Constrained School Choice: An Experimental Study

By Caterina Calsamiglia, Guillaume Haeringer, and Flip Klijn*

School choice programs offer families a say in the assignment of their children to public schools. Inspired by the successes in the design of markets for physicians (Alvin E. Roth and Elliott Peranson 1999), the matching literature has shown to be able to propose concrete solutions to design school choice mechanisms—Atila Abdulkadiroğlu and Tayfun Sönmez (2003).

In this paper we show that a seemingly unimportant aspect of school choice mechanisms turns out to have strong effects on their performance. The literature traditionally assumes that families can submit a list of all schools that are acceptable for them. However, in many actual applications families are allowed to submit a list containing only a limited number of schools. For instance, in the school district of New York City students are assigned to about 500 school programs, and parents are asked to compose a preference list containing only a maximum number of schools (currently 12). Other examples include college admissions in Spain and Hungary where students cannot submit a choice list containing more than eight and four academic programs, respectively. This apparently innocuous restriction is reason for concern. When an individual’s choice is constrained by the number of schools he can include in his choice list, the risk that the mechanism may exhaust the options listed becomes nonnegligible. In particular, if he fears rejection by his most preferred programs, it can be advantageous not to apply to these programs and use the allowed application slots for less preferred programs. These decisions are made under uncertainty and therefore ex ante safe optimal strategies may lead to inefficiencies if individuals do not apply to schools that would have accepted them. The literature provides clear predictions in terms of stability, efficiency, and behavior only when choice is not constrained. Imposing a curb on the length of the submitted lists, though having the apparent merit

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1 See also Abdulkadiroğlu, Parag A. Pathak, and Roth (2005) and Abdulkadiroğlu, Pathak, Roth, and Sönmez (2005).

2 Students that are not assigned a seat are asked to produce a second list containing only schools with vacant seats.

3 In Spain and Hungary colleges are not strategic, for the priority orders are determined by students’ grades. So college admission in these countries is, strictly speaking, akin to school choice.

4 Obviously, if the number of acceptable schools for individuals is smaller than the number of slots in the list there is no problem. However, in case the number of slots is restrictive, individuals will have to think out which schools to include and in which order. Abdulkadiroğlu, Pathak, and Roth (2005) report that in New York City about 25 percent of the students exhausted their list in academic year 2003–2004, which suggests that the constraint was indeed binding for a substantial proportion of students.

5 Haeringer and Klijn (2009) provide an equilibrium analysis for the constrained setting which makes clear that the predictions from the unconstrained setting do not carry over in a straightforward way.
of simplifying matters, has the perverse effect of forcing participants not to be truthful, reducing the efficiency and stability of the mechanisms.\textsuperscript{6} Perhaps more important is the fact that when choice is constrained the risk of remaining unassigned or being assigned to a school one likes very little makes the game more risky, which may cause families to complain and ultimately the market to fail.\textsuperscript{7}

The goal of our experimental study is to propose an overhaul of the most prominent matching mechanisms when we impose a maximal length of the submitted preference lists. We focus on three matching mechanisms: the Boston (BOS), the Student Optimal Stable Matching (SOSM), and the Top Trading Cycles (TTC) mechanisms. These mechanisms (and their variants) are or have been employed or proposed in many US school districts. They are also the subject of many theoretic, experimental, and empirical studies, virtually all of them assuming the absence of a constraint on the length of submitted preference lists.

We experimentally study the school choice problem using the experimental design initially proposed by Yan Chen and Sönmez (2006) but considering two cases, one in which subjects are not constrained by the number of schools they can include in the list, and one in which they can submit a list containing no more than three schools. When choice is constrained individuals face the risk of not being assigned to any school, in which case they receive only their participation fee.\textsuperscript{8} An interesting feature of the experimental design we consider is the presence of district schools. For each student in the matching game one of the schools is his district school, i.e., the school for which he has the highest priority. For SOSM and TTC the district school for a student plays the role of a safety school, i.e., a school to which he is sure to be matched whenever he asks it. Accordingly, the stakes differ across individuals, depending on how high their safe option is ranked in their preferences. To streamline the analysis we thus partition the subjects into two samples. The first sample, which we call the high-district sample, contains all subjects for whom the district school is ranked high in their preferences. For these subjects constraining their choice does not affect their incentives to be truthful. Furthermore, they are ensured to obtain a high payoff since their own district school is the easiest to be assigned to in any of the mechanisms. The low-district sample consists of the remaining subjects. For them the constraint hampers their prospects, and choosing an appropriate strategy is far more complex.

