspiring this discussion, including the example.

Note that the argument goes through for any lottery on two prices, and can be replicated for any value of A . Also, note that the nature of the lottery, whether based on other contractors' types or any exogenous device, is of no importance for this result.

On the other hand, the result requires sufficient smoothness on G , guaranteed by log-concavity. Assume that G presents an atom at P_1 , so that the probability that $c = P_1$ is $\alpha > 0$, and for simplicity assume that the probability that the cost is between P_1 and P_2 is (almost) zero. In this case the sponsor may design a lottery so that with probability μ the price is (just above) P_1 and with probability $(1 - \mu)$ the price is P_2 . The lottery should satisfy $(P - P_2)G(P_2) = \mu(P_1 - P)G(P_2)$: if the cost is $c = P_1$ the contractor makes zero profits in both cases. Note that the number μ exists in $(0, 1)$ if $P - P_2 < P_1 - P$. With respect to the fixed price P , such a lottery reduces the probability of bankruptcy by $(1 - \mu)\alpha$, the probability that the cost is $c = P_1$ and the price realization is (just above) P_1 .

Incentive compatibility typically imposes monotonicity in the allocation of the contract: stronger competitors necessarily obtain the contract with (weakly) higher probability. This is also the case in this setting. However, here a "stronger" competitor is one that is better protected against downside losses, that is, less solvent firms. Indeed,

Proposition 2. *If (σ, P) is IC, then $U_i(A_i; \sigma, P)$ is continuous and monotone decreasing in A_i . Monotonicity is strict if $\bar{c} = 1 > P_i(A_i) + A_i$.*

Proof. See online appendix. \square

The intuition for this result is simple: a contractor can always "imitate" the equilibrium behavior of a financially stronger contractor, and obtain the same probability of winning the contract and at the same price. The incremental profits of the former contractor in case of winning are larger, since the financially weaker contractor loses less in expected terms when this price is not sufficient to cover the realized cost. Indeed, the maximum losses coincide with the net worth of the firm. That is, informational rents are linked to low asset holdings, not to solvency. Our next proposition is another consequence of this. It unveils the "perverse effect" of limited liability.

Proposition 3. *In any IC mechanism $\Psi_i(A_i)$ is (weakly) monotonically decreasing and $P_i(A_i)$ (weakly) monotonically increasing.*

Proof. See online appendix. \square

When faced with a menu of contracts (price and probability of winning) financially weaker types will opt for contracts with a higher probability of winning and lower prices, and financially stronger types will choose contracts with a lower probability of winning and higher prices.

Besides imposing monotonicity, IC eliminates one degree of freedom in the procurer's choice. That is, a "revenue equivalence" result holds in this setting.

Proposition 4. *Two mechanisms that share $\Psi_i(A_i)$ and give the same rents to bidders of the financially strongest type also share $P_i(A_i)$.*

Proof. See online appendix. \square

Proposition 4 tells us that $U_i(A_i; \sigma, P)$ is completely determined by $\Psi_i(A_i)$ and $U_i(\bar{A}; \sigma, P)$ as follows:

$$U_i(A_i; \sigma, P) = U_i(\bar{A}; \sigma, P) + \int_{A_i}^{\bar{A}} [1 - G(\min\{1, P_i(x) + x\})] \Psi_i(x) dx. \quad (3)$$

Recall that the objective function for the sponsor can be written as the sum of the expected cost of the project, contractors' expected utility, and expected bankruptcy costs:

$$E\{c\} + \sum_i E_A U_i(A_i; \sigma, P) + E_A \sum_i \sigma_i(A) [1 - G(\min\{1, P_i(A_i) + A_i\})] C_B. \quad (4)$$

Using (3), we can rewrite (4) as:

$$E\{c\} + \sum_i U_i(\bar{A}; \sigma, P) + \sum_i E_{A_i} \left[\int_{A_i}^{\bar{A}} \Psi_i(x) [1 - G(\min\{1, P_i(x) + x\})] dx + \Psi_i(A_i) [1 - G(\min\{1, P_i(A_i) + A_i\})] C_B \right].$$

Notice that the third term of this expression captures the informational rents and the bankruptcy costs incurred. This third term can be written as

$$\sum_i \int_{\underline{A}}^{\bar{A}} \left[\int_{A_i}^{\bar{A}} \psi_i(x) [1 - G(\min\{1, P_i(x) + x\})] dx + \psi_i(A_i) [1 - G(\min\{1, P_i(A_i) + A_i\})] C_B \right] dF(A_i)$$

or, changing the order of integration,

$$\begin{aligned} & \sum_i \int_{\underline{A}}^{\bar{A}} \psi_i(A_i) [1 - G(\min\{1, P_i(A_i) + A_i\})] \left(\frac{F(A_i)}{f(A_i)} + C_B \right) f(A_i) dA_i \\ &= \sum_i \int_{\underline{A}}^{\bar{A}} E_{A_i} \sigma_i(A) [1 - G(\min\{1, P_i(A_i) + A_i\})] \left(\frac{F(A_i)}{f(A_i)} + C_B \right) f(A_i) dA_i \\ &= E_A \sum_i \sigma_i(A) [1 - G(\min\{1, P_i(A_i) + A_i\})] \left(\frac{F(A_i)}{f(A_i)} + C_B \right). \end{aligned} \tag{5}$$

Hence the sponsor's objective function is

$$E\{c\} + \sum_i U_i(\bar{A}; \sigma, P) + E_A \sum_i \sigma_i(A) [1 - G(\min\{1, P_i(A_i) + A_i\})] \left(\frac{F(A_i)}{f(A_i)} + C_B \right).$$

This reformulation allows us to understand some aspects of the optimal mechanism:

Remark 2. For sufficiently high C_B it will be optimal to leave some rents to the financially strongest type.

This might seem counterintuitive, since the financially strongest type poses no incentive problems and individual rationality would be satisfied for $U_i(\bar{A}; \sigma, P) = 0$. For $C_B = 0$, it is indeed optimal to set $U_i(\bar{A}; \sigma, P) = 0$. However, $U_i(\bar{A}; \sigma, P)$ also determines the probability of default, namely $[1 - G(\min\{1, P_i(A_i) + A_i\})]$, and hence it affects bankruptcy costs. Thus, it may be in the sponsor's interest to leave some rents to the financially strongest type if this reduces the probability of default sufficiently and bankruptcy costs are high enough. To see this assume that $C_B \rightarrow \infty$. In this case it is optimal to avoid bankruptcy almost surely, which can be achieved only if $P_i(\underline{A})$ is sufficiently close to $1 - \underline{A}$. This in turn implies that $P_i(\bar{A}) + \bar{A} > 1$, so that, by IC, $U_i(\bar{A}) > 0$.

Remark 3. Informational rents need not be monotone and depend on the mechanism itself.

To see this notice that expression (5) is reminiscent of the “virtual cost” in the standard procurement problem once IC constraints are taken into account and captures informational rents only when $C_B = 0$. We know from Proposition 3 that $P_i(A_i)$ is monotonically increasing, hence $P_i(A_i) + A_i$ also increases in A_i . Therefore, the probability of default, namely $[1 - G(\min\{1, P_i(A_i) + A_i\})]$, is decreasing in A_i . So financially lower types are always associated with higher bankruptcy costs. However, this monotonicity may be lost in the term that represents informational rents since $[1 - G(\min\{1, P_i(A_i) + A_i\})] \frac{F(A_i)}{f(A_i)}$ may not be monotone even if the inverse hazard rate of F is monotonically increasing.¹⁴ Moreover, whether the term is monotone or not depends on the mechanism itself, i.e., on $P_i(A_i)$, and not only on exogenous primitives.¹⁵ An important consequence of this is that, even when $C_B = 0$ the optimal mechanism need not be the one that selects the financially lowest type, as we will show in the next section.