Our experimental study aims at contributing to both the behavioral and the matching literatures. From the behavioral perspective our experiment provides evidence of several stylized facts. We first observe that when individuals are constrained, the risk of being unassigned if they do not pick an appropriate list induces them to use less often a (weakly) dominated strategy.\textsuperscript{9} But on the other hand high-district sample subjects use the (weakly) dominant strategy less often when the constraint is imposed. Second, we observe in general more signs of optimizing behavior from the low-district subjects than from the high-district subjects.\textsuperscript{10} Third,
we introduce a taxonomy of biases to characterize strategic choices (e.g., altering the relative position of the district school). Not surprisingly, the introduction of a constraint on the choice exacerbates the intensities of the different biases we consider. However, a more startling observation is that the predominant bias, and whether biases are correlated or not, change with the experimental treatment. We find that the structure of biases does not only differ between the low and high-district samples but also between the constrained and unconstrained settings. This suggests that constraining choice affects the way individuals apprehend the game and thus the way they elaborate their strategic choices. Also, observed differences between high- and low-district subjects suggest that the size of stakes may significantly affect individual behavior.

From the matching literature perspective, our experiment aims at probing the mechanisms when played with and without the constraint on the size of the submitted preference lists. When choice is constrained we observe that for SOSM and TTC the number of participants who reveal their preferences truthfully up to their district school in the constrained case is reduced very significantly. However, we also observe that not all subjects use their (weakly) dominant strategy whenever available, and many of the nonoptimal manipulations turn out to be due to the presence of asymmetries in school capacities. The latter suggests that details such as the differences in competitiveness across schools seem important elements influencing individual behavior.

In spite of a loss of predictability of subjects’ behavior in the constrained setting we find that the relative ranking of the mechanisms in terms of efficiency is quite robust. That is, TTC outperforms SOSM, which in turn is superior to BOS. As for the stability of the assignments obtained, the results are less encouraging. In general, assignments are not stable. The picture becomes clearer if we consider the average number of blocking pairs, which can be interpreted as a measure of discontent or inviability of the market institution. SOSM emerges as the best mechanism since it leads to significantly fewer blocking pairs than BOS and TTC. Also, constraining choice clearly increases the number of blocking pairs for SOSM (but not necessarily for BOS and TTC).

Finally, when investigating the extent to which the three mechanisms can counter the segregation in neighborhoods, we observe that more students are assigned to their district school in the constrained setting. However, we observe that segregation rates under SOSM and BOS are not statistically different but always higher than those under TTC.

The remainder of the paper is organized as follows. The school choice problem and the three mechanisms analyzed in this paper are presented in Section I. The experimental design is explained in Section II. Experimental analysis and results are in Section III. Section IV concludes.

I. School Choice: Three Competing Mechanisms

A school choice problem (Abdulkadiroğlu and Sönmez 2003) is defined by a set of schools and a set of students, each of which has to be assigned a seat at no more than one school. Each student is assumed to have strict preferences over schools and the option of remaining unassigned. Each school is endowed with a strict priority ordering over students and a fixed capacity of seats. If a student prefers the option of remaining unassigned to being assigned a seat at a given school, then this school is said to be unacceptable for the student. Otherwise, a school is acceptable.

An outcome of a school choice problem is a matching, i.e., an assignment of students to school seats such that each student is assigned at most one seat and each school receives no more students than its capacity. At a given matching, a student is unassigned, or assigned to himself, if he is not assigned a seat in any school. In the context of school choice, schools are institutions

5,040 different strategies in the constrained and unconstrained case, respectively. Hence, approaches like k-level thinking (Miguel A. Costa-Gomes, Vincent P. Crawford, and Nagore Iriberri 2009) or quantal response equilibrium (Richard D. McKelvey and Thomas R. Palfrey 1995) are not suitable in our case.
serving society and so only the welfare of the students is considered. Hence, a matching is *Pareto efficient* if there is no matching that gives every student a weakly better assignment and at least one student is strictly better off.

A matching is *stable* if: (i) it is *individually rational*, i.e., each student is unassigned or assigned a seat at some acceptable school; (ii) it is *non wasteful* (Michel Balinski and Sönmez 1999), i.e., no student prefers a vacant seat to his assigned seat; and (iii) there is no *justified envy*, i.e., there is no student-school pair \((i, s)\) such that \(i\) prefers \(s\) to his assignment and \(i\) has higher priority at \(s\) than some other student who is assigned a seat at \(s\).

A (student assignment) *mechanism* systematically selects a matching for each school choice problem. A mechanism is stable if it always selects a stable matching. Similarly, an efficient mechanism is one that always selects a Pareto efficient matching. Finally, a mechanism is *strategy proof* if truth telling is a weakly dominant strategy.

We now briefly describe the mechanisms that we analyze in this paper: the Boston (BOS), the Student Optimal Stable Matching (SOSM), and the Top Trading Cycles (TTC) mechanisms. The three mechanisms work as follows. Each school has a strict priority ordering of the students, usually determined by law (giving priority to students from the same neighborhood, having siblings already in the school, etc.). Each student is asked to submit a *choice list*, i.e., an ordered list of schools. Given the priority ranking and the choice lists the three mechanisms determine the final assignment in the following manner.

From the students’ point of view the mechanisms work identically in the following sense:

**Round** \(k, k \geq 1\) [students]: Every student that has not been assigned a school in the previous round applies to the highest ranked available school in his choice list that has not rejected him yet (if there is no such school then the student “applies” to himself).

The three mechanisms differ in who gets rejected by a school:

- **[BOS]** **Round** \(k, k \geq 1\): Each school assigns seats one at a time to the students that apply to it following its priority order. If the school capacity is or was attained, the school rejects any remaining or future applicants. If a student “applies” to himself, he is assigned to himself. *BOS terminates when all students have been assigned.*

- **[SOSM]** **Round** \(k, k \geq 1\): Each school tentatively assigns seats one at a time to the students that apply to it or that were tentatively assigned a seat in a previous round, following its priority order. When the school capacity is attained the school rejects any remaining students that apply to it. If a student “applies” to himself, he is tentatively assigned to himself. *SOSM terminates when no student is rejected. Then the tentative matching becomes final.*

- **[TTC]** **Round** \(k, k \geq 1\): Each school with vacant seats “points” to the student with highest priority among the students that have not been assigned a seat yet. This procedure, together with the above described procedure for the students, induces a cycle or cycles of students and schools. If a student is in a cycle he is assigned a seat at the school he applies to (or to himself if he is in a self-cycle). If a school is in a cycle then its number of vacant seats is decreased by one. If a school has no more vacant seats then it is no longer available and students that applied to it are rejected. *TTC terminates when all students have been assigned.*

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11 Priorities are the counterpart of schools’ preferences over students in a college admission problem.
We consider two cases: an unconstrained case, where students can submit a list containing all schools, and a constrained case, where students can only submit a choice list that contains a small number of schools. In this case if the mechanism exhausts one of the choice lists, the corresponding student will be removed from the system and be left unassigned.

In the unconstrained case SOSM and TTC are strategy proof, but BOS is not. SOSM is also stable. However, it is not Pareto efficient. In contrast, TTC is Pareto efficient, but not stable. BOS is Pareto efficient, but since it is not strategy proof this does not necessarily mean that it is Pareto efficient with respect to the true preferences. Under SOSM and TTC revealing your preferences truthfully up to the district school is a weakly dominant strategy in the unconstrained case. In the constrained case, Haeringer and Klijn (2009) explain that in general there is no dominant strategy and that for TTC and SOSM reversing the relative true ranking of schools is a dominated strategy.

II. Experimental Design

Our experimental design is based on that of Chen and Sönmez (2006), i.e., a matching game between 36 students and 36 school seats across seven schools. Two schools, called A and B, have only three seats, while the five other schools, C, D, E, F, and G, have six seats each. To each student corresponds a district school, and each school is the district school of as many students as its capacity.

Schools’ priorities are obtained as follows. We first generate a random order of all students. For each school, the students living in the district of that school are put in the highest position of the priority order of the school, in the order given by the random draw. The other students are then ranked below, following again the order of the random draw. As for the students’ preferences, we consider two payoff environments, designed and random. The designed environment builds on three hypothetical factors: a school’s quality, its proximity (i.e., whether or not the school is the district school), and a random factor. More precisely, the competitive schools A and B as well as the district school are more likely to be among the most preferred schools. Preferences in the random environment are generated randomly. The two different payoff environments and the heterogeneity of schools in the experimental protocol enable us to disentangle the extent to which the payoff structure and the distribution of capacities across schools affect subjects’ behavior.

The matching game studied in this experiment consists of asking subjects to submit an ordered list of schools (the choice list hereafter). The final matching is obtained using either BOS, SOSM, or TTC, and the payoff accrued to a subject is given by the school to which he is matched. The experiment consists of a one-shot game with incomplete information, where each subject knows only his own payoff table and his district school (the school for which he has the highest priority), but not the other participants’ payoff tables.