5. The (sub)optimality of auctions and the design of good and simple mechanisms

The main trade-off the sponsor faces is between informational rents and bankruptcy costs. If bankruptcy costs are important, it is easy to see that avoiding price competition might be attractive for the sponsor. In this case it may be optimal for the sponsor to pool all types and hence give financially stronger bidders the same probability of winning the contract

¹⁴ This is the main difference between our problem and the problem analyzed by Manelli and Vincent (1995) where price competition also may select bad types. However, in Manelli and Vincent (1995) informational rents are monotone and can be represented by the inverse hazard rate.

¹⁵ That is, “ironing” techniques a la Myerson (1981) are not an instrument to deal with this lack of monotonicity. This makes the design of an optimal mechanism complex even in the absence of default costs, and implies that the optimal mechanism will be also complex and will depend on fine details of the model.

as financially weak bidders. However, we will now show that even with extremely high default costs it is not optimal to avoid bankruptcy completely. This can be attained by posting the (minimum) price sufficient to avoid default by even the financially weakest contractor \underline{A} , namely $\bar{P} = 1 - \underline{A}$. Notice that the expected cost for the buyer when using an arbitrary posted price $P \leq 1 - \underline{A}$ is

$$P + \int_{\underline{A}}^{1-P} (1 - G(P + A)) \left(\frac{F(A)}{f(A)} + C_B \right) f(A) dA,$$

with derivative with respect to P

$$1 - \int_{\underline{A}}^{1-P} g(P + A) \left(\frac{F(A)}{f(A)} + C_B \right) f(A) dA,$$

which is positive at $P = \bar{P} = 1 - \underline{A}$. Thus, even for large C_B , allowing for some probability of bankruptcy is better than insisting on avoiding it completely.¹⁶

If bankruptcy costs are not important, the sponsor is mainly interested in minimizing informational rents. Surprisingly, even in this case reducing price competition might be an attractive strategy. Before proving this, we establish that a first price auction maximizes price competition and hence selects the contractor with the lowest net worth.

Proposition 5. For simplicity, assume $\bar{A} > E\{c\} = \frac{1}{2}$. A first price auction has a symmetric equilibrium, $b(A)$, where $b(A)$ is strictly increasing in A for $A < \frac{1}{2}$, and $b(A) = \frac{1}{2}$ for $A \geq \frac{1}{2}$.

Proof. See online appendix. \square

The condition $\bar{A} > E\{c\} = \frac{1}{2}$ simplifies the proof of existence but is not crucial for the result.¹⁷ Contractors with assets in excess of $\frac{1}{2}$ will never default in any IR mechanism, and then will always be pooled (i.e., will be selected with the same probability at the same price) in any IC mechanism. Thus, an auction indeed maximizes price competition. The following proposition shows that under weak conditions an auction will be suboptimal.

Proposition 6. For any $C_B \geq 0$, there exist non-strictly monotone, IC mechanisms that result in higher surplus for the sponsor than mechanisms that assign the contract to the contractor with lowest A (auctions) if $f'(A)$ exists and is negative for A close to \bar{A} and $g'(c) \leq 0$.¹⁸

Proof. See online appendix. \square

The proof of Proposition 6 is constructive: the mechanism that outperforms the auction pools an interval of the financially strongest types. We can gain some further intuition for this result by looking at the difference between contract price and realized price, i.e., the price paid to the contractor that finalizes the contract. When increasing the probability that financially stronger types win the contract by, say, pooling a certain interval of types, the mechanism does not only result in a higher contract price but also in a higher expected value of the winner's net worth. Thus, it increases the probability that the contract price will coincide with the realized cost for the sponsor. In contrast, the auction selects the contractor with lowest net worth, so that although it minimizes the contract price it also maximizes the probability that the realized cost for the sponsor exceeds that price. This is so even in the absence of bankruptcy costs, $C_B = 0$.