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12 In real-life settings unassigned students can usually submit a second list, opting only for schools that have vacant seats.
13 In fact, it always generates the best stable matching for the students.
14 See Abdulkadiroğlu and Sönmez (2003) for details.
15 Our experiment is carried out under incomplete information. Apart from a recent paper by Lars Ehlers and Jordi Massó (2007), which links equilibria under complete information with Bayesian equilibria under incomplete information, there are yet no theoretical results that can offer a clear insight into individuals’ optimal behavior under incomplete information if there is no dominant strategy under complete information.
16 See Chen and Sönmez (2006) for details on the design of the rankings and monetary payoffs.
17 Since subjects did not know about others’ preferences their behavior is not expected to change from the random to the designed environment, but the assignment will.
18 A recent experiment study by Joana Pais and Ágnes Pinter (2008) shows that, in the unconstrained case, the less information subjects have about others’ payoffs the more likely they are to be truthful for BOS, SOSM, and TTC. That is, providing subjects with virtually no information about their opponents’ payoffs represents the least favorable situation to reject that the mechanisms induce truthful behavior.
We consider two different treatments, one in which subjects are not constrained in their choices (as in Chen and Sönmez 2006), and one in which subjects are asked to submit no more than three schools.

In the experiment, each participant is randomly assigned an ID number and is seated in a chair in a classroom. The experimenter reads the instructions aloud. Subjects are allowed to ask questions, which are answered in public. Subjects are then given 15 minutes to read the instructions again at their own pace and to make their decisions, which consist of an ordered list of schools. Next, the experimenter collects the decisions and asks a volunteer to draw numbers out of an urn, which generates the random order. The experimenter then introduces the subjects’ decisions and the random order in a computer program with the appropriate algorithm to compute the assignment, announces the results, and hands out the corresponding payments to the subjects.

We use a $3 \times 2 \times 2$ design: each of the three mechanisms (BOS, SOSM, and TTC) is examined for a designed as well as a random payoff environment, and in a constrained and unconstrained setting. For each of the 12 treatments, two independent sessions were carried out (one at the Universitat Autònoma de Barcelona and one at the Universitat Pompeu Fabra) between May and November 2006 and May and June 2008 for the constrained and unconstrained cases, respectively. Altogether, 864 students from a wide range of disciplines (economics, psychology, humanities, etc.) participated in the experiments, and the average payoff was €11.8 (not including the €3 participation fee). The sessions last approximately 45 minutes, with the first 20–25 minutes being used for instructions.

III. Results on Behavior and Allocations

A. Suboptimal Play

A designer usually advocates a particular mechanism because of some of its theoretical properties. It is a primary concern in the actual implementation of matching mechanisms whether individuals understand the game being played and respond to the incentives induced by the mechanism.19

Preserving the Original Ranking.—We start our analysis by comparing the degree of optimal play exhibited by subjects in the different treatments. In the unconstrained setup, for SOSM and TTC, playing truthfully up to the district school is a weakly dominant strategy. Such a strategy is still dominant in the constrained setup if the district school is one of the three most preferred schools. But if the district school is not one of the three most preferred schools then playing truthfully up to the district school will not be a feasible strategy. In that case, as shown by Haeringer and Klijn (2009), in the constrained SOSM and TTC, although there is no dominant strategy, reversing the original ranking is a weakly dominated strategy. Hence, not preserving the original ranking indicates lack of optimal behavior for all treatments when SOSM and TTC are implemented. Consequently we start by comparing the number of subjects that violate the original ranking, which is a minimal requirement for optimality common to all subjects under TTC and SOSM. To make the analysis comparable we look at the preservation of ranking only for the first three choices.

More Preservation of Ranking. For SOSM and TTC the proportion of subjects reversing the relative ranking is significantly higher in the unconstrained case. The reversal concerns small schools.

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Table 1 displays the proportion of subjects preserving the original ranking. Throughout the paper the subscript $d$ and $r$ attached to the name of a mechanism correspond to the designed and random environment, respectively. As shown by the table, not only do most subjects preserve ranking in the constrained case, but in most cases a significantly higher proportion of subjects do so in the constrained versus the unconstrained case.

Moreover, it turns out that reversals of relative rankings are mainly due to the small schools. If we consider the relative ranking of all schools up to the district school except schools A and B we find that under TTC and SOSM all subjects respect the original relative ranking.

Distortions of the original ranking in the constrained setting are not significantly different for the designed and the random environment. This suggests that what drives the distortions on small schools is the asymmetry in school capacities, and not the fact that the small schools generally give a higher payoff, which is true only in the designed environment. This is due to the fact that students do not know whether preferences are correlated or not and therefore need not necessarily infer that A and B are the most wanted schools in the designed environment.

**Truncated Truth Telling.**—We now continue to analyze optimal play by looking at the proportion of subjects using a truncated truth telling strategy, which is defined as a strategy that consists of submitting a choice list whose first three choices coincide with the preferences. For SOSM and TTC in the unconstrained case and for subjects for whom the district school is one of the three most preferred options truncated truth telling is a weakly dominant strategy. For BOS truth telling is not a dominant strategy, and therefore little truth telling is expected.

For SOSM and TTC we restrict our attention to the schools that are ranked above the district school (including the latter), whereas for BOS we consider the first three choices except when the district school is ranked first, in which case we compare only the first choice.

Since the predicted behavior is different for those individuals with the district school above the third position or below, it proves useful to partition the sample of subjects into two subsamples, the high-district sample and the low-district sample. The high-district sample contains all those

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Table 1—Preserving Original Ranking—Full Sample

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Constrained</th>
<th>Unconstrained</th>
<th>t-stat (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOSM, $d$</td>
<td>95.8</td>
<td>73.6</td>
<td>3.87 (0.0001)</td>
</tr>
<tr>
<td>SOSM, $r$</td>
<td>91.7</td>
<td>81.9</td>
<td>1.73 (0.043)</td>
</tr>
<tr>
<td>TTC, $d$</td>
<td>93.1</td>
<td>84.7</td>
<td>1.59 (0.057)</td>
</tr>
<tr>
<td>TTC, $r$</td>
<td>90.3</td>
<td>88.9</td>
<td>0.27 (0.4)</td>
</tr>
</tbody>
</table>

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20 As for BOS, there is no theoretical prediction about the preservation of the original ranking. For this reason we omit the results concerning the suboptimal play for this mechanism.

21 This explains why we observe (Table 1 in the Web Appendix) a higher proportion of subjects in the low-district sample reverse relative rankings in the unconstrained setting for the designed compared to the random environment. In the designed environment all subjects have one of the small schools as one of their three most preferred schools, while in the random environment only 52 out of 72 do. That is, the source of reversal is more present in the designed than in the random environment. For BOS we find that competitiveness, or in this case size of the school, is not fully determining individual manipulation, since around two thirds of the subjects still do not preserve the ranking when we ignore small schools.

22 Considering only the first three choices is to obtain a meaningful comparison between the constrained and unconstrained cases.

23 The reason for that is that for SOSM and TTC once the district school is reached, the subject is ensured a spot in it independently of what is included in the list after the district school. But for BOS this is not the case, and we consequently look at the first three choices. The only case in BOS for which a spot in the district school can be ensured is when the district school is first in the choice list, and therefore only in this case truth telling up to the district school is a dominant strategy.
subjects for which the district school is one of the three most preferred schools and the low-
district sample consists of all the other subjects. Table 2 displays the percentage of subjects using
a truncated truth telling strategy in the full sample as well as in the high- and low-district samples.
The following summarizes our findings for the full sample.

**Less Truth Telling.** For SOSM and TTC, the proportion of truncated truth telling is significantly
higher in the unconstrained than in the constrained case. The proportion of truncated truth telling
does not change significantly between the three mechanisms in the constrained case.

Comparing mechanisms in the unconstrained case, Boston has a significantly lower proportion
of subjects truth telling, which is one of the main reasons why Boston Public School authorities
were persuaded to change from Boston to either SOSM or TTC. For Boston there is no increase
in truth telling when including the constraint.

It is instructive to go deeper in the analysis of truncated truth telling by comparing the behavior
of low- and high_district subjects. As can be seen in Table 2 the vast majority of subjects in the
low_district sample do not truth tell in the constrained environment. Surprisingly though, a very
large proportion of subjects from the high_district sample do not truncated-truth-tell. The high-
district sample seems to adhere less to the general predictions. They are the group that has less to
lose since their secure pay, corresponding to their district school, is high.

**B. Misrepresentations**

To further understand what drives the subjects’ strategic choices we consider two types of mis-
representations introduced by Chen and Sönmez (2006): Small School Bias (SSB) and District
School Bias (DSB). SSB consists of lowering the position of a more competitive school in the
submitted list. DSB consists of raising the ranking of the district school in the submitted list.

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24 Comparing the constrained and unconstrained cases, the \( t \)-statistics (\( p \)-value) are 0 (0.5) for BOS\(_d\), 2.3443 (0.0102) for BOS\(_r\), 4.2805 (0.000) for SOSM\(_d\), 5.2262 (0.000) for SOSM\(_r\), 5.3184 (0.000) for TTC\(_d\), and 7.7051 (0.000) for TTC\(_r\).
25 See Roth (2008) quoting the superintendent of schools of Boston.
26 For BOS we have that both in the unconstrained and in the constrained setting reporting truthfully is not a dominant
strategy. Given that the chances of being admitted in a given school get smaller as we go down the choice list, individuals
have high incentives to manipulate the submitted lists in both the constrained and the unconstrained case by putting
a school for which they have chances of being admitted in the first position.
27 Note also that for the low_district sample in the designed treatment there seems to be slightly less truncated truth
telling resulting, as to the ranking reversals, from the fact that in the designed environment there is a higher proportion of
individuals that have small schools in the top three positions in their ranking, which gives them incentives to manipulate
their preferences to avoid risks.
28 In this case, the designed environment leads to more truth-telling, which does not have a clear explanation.
29 In the experiment the most competitive schools are the small schools (A and B) since subjects do not know the other
players’ preferences.
(for example, ranking the district school in the third position when it is actually fourth or lower in preferences). This second effect is particularly interesting for SOSM and TTC because it signals whether subjects tend to secure their prospects, and thus should be more important for the low-district subjects in the constrained case.