In the proof of Proposition 6 we construct a mechanism that outperforms auctions and that pools the financially stronger bidders, hence a mechanism that pools at the top. To get a better intuition why this might work, it will be instructive to substitute a discrete type space for the continuous type space that we have been studying up to this point. In particular, assume that A can take three values, A_k , $k = 0, 1, 2$, and for simplicity assume that $A_0 = 0$, $A_1 \in (0, \frac{1}{2})$ and $A_2 = \frac{1}{2}$. We denote the probability of the three types, respectively as α_0 , α_1 , and $\alpha_2 = 1 - (\alpha_0 + \alpha_1)$. Also for simplicity, we will assume that there are only two contractors and the cost is uniformly distributed over $[0, 1]$, $G(c) = c$.¹⁹

¹⁶ Posted prices do not select based on the type. Therefore as the number of contractors grows the probability of bankruptcy under the optimal posted price does not approach zero.

¹⁷ On the one hand, \bar{A} can be arbitrarily close to $\frac{1}{2}$. Also, the assumption $\bar{A} > \frac{1}{2}$ is virtually equivalent to an atom at $\frac{1}{2}$. On the other hand, we assume $E\{c\} = \frac{1}{2}$ for determining the support of the equilibrium bidding function. An arbitrary $E\{c\}$ may affect such support but not the equilibrium existence.

¹⁸ The condition $g'(c) \leq 0$ is stronger than necessary. The proof requires that $g'(c) < 0$ is satisfied only in the upper tale and hence Proposition 6 should work for distributions as, e.g., the truncated normal.

¹⁹ It will become clear below that none of these assumptions, not even the three-type assumption, is restrictive.

A mechanism in this setting can be represented by a set of six values, (ψ_0, ψ_1, ψ_2) and (P_0, P_1, P_2) that denote the probability of winning and the price for each of the three types. Notice that we are only considering symmetric mechanisms. Also, notice that ψ_k is positive for all k , since in case both contractors draw the same type each is assigned the contract with probability one half (the contract is always allocated). That is, $\psi_k \geq \frac{\alpha_k}{2}$. Individual rationality for type 2 amounts to $P_2 \geq 1/2$. The fact that the probability of allocation is one also eliminates one degree of freedom in the choice of ψ_k . We begin our analysis by isolating how informational rents behave.²⁰

The key to understanding how informational rents of the different types change when we move from an auction to a pooled mechanism is to keep in mind that type 0 obtains informational rents due to the possibility of imitating type 1, while type 1 obtains informational rents due to the possibility of imitating type 2. When we move from an auction to pooling at the top, the informational rents of type 1 increase because the probability that type 2 obtains the contract is larger in the pooled mechanism and the price is the same in both mechanisms. This argument is independent of the number of types and in particular of the existence of type 0.²¹ However, pooling the two financially strongest types also has an effect on the rents of type 0 by reducing the probability that type 1 obtains the contract but increasing the price type 1 receives. Therefore, the probability that type 0 still obtains the contract if she decides to imitate type 1 is also reduced although the price she gets in that case is higher. We know that this change in probability and price is profitable for type 1 itself, but type 0 is relatively more interested in probability of winning than in price. So, depending on the underlying parameters pooling at the top may well reduce bidder 0's rents. And in that case the increase in the rents of type 1 may be outweighed by the decrease in the rents of type 0.²² Thus, pooling of the two financially strongest types may reduce informational rents and the resulting expected price for the sponsor.²³

On the other hand, pooling at the bottom cannot lead to a reduction in informational rents compared to an auction. If the two financially weakest types are pooled, then type 1 would still expect the same rents as in the auction, since its source of informational rents, namely type 2's price and probability of winning, is not affected. Moreover, these rents would now come from an increased probability of obtaining the contract and a lower price. If that trade-off leaves type 1 indifferent, then an imitating type 0 will prefer the higher probability. Thus, the rents of type 0 would certainly be higher in the mechanism that pools the two low types. Moreover, this effect of pooling the two financially weakest types is independent of whether type 1 is itself pooled to type 2 or not. That is, total pooling (a posted price $P = \frac{1}{2}$) always leaves more rents to contractors than a mechanism that only pools the two financially strongest types.²⁴

This discussion tells us that, in a procurement setting with limited liability but *small default costs* blunting price competition may be desirable at the top, but not at the bottom. Therefore, price floors, which are often used by public administrations, would tend to blunt competition at the wrong side.²⁵ In practice, pooling at the top could be obtained by requiring a minimum discount in the auction. Consider the continuous case again: assume that the sponsor announces (i) a maximum price of $\frac{1}{2}$ and (ii) that all bids that are above some price $\hat{P} < \frac{1}{2}$ will be treated as the same bid. It is simple to extend the proof of Proposition 5 to show that to each such \hat{P} corresponds a cut-off value \hat{A} such that types above \hat{A} bid $\frac{1}{2}$ and types below \hat{A} use a strictly monotone bidding function. The effect of this "distortion" is to reduce the probability of bankruptcy for these financially strong types. In a standard auction problem distorting the allocation for low valuation types by reducing their probability of winning causes a reduction in the rents of all higher valuation types. In a similar way, here "distorting" the allocation of high A types by reducing their probability of default causes a reduction in the rents of lower A types.

Not surprisingly, as the cost of bankruptcy C_B increases, pooling of types becomes even more attractive. Also, the relative impact of pooling at the top and at the bottom becomes less clear cut. Our discussion above has shown that pooling at the top increases the price (and thereby decreases the probability of default) for the pooled types but might reduce all prices (and hence increase the probability of bankruptcy) for financially weaker types. This latter effect is absent when pooling at the bottom. However, the prices of the pooled types move in different directions. On the one hand, pooling at the top

²⁰ For an analytical derivation of informational rents see the working paper version of the present article, Burguet et al. (2009).

²¹ This immediately implies that when $C_B = 0$ the optimal mechanism with only two possible types must minimize the probability that the financially strong type obtains the contract, i.e., assign the contract to the financially weak type.

²² Numerical examples for both possibilities can be found in the working paper version of this article (Burguet et al., 2009). In a sense this pooling at the top is similar to reserve prices in normal auctions in that it may reduce the rents of "higher" types by changing the allocation for "lower" types. However, as the above discussion shows, the trade-offs in our context are more subtle.

²³ It is easy to see that this argument extends to a general K -type case. Assume that there are additional types A_3, \dots, A_K , with $A_k > A_{k-1}$. Pooling the two financially strongest types will always increase the rents of the second financially strongest type: the value of ψ_K goes up and P_K remains unchanged, determined by the IR constraint for type K , so that by imitating type K type $K-1$ can guarantee higher rents. However, every other type below $K-1$ might see her rents reduced. On the one hand, the values of ψ_k for $k < K-1$ are unaffected. On the other hand, using exactly the same argument as in the three-type case for type 0, the rents for type $K-2$ might be lower, which means that P_{K-2} would also be lower. Iterating this argument, we conclude that in such case P_k would be lower for all types $k < K-1$.

²⁴ This argument generalizes to a k -type space where there are additional types A_3, \dots, A_K , with $A_k > A_{k-1}$. Since the win probabilities of the other types $k > 1$ are unaffected, pooling of types 0 and 1 (and consequently, changing both P_0 and P_1 so that the IC constraints for these types still hold with equality), would result in the same effects discussed in the three-type case. Thus, for arbitrary discrete type spaces we conclude that pooling the financially weakest types will never be optimal when $C_B = 0$. A trivial corollary is that total pooling will never be optimal either under these circumstances.