We will define the biases in the constrained and the unconstrained case in the same way so that measures are comparable across treatments. More precisely, we will look at the behavior regarding the first three positions in the ranking. Therefore, an individual exhibits a DSB if the district school is below the third position in the ranking but appears in the choice list, or if the district school is in the first three positions but is moved up in the ranking in the choice list. In particular, if an individual in the unconstrained case has the district school ranked fifth in preferences but puts it in the fourth position in the choice list, the individual will not be considered to exhibit a DSB. Similarly, an individual exhibits a SSB if a competitive school was ranked in the first three positions in the original preferences and either has not been included in the choice list or has been included in a lower position than in the original preferences.

Both distortions are a natural reaction to the added risk induced by the constraint. If the chances of being unassigned are high, it is very likely that subjects secure their prospects by raising the relative rank of the district school and lowering that of the most competitive schools. While this is very clear for the subjects in the low-district sample, there is no reason for this to happen for the high-district sample individuals under SOSM and TTC. For BOS the difference in incentives from moving to the constrained environment is smaller since there was already a high risk of exhausting the first positions in the list in the unconstrained case. $^30$

**Misrepresentations.** For TTC and SOSM, a significantly higher proportion of individuals exhibit a DSB and a SSB in the constrained compared to the unconstrained case. For BOS, the differences are small. $^31$

The misrepresentations predominant for the low-district sample are different in the constrained and the unconstrained set-up.

Also, the degree of DSB and SSB is significantly lower for TTC and SOSM than for BOS in the unconstrained case, but not in the constrained case. Although there are some differences between the designed and random environment, none of them are significant.

When considering the high- and low-district samples separately (tables 3 and 4) we observe that for the low-district sample there is a considerable increase in the DSB observed and, although smaller, a considerable increase in the SSB. For the high-district sample we see that the increase of both types of misrepresentations is smaller, especially for SSB, where the differences are not significant.

To complete our identification of preference patterns that drive the two main misrepresentations (DSB and SSB) we provide in tables 5 and 6 a decomposition of DSB and SSB, separating the subjects’ choices that exhibit only DSB, only SSB and both DSB and SSB. The most striking observation is that in the low-district sample DSB is predominant in the constrained case, while SSB is in the unconstrained case (especially for the designed environment). This is not the case for the high-district sample, for we seldom observe SSB without DSB. Note that when payoffs are random most of the misrepresentations are done through DSB or DSB and SSB, while in the

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$^30$ Further scrutiny shows that for almost all subjects in the low-district sample in the unconstrained case DSB consists of putting the district school in one of the first three positions in their choice lists. That is, the first three positions seem to be focal to most subjects, which ex post could have justified constraining subjects to list only three schools in the constrained setup—see the section on Safety school effect in the Web Appendix.

$^31$ The $p$-values for SOSM and TTC are 0.000 and for BOS 0.07 for the designed and 0.02 for the random environments.
designed environment DSB and SSB go together, but this is a result of the small schools having generally higher payoffs in the correlated environment.

These results suggest that the constraint significantly affects how subjects elaborate their strategies. The district school becomes focal in the constrained case and significantly less so in the unconstrained case.

<table>
<thead>
<tr>
<th>Table 3—District School Bias—High- and Low-District Samples</th>
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<tbody>
<tr>
<td><strong>Low-district sample</strong></td>
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<tr>
<td></td>
</tr>
<tr>
<td>BOS_d</td>
</tr>
<tr>
<td>BOS_r</td>
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<td>SOSM_d</td>
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<td>SOSM_r</td>
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<thead>
<tr>
<th>Table 4—Small School Bias—High- and Low-District Samples</th>
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<tbody>
<tr>
<td><strong>Low-district sample</strong></td>
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<tr>
<td>BOS_d</td>
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<td>BOS_r</td>
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<th>Table 5—Decomposition of Biases (Low-District Sample)</th>
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<tr>
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<td>(1)</td>
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<tr>
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<td>TTC_d</td>
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Notes: (1) DSB and SSB; (2) DSB Only; (3) SSB Only

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<tr>
<th>Table 6—Decomposition of Biases (High-District Sample)</th>
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<tr>
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<td>(1)</td>
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<td>TTC_d</td>
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<tr>
<td>TTC_r</td>
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</tbody>
</table>

Notes: (1) DSB and SSB; (2) DSB Only; (3) SSB Only
Also, for low-district subjects the school district appears as a safety school, and most of them use it as such, i.e., around 70–80 percent put the district school in the third position in their choice list in the constrained set-up.32

C. Efficiency and Stability

To compare efficiency across mechanisms and environments we employ mean payoffs. For stability we look at the number of stable matchings and at the number of students that constitute a blocking pair in the final assignment.33 Since we have two identical sessions for each treatment in principle we have only two observations to make claims on relative performance of the mechanisms. But thanks to the fact that the two identical sessions for each treatment are of a one-shot, incomplete information game we can assume that the behavior of each individual in the two sessions is independent of the particular distribution of the 72 subjects over the two sessions. That is, the behavior of a particular subject does not depend on the particular colleagues in the session, and therefore we can assume that his behavior would have been identical if the group of subjects he played against were different. We exploit this fact to obtain more information on what the possible outcomes of the mechanisms could have been. Potentially we could construct $2^{72}$ different virtual sessions with the 72 individuals from the two sessions. However, to avoid this computational impossibility we recur, just as Chen and Sönmez (2006), to the recombinant estimator discussed by Charles H. Mullin and David H. Reiley (2006), which requires running fewer recombinations.34

Efficiency.—As mentioned earlier, in the unconstrained setting TTC is Pareto efficient (provided individuals are truthful) and SOSM on the contrary is not. BOS is Pareto efficient with respect to the submitted preferences, but since we expect little truth telling, the outcome should not be efficient either. We therefore expect TTC to outperform SOSM and BOS in terms of efficiency, although we look only at mean payoffs.

In the constrained case no theoretical predictions can be made, since efficiency of neither mechanism is guaranteed. But with the high degree of misrepresentation in the three mechanisms we do not expect large differences in mean payoffs.

Efficiency Loss. The efficiency of SOSM, TTC and Boston is significantly reduced in the constrained compared to the unconstrained case. Table 7 shows that for the three mechanisms and the two environments there is a significant efficiency loss when we impose a constraint.

In the unconstrained case TTC is significantly more efficient than BOS ($p$-values of 0.009 and 0.017 for the designed and random environments, respectively). We must however reject that SOSM is more efficient than BOS ($p$-values of 0.19 and 0.21 for the designed and random environments, respectively). TTC is significantly more efficient than SOSM in the random

32 The proportion of low-district subjects putting the district school in the third position in their choice lists are: For SOSM, 78.6 percent (designed) and 68.2 percent (random); for TTC, 78.6 percent (designed) and 81.8 percent (random).
33 A blocking pair consists of a student and a school where the student prefers the school to his assigned school and the school either has a vacant seat or has accepted an individual with lower priority. For a given assignment a student may have an opportunity to block with several schools. In other words, the data reports the number of students who have the possibility to block and not the total number of blocking pairs.
34 See the Web Appendix for details on the recombinant estimator.
environment and at only a ten percent significance level in the designed environment (with a \( p \)-value of 0.03 and 0.052 in the random and the designed environment, respectively).

The comparison between the mechanisms changes in the constrained case. Now both SOSM and TTC are significantly more efficient than BOS for both environments (with \( p \)-values of 0.02 and 0.002 for the designed environment and 0.04 and 0.001 for the random environment). But TTC is not significantly better than SOSM for neither the designed nor the random environment (with respective \( p \)-values of 0.1 and 0.09).

In summary then, the constraint clearly affects the efficiency of any of the three mechanisms but mainly leaves unchanged the relative ranking of the three mechanisms. However, the constraint tends to equalize SOSM and TTC in terms of efficiency.

**Stability.**—Stability is an important issue when considering the sustainability of a mechanism. It avoids potential lawsuits or the appearance of “alternative” markets or mechanisms to perform the matching. Stability guarantees that in a matching, for every individual, the schools he prefers to the one he has been assigned to are filled with higher priority students. In the unconstrained setting we know that SOSM is stable as long as subjects are truthful, but TTC and BOS are not. But in the constrained case no relative ranking of the mechanisms with respect to stability can be theoretically predicted.

**Low Stability.** Stability is low for all mechanisms. SOSM is significantly more stable than TTC or BOS in both the constrained and the unconstrained cases.

The artificial sessions rarely induce a stable matching, for any of the three mechanisms. For instance, under SOSM about 0.3 percent of the matchings are stable. There are no significant differences between mechanisms or treatments.

The likelihood that a market mechanism fails is related to the number of blocking pairs in a given assignment since the pairs will be filing complaints, and the more complaints, the more trouble in practice. Table 8 presents the number of blocking pairs in each of the mechanisms, and Table 9 provides a comparison across mechanisms. It shows that for BOS and TTC there is no evidence that the constrained or the unconstrained set-up is better. Results are either reversed within a given mechanism or not significant. An increase in the number of blocking pairs (for the designed environment) seems to go against the fact that in the constrained case there are more misrepresentations (DSB and SSB). However, we must take into account that both BOS and TTC

\[35\] Chen and Sönmez (2006) and later a Corrigendum to their paper report that in the designed environment SOSM is significantly more efficient than both BOS and TTC, and that TTC is not significantly more efficient than BOS. In the random environment only BOS performs significantly better than TTC. But Calsamiglia, Haeringer, and Klijn (2010) show that their results are not robust given the number of recombinations they did and show that if a larger number of recombinations is done using their data, SOSM is not significantly more efficient than TTC in the designed environment, and BOS is not significantly more efficient than TTC.
are not designed to produce stable assignments. For SOSM though, the unconstrained environment is more stable than the constrained environment, whether we consider the designed or the random environment. SOSM has a significantly lower number of blocking pairs than both TTC and BOS. The other differences between mechanisms are not significant.

D. Segregation

School choice can potentially reduce the segregation in schools generated by segregation in neighborhoods by allowing individuals to go to schools that are not in their districts. In the constrained setting, since individuals exhibit more DSB they are likely to be allocated to their district school more often than in the unconstrained case. This implies that the degree of segregation of neighborhoods and schools will be more similar when the constraint is present.

**Segregation Increases.** Individuals are assigned to their district school significantly more often in the constrained than in the unconstrained case.

Table 10 displays the proportion of subjects assigned to their district school. The increase in DSB resulting from the constraint has caused the percentage of subjects assigned to their district school to increase significantly. This implies that the constraint reduces the capacity of school choice to counter the segregation in neighborhoods. But note that the percentage increase in the number of subjects assigned to their district school is not as large as the percentage increase we observed in the number of subjects exhibiting a DSB. Also no ranking of mechanisms with respect to their level of segregation can be established within either the constrained or the unconstrained environments.

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36 See Muriel Niederle and Roth (2003) for empirical evidence about the mobility generated by a centralized mechanism in the gastroenterology market.
IV. Conclusions

This paper analyzes the effect of a seemingly irrelevant characteristic that a large number of actual school choice mechanisms have: students are allowed to submit a list of preferences with only a small number of schools. We experimentally show that this has large negative effects on the manipulability of the mechanisms, since for many subjects the existence of a dominant strategy to truth tell disappears. The introduction of the constraint is also shown to reduce efficiency and stability and to increase segregation.

But the inclusion of this constraint induces a smaller fraction of the individuals to use a dominated strategy, although a larger fraction of the high district sample uses the dominant strategy less. Quite surprisingly, we find that much of the use of dominated strategies in both the constrained and the unconstrained case derives from aspects that are not part of the design of the matching mechanism itself, like asymmetries in school capacities, i.e., differences in the degree of competitiveness between schools.

Our experiment results also highlight the role played by the presence of district schools, which appear to be particularly important when choice is constrained. Having a safety net in one’s choice list clearly alleviates the risk of making an inappropriate choice, and thus leaves the participants some sort of control over their final assignment (although it may lead individuals to be excessively cautious). Stopping here we would conclude that district priorities could be a desirable policy in a constrained environment. However, the emphasis on the district school increases segregation significantly (although less than the increase in DSB) and plausibly increases segregation among neighborhoods too since families, knowing the importance of the district school as the safe option, may choose their residence accordingly.

Probing the robustness of a matching mechanism is a difficult issue, for we seldom observe participants’ true preferences. But there is confidence that over the years participants increasingly use dominant strategies when the mechanism in use is strategy proof—Abdulkadiroğlu, Pathak, and Roth (2009) and Roth (2008). The analysis presented in the paper leads us to conclude that removing the constraint will come at a small cost but will clearly improve the performance of the school choice mechanisms.

More important, this paper points out the need to carefully consider the small details of actual mechanisms that the literature has overlooked so far, but which are crucial in the provision of correct predictions or adequate advice.

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<tr>
<th></th>
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37 Gabrielle Fack and Julien Grenet (2008) show that housing prices in neighborhoods with good public schools are significantly higher.
REFERENCES