²⁵ Once we have pooled the two financially weakest types, we could consider pooling the third financially weakest type as well. The argument used to show that pooling the two financially weakest types increases the rents of contractors shows that this could only further increase the contractors' expected rents. Thus, a price floor, whether it only pools the two financially weakest types or some larger set of types at the bottom, is never optimal.

increases the probability of obtaining the contract for higher types, and therefore these types see their price reduced (and hence their probability of default increased). On the other hand, when pooling at the bottom the price for the financially weakest type, who now has a lower probability of obtaining the contract but has to be guaranteed higher rents, must increase (and consequently its probability of default must decrease). It is not difficult to construct examples where one or the other partial pooling is more effective in reducing the probability of default.²⁶

6. Conclusions

We have shown that limited liability results in a perverse selection in procurement when the cost of the project is common but uncertain, and contractors differ in their financial strength. Indeed, in this case incentive compatibility implies that selecting the financially more sound firm is not feasible. In fact, the stronger the price competition the more likely it is to select the financially weakest contractor. This is an unfortunate aspect of price competition when the costs of default are high. Perhaps more surprisingly, even if these costs are inexistent, fiercer price competition, and so a higher likelihood of selecting financially weaker firms at lower contract prices, may be against the interest of the sponsor. This happens because informational rents, and not only default costs, are linked to the probability of default. Thus, mechanisms that give rise to higher probability of default may also result in higher informational rents. We have provided sufficient conditions under which the sponsor will always prefer to curtail price competition somehow when the space of types is a continuum. These are far from necessary. By considering a finite type space, we have argued that limited liability may give some foundations to the usual practice in public procurement of blunting price competition even if default costs are low. However just what sort of limits to price competition are appropriated is a delicate issue. For instance, our discussion shows that price floors, which are commonly used by some public procurers and tend to pool firms at the lower levels of financial strength, may be counterproductive.²⁷

We can consider several extensions of our basic model. We have assumed that the default has no costs for the contractor beyond losing her assets. This is the nature of limited liability. However, there is a difference to be made here between firm and owner. Perhaps the owner incurs further opportunity costs associated with dismantling the firm's organization in the form of an option value, in case things get better in the future and there are costs associated to creating a new firm. If the owner (not the firm) possesses resources in excess of these costs, our analysis still applies to this case. Indeed, the owner would prefer recapitalizing the firm and not declaring bankruptcy unless these extra costs are lower than the difference between completion cost and value of assets. Thus, it suffices to redefine A_i as the sum of the value of firm's assets and the opportunity cost of keeping a firm to apply our propositions to this case. The analysis will be equivalent to assuming that the procurer does not fetch the whole value of the firm's assets under default. That is, the analysis is equivalent to having a larger C_B . Things would be slightly different if the owner does not have or cannot raise financial resources so as to avoid bankruptcy even if she wanted to. In that case, things may be more complex, and in particular IC may not even imply monotonicity of implementable mechanisms.

In our paper, we have focused on the role of a firm's assets as guarantee against adverse realizations of production costs. Financial assets also play a role as budget constraints when firms have to pay up-front in order to win a contract. These two roles appear together, for instance, in competition for concessions, where the winner pays in order to receive the rights for future business profits. As in our setting, financially weaker bidders may have larger willingness to pay for the concession in an auction, but they are more financially constrained in the bidding process than solvent firms. Therefore, in such setup the perverse effect of bidder's wealth (Proposition 3) is no longer a necessary condition for incentive compatibility. Indeed, Zheng (2001) has shown in a first price auction where wealth plays both roles of budget constraint and a limit to liabilities that there may be equilibria in which the wealthiest bidder bids most aggressively whereas in other equilibria bidders with lower wealth bid higher.²⁸

Finally, our analysis has abstracted from efficiency differentials. Allocational efficiency is usually appropriately managed by price competition. Thus, our results should be handled with care. When efficiency differentials are suspected to be important, one needs to search for the optimal balance between the two goals of selecting the most efficient firm and the financially fittest. This is an issue left for further research.

Supplementary material

The online version of this article contains additional supplementary material.
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²⁶ See Burguet et al. (2009).

²⁷ Another mechanism that is commonly used to limit price competition is the average bid auction in which the bidder closest to the average bid wins. Decarolis (2009) provides empirical evidence using data on the road construction industry in Italy that the average bid auctions have generated both significant inefficiencies in contract's allocation and high costs of procurement.

²⁸ We thank an anonymous referee for pointing this out.

